Unit 6 Practice Problems

Lesson 1

Problem 1
(from Unit 3, Lesson 7)
Lin and Tyler are drawing circles. Tyler’s circle has twice the diameter of Lin’s circle. Tyler thinks that his circle will have twice the area of Lin’s circle as well. Do you agree with Tyler?

Solution
No, radius and area are not proportional. The area of Tyler’s circle will be 4 times as big as the area of Lin’s circle.

Problem 2
(from Unit 5, Lesson 15)
Jada and Priya are trying to solve the equation \( \frac{2}{3} + x = 4 \).

Jada says, “I think we should multiply each side by \( \frac{3}{2} \) because that is the reciprocal of \( \frac{2}{3} \).”

Priya says, “I think we should add \(-\frac{1}{3}\) to each side because that is the opposite of \( \frac{2}{3} \).”

1. Which person’s strategy should they use? Why?
2. Write an equation that can be solved using the other person’s strategy.

Solution
1. Priya is correct. The operation in the expression \( \frac{2}{3} + x \) is addition. Adding the additive inverse of \( \frac{2}{3} \) to both sides of the equation will change the equation to the form “\( x = \ldots \)”.

2. Answers vary. Sample response: \( \frac{2}{3}x = 4 \).

Problem 3
(from Unit 5, Lesson 13)
What are the missing operations?

1. \( 48 \ ? (-8) = (-6) \)
2. \( (-40) \ ? 8 = (-5) \)
3. \( 12 \ ? (-2) = 14 \)
4. \( 18 \ ? (-12) = 6 \)
5. \( 18 \ ? (-20) = -2 \)
6. \( 22 \ ? (-0.5) = -11 \)

Solution
1. Divide
2. Divide
3. Subtract
4. Add

5. Add

6. Multiply

**Problem 4**

Unit 6 Practice Problems

The team that has the ball has four chances to gain at least ten yards. If they don’t gain at least ten yards, the other team gets the ball. Positive numbers represent a gain and negative numbers represent a loss. Select all of the sequences of four plays that result in the team getting to keep the ball.

1. 8, -3, 4, 21
2. 30, -7, -8, -12
3. 2, 16, -5, -3

**Solution**

A, C

**Problem 5**

A sandwich store charges $20 to have 3 turkey subs delivered and $26 to have 4 delivered.

1. Is the relationship between number of turkey subs delivered and amount charged proportional? Explain how you know.
2. How much does the store charge for 1 additional turkey sub?
3. Describe a rule for determining how much the store charges based on the number of turkey subs delivered.

**Solution**

1. No. Sample reasoning: If they deliver 3 turkey subs, they charge $6.67 per sub, but for 4 subs, they charge $6.50 per sub.
2. $6
3. The rule could be $6 per sub plus a $2 delivery fee. 6 times 3 is 18, but they charged $2 more than that for 3 subs. 6 times 4 is 24, but they charged $2 more than that for 4 subs.

**Problem 6**

Which question cannot be answered by the solution to the equation $3x = 27$?

1. Elena read three times as many pages as Noah. She read 27 pages. How many pages did Noah read?
2. Lin has 27 stickers. She gives 3 stickers to each of her friends. With how many friends did Lin share her stickers?
3. Diego paid $27 to have 3 pizzas delivered and $35 to have 4 pizzas delivered. What is the price of one pizza?
4. The coach splits a team of 27 students into 3 groups to practice skills. How many students are in each group?

**Solution**

C

**Lesson 2**

**Problem 1**

(from Unit 3, Lesson 1)

The table shows the number of apples and the total weight of the apples.
Lesson 11

There are 87 children and 39 adults at a show. The seating in the theater is split into 4 equal sections.

Problem 2

Select all stories that the tape diagram can represent.

Solution

<table>
<thead>
<tr>
<th>number of apples</th>
<th>weight of apples (grams)</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>511</td>
</tr>
<tr>
<td>5</td>
<td>1200</td>
</tr>
<tr>
<td>8</td>
<td>2016</td>
</tr>
</tbody>
</table>

Estimate the weight of 6 apples.

Solution

About 1500 grams.

Problem 2

Select all stories that the tape diagram can represent.

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<td>2016</td>
</tr>
</tbody>
</table>

There are 87 first graders in after-care. After 39 students are picked up, the teacher put the remaining students into 4 groups for an activity.

Lesson 3

Lin buys a pack of 87 pencils. She gives 39 to her teacher and shared the remaining pencils between herself and 3 friends.

Lesson 4

Andre buys 4 packs of paper clips with 39 paper clips in each. Then he gives 87 paper clips to his teacher.

Lesson 5

Diego’s family spends $87 on 4 tickets to the fair and a $39 dinner.

Solution

B, C, E

Problem 3

Andre wants to save $40 to buy a gift for his dad. Andre’s neighbor will pay him weekly to mow the lawn, but Andre always gives a $2 donation to the food bank in weeks when he earns money. Andre calculates that it will take him 5 weeks to earn the money for his dad’s gift. He draws a tape diagram to represent the situation.

Solution

1. Explain how the parts of the tape diagram represent the story.

Lesson 14

Lesson 15

Lesson 16

Lesson 18

2. How much does Andre’s neighbor pay him each week to mow the lawn?

Lesson 19

Lesson 22

2. $10

Problem 4

(from Unit 5, Lesson 13)

Without evaluating each expression, determine which value is the greatest. Explain how you know.

1. $7 \frac{3}{8} - 9 \frac{1}{8}$

2. $(-7 \frac{3}{8}) + (-9 \frac{1}{2})$

3. $(-7 \frac{3}{8}) \cdot 9 \frac{1}{2}$

4. $(-7 \frac{3}{8}) \div (-9 \frac{1}{2})$

Solution

$(-7 \frac{3}{8}) \div (-9 \frac{1}{2})$ is the greatest because it is the only expression with a positive value.
Problem 5
(from Unit 5, Lesson 15)
Solve each equation.

1. $(8.5) \cdot (-3) = a$
2. $(-7) + b = (-11)$
3. $c - (-3) = 15$
4. $d \cdot (-4) = 32$

Solution
1. -25.5
2. -4
3. 12
4. -8

Lesson 3

Problem 1
(from Unit 5, Lesson 15)
Solve each equation mentally.

1. $2x = 10$
2. $-3x = 21$
3. $\frac{1}{3}x = 6$
4. $-\frac{1}{2}x = -7$

Solution
1. 5
2. -7
3. 18
4. 14

Problem 2
(from Unit 5, Lesson 3)
Complete the magic squares so that the sum of each row, each column, and each diagonal in a grid are all equal.

Solution
Problem 3

Draw a tape diagram to match each equation.

1. $5(x + 1) = 20$
2. $5x + 1 = 20$

**Solution**

1. A diagram showing 5 equal parts of $x + 1$ for a total of 20
2. A diagram showing 5 equal parts of $x$ and one part of 1 for a total of 20

Problem 4

Select all the equations that match the tape diagram.

1. $35 = 8 + x + x + x + x + x$
2. $35 = 8 + 6x$
3. $6 + 8x = 35$
4. $6x + 8 = 35$
5. $6x + 8x = 35x$
6. $35 - 8 = 6x$

**Solution**

A, B, D, F

Problem 5

(from Unit 4, Lesson 2)

Each car is traveling at a constant speed. Find the number of miles each car travels in 1 hour at the given rate.

1. 135 miles in 3 hours
2. 22 miles in $\frac{1}{2}$ hour
3. 7.5 miles in $\frac{1}{2}$ hour
4. $\frac{100}{3}$ miles in $\frac{3}{3}$ hour
5. $97\frac{1}{2}$ miles in $\frac{3}{2}$ hour

**Solution**

1. 45 miles
2. 44 miles
3. 30 miles
4. 50 miles
5. 65 miles
Lesson 4

Problem 1
(from Unit 3, Lesson 1)
Draw a square with side length 7 cm.

1. Predict the perimeter and the length of the diagonal of the square.
2. Measure the perimeter and the length of the diagonal of the square.
3. Describe how close the predictions and measurements are.

Solution
1. Perimeter: 28 cm. Length of diagonal: Approximately 9.9 cm.
2. Answers vary.
3. Answers vary.

Problem 2
(from Unit 5, Lesson 9)
Find the products.

1. \((100) \cdot (-0.09)\)
2. \((-7) \cdot (-1.1)\)
3. \((-7.3) \cdot (5)\)
4. \((-0.2) \cdot (-0.3)\)

Solution
1. -9
2. 7.7
3. -36.5
4. 0.06

Problem 3
Here are three stories:
- A family buys 6 tickets to a show. They also pay a $3 parking fee. They spend $27 to see the show.
- Diego has 27 ounces of juice. He pours equal amounts for each of his 3 friends and has 6 ounces left for himself.
- Jada works for 6 hours preparing for the art fair. She spends 3 hours on a sculpture and then paints 27 picture frames.

Here are three equations:
- \(3x + 6 = 27\)
- \(6x + 3 = 27\)
- \(27x + 3 = 6\)

1. Decide which equation represents each story. What does \(x\) represent in each equation?
2. Find the solution to each equation. Explain or show your reasoning.
3. What does each solution tell you about its situation?

Solution
1. Tickets to the show: \(6x + 3 = 27\), \(x\) represents the cost of a ticket. Diego’s juice: \(3x + 6 = 27\), \(x\) represents the number of ounces of juice he gave each friend. The art fair: \(27x + 3 = 6\), \(x\) represents the number of hours spent on each picture frame.
2. \(6x + 3 = 27\): \(x = 4\). \(3x + 6 = 27\): \(x = 7\). \(27x + 3 = 6\): \(x = \frac{1}{3}\). Explanations vary.
3. Each ticket to the show cost $4. Diego gave each friend 7 ounces of juice. Jada spent \( \frac{1}{2} \) of an hour painting each picture frame.

**Problem 4**

Here is a diagram and its corresponding equation. Find the solution to the equation and explain your reasoning.

![Diagram](image)

\[ 6x + 11 = 21 \]

**Solution**

\[ x = \frac{10}{6} \] (or equivalent). Explanations vary.

**Problem 5**

(from Unit 5, Lesson 7)

1. Plot these points on the coordinate plane:
   \[ A = (3, 2), B = (7.5, 2), C = (7.5, -2.5), D = (3, -2) \]

![Coordinate Plane](image)

2. What is the vertical difference between \( D \) and \( A \)?

3. Write an expression that represents the vertical distance between \( B \) and \( C \).

**Solution**

The vertical difference between \( D \) and \( A \) is -4 units.
Lesson 5

Problem 1
(from Unit 4, Lesson 2)
Here are some prices customers paid for different items at a farmer’s market. Find the cost for 1 pound of each item.

1. $5 for 4 pounds of apples
2. $3.50 for 1/2 pound of cheese
3. $8.25 for 1 1/2 pounds of coffee beans
4. $6.75 for 1 1/2 pounds of fudge
5. $5.50 for a 6 1/2 pound pumpkin

Solution

1. $1.25
2. $7
3. $5.50
4. $9
5. $0.88

Problem 2
(from Unit 5, Lesson 9)
Find the products.

1. 4/3 · (4/5)
2. (2/3) · (2/5)
3. (2/3) · 39
4. (3/4) · (4/3)

Solution

1. 4/15
2. 1
3. -2
4. 3/10 or equivalent

Problem 3
Here are two stories:

• A family buys 6 tickets to a show. They also each spend $3 on a snack. They spend $24 on the show.
• Diego has 24 ounces of juice. He pours equal amounts for each of his 3 friends, and then adds 6 more ounces for each.

Here are two equations:

• 3(x + 6) = 24
• 6(x + 3) = 24

1. Which equation represents which story?
2. What does x represent in each equation?
3. Find the solution to each equation. Explain or show your reasoning.
4. What does each solution tell you about its situation?
Solution

1. Family at the show: \(6(x + 3) = 24\), Diego's juice: \(3(x + 6) = 24\)

2. Family at the show: \(x\) represents the cost of a ticket. Diego's juice: \(x\) represents the number of ounces of juice Diego originally poured for each friend.

3. \(6(x + 3) = 24\): \(x = 1\), \(3(x + 6) = 24\): \(x = 2\)

4. Tickets to the show cost $1. Diego originally poured 2 ounces of juice.

Problem 4

Here is a diagram and its corresponding equation. Find the solution to the equation and explain your reasoning.

\[
\begin{align*}
\text{\# of units} & \quad \text{Value per unit} \\
\hline
& x + 1 & x + 1 & x + 1 & x + 1 & x + 1 \\
24 & & & & & \\
\end{align*}
\]

\[6(x + 1) = 24\]

Solution

3. Sample response: in the tape diagram, there are six units of \(x + 1\) that make 24, so \(x + 1\) must be \(24 \div 6\), which is 4. Since \(x + 1 = 4\), \(x = 3\).

Problem 5

(from Unit 5, Lesson 7)

Below is a set of data about temperatures. The range of a set of data is the distance between the lowest and highest value in the set. What is the range of these temperatures?

\(9^\circ\text{C}, -3^\circ\text{C}, 22^\circ\text{C}, -5^\circ\text{C}, 11^\circ\text{C}, 15^\circ\text{C}\)

Solution

27

Problem 6

(from Unit 4, Lesson 11)

A store is having a 25% off sale on all shirts. Show two different ways to calculate the sale price for a shirt that normally costs $24.

Solution

Answers vary. Possible strategies:

- \((0.25) \cdot 24 = 6\), and \(24 - 6 = 18\) (find 25% of $24 and subtract this from $24)

- \(1 - 0.25 = 0.75\), and \((0.75) \cdot 24 = 18\) (find 75% of $24)

- \(24 \div 4 = 6\), and \(3 \cdot 6 = 18\) (find 25% of $24 and multiply this by 3)

Lesson 6

Problem 1

(from Unit 6, Lesson 2)

A school ordered 3 large boxes of board markers. After giving 15 markers to each of 3 teachers, there were 90 markers left. The diagram represents the situation. How many markers were originally in each box?

\[
\begin{align*}
\text{\# of units} & \quad \text{Value per unit} \\
\hline
x - 15 & x - 15 & x - 15 \\
90 & & \\
\end{align*}
\]

Solution
Problem 2
(from Unit 6, Lesson 3)
The diagram can be represented by the equation $25 = 2 + 6x$. Explain where you can see the 6 in the diagram.

Solution
There are 6 equal parts labeled $x$.

Problem 3
Elena walked 20 minutes more than Lin. Jada walked twice as long as Elena. Jada walked for 90 minutes. The equation $2(x + 20) = 90$ describes this situation. Match each amount in the story with the expression that represents it.

A. The number of minutes that Jada walked
B. The number of minutes that Elena walked
C. The number of minutes that Lin walked
1. $x$
2. $x + 20$
3. $2(x + 20)$
4. 90

Solution
1. C
2. B
3. A
4. A

Problem 4
Match each equation to a story. (Two of the stories match the same equation.)

1. $3(x + 5) = 17$
2. $3x + 5 = 17$
3. $5(x + 3) = 17$
4. $5x + 3 = 17$

1. Jada’s teacher fills a travel bag with 5 copies of a textbook. The weight of the bag and books is 17 pounds. The empty travel bag weighs 3 pounds. How much does each book weigh?
2. A piece of scenery for the school play is in the shape of a 5-foot-long rectangle. The designer decides to increase the length. There will be 3 identical rectangles with a total length of 17 feet. By how much did the designer increase the length of each rectangle?
3. Elena spends $17 and buys a $3 book and a bookmark for each of her 5 cousins. How much does each bookmark cost?
4. Noah packs up bags at the food pantry to deliver to families. He packs 5 bags that weigh a total of 17 pounds. Each bag contains 3 pounds of groceries and a packet of papers with health-related information. How much does each packet of papers weigh?
5. Andre has 3 times as many pencils as Noah and 5 pens. He has 17 pens and pencils all together. How many pencils does Noah have?

Solution
1. D
2. A
3. C
4. C
5. B

Lesson 7

Problem 1
(from Unit 2, Lesson 11)
There is a proportional relationship between the volume of a sample of helium in liters and the mass of that sample in grams. If the mass of a sample is 5 grams, its volume is 28 liters. (5, 28) is shown on the graph below.

1. What is the constant of proportionality in this relationship?
2. In this situation, what is the meaning of the number you found in part a?
3. Add at least three more points to the graph above, and label with their coordinates.
4. Write an equation that shows the relationship between the mass of a sample of helium and its volume. Use \( m \) for mass and \( v \) for volume.

Solution
1. 5.6 liters per gram
2. The volume of 1 gram of helium is 5.6 liters.
3. Answers vary. Sample answer:
Problem 2
Explain how the parts of the balanced hanger compare to the parts of the equation.

\[ 7 = 2x + 3 \]

Solution
Responses vary. Sample response: The fact that the hanger is balanced (equal weights on each side) matches the equal sign in the equation (equal expressions on each side). On the left of the hanger there are 7 equal weights. The equation shows 7 on the left side, so we can assume that each square represents 1 unit. The right side of the hanger has 2 circles of unknown weight, which matches the \( 2x \) in the equation - twice an unknown amount. The right side of the hanger also has 3 squares of unit weight, which matches the 3 on the right side of the equation. The weight of the 2 circles and 3 squares added together (the plus sign in the equation) is the same as (equal sign) the weight of the 7 squares.

Problem 3
Here is a hanger:

1. Write an equation to represent the hanger.
2. Draw more hangers to show each step you would take to find \( x \). Explain your reasoning.
3. Write an equation to describe each hanger you drew. Describe how each equation matches its hanger.

**Solution**

1. \(5x + 2 = 17\)
2. Subtract 2 from each side to get a hanger with 5 circles on the left and a rectangle labeled 15 on the right. Then divide both sides by 5 to get a hanger with one circle on the left and a rectangle labeled 3 on the right.
3. \(5x - 15, x - 3\)

**Lesson 8**

**Problem 1**

Here is a hanger:

1. Write an equation to represent the hanger.
2. Solve the equation by reasoning about the equation or the hanger. Explain your reasoning.

![Hanger](image)

**Solution**

1. \(5(x + 2) = 11\)
2. Explanations vary. Sample explanation: Divide both sides by 5 to get a circle labeled \(x\) and a rectangle labeled 2 on the left, and a rectangle labeled 2.2 on the right. Then subtract 2 from each side to get a circle on the left and rectangle with 0.2 on the right.
3. \(x + 2 = 2.2, x = 0.2\)

**Problem 2**

Explain how each part of the equation \(9 = 3(x + 2)\) is represented in the hanger.

- \(x\)
- 9
- 3
- \(x + 2\)
- \(3(x + 2)\)
- the equal sign
Solution

Answers vary. Sample response:

- The circle has an unknown weight, so use x to represent it.
- The left side has 9 squares, each weighing 1 unit.
- There are 3 identical groups on the right side.
- Each group on the right side is made up of one circle with weight x units and 2 squares of weight 1 unit each.
- The total weight of those 3 identical groups is the total weight of the right side.
- The equal sign is seen in the hanger being balanced.

Problem 3
(from Unit 4, Lesson 11)
Select the word from the following list that best describes each situation.

A. Tax
B. Commission
C. Discount
D. Markup
E. Tip or gratuity
F. Interest

1. You deposit money in a savings account, and every year the amount of money in the account increases by 2.5%.
2. For every car sold, a car salesman is paid 6% of the car’s price.
3. Someone who eats at a restaurant pays an extra 20% of the food price. This extra money is kept by the person who served the food.
4. An antique furniture store pays $200 for a chair, adds 50% of that amount, and sells the chair for $300.
5. The normal price of a mattress is $600, but it is on sale for 10% off.
6. For any item you purchase in Texas, you pay an additional 6.25% of the item’s price to the state government.

Solution

1. F
2. B
3. E
4. D
Problem 4
(from Unit 6, Lesson 3)
Clare drew this diagram to match the equation $2x + 16 = 50$, but she got the wrong solution as a result of using this diagram.

1. What value for $x$ can be found using the diagram?
2. Show how to fix Clare's diagram to correctly match the equation.
3. Use the new diagram to find a correct value for $x$.
4. Explain the mistake Clare made when she drew her diagram.

Solution
1. $x$ can be found by subtracting 2 and 16 from 50 since the three parts 2, $x$, and 16 sum to 50 in the diagram.
2. The diagram correctly represents the equation if the first block is changed from 2 to $x$. Then the three parts of the diagram are $x$, $x$, and 16, for a total of $2x + 16$.
3. Since the corrected diagram shows that the number 50 is divided into parts of size $x$, $x$ and 16, the two $x$'s must together equal 16 less than 50, which is 34. This means that one $x$ is 17.
4. Sample explanation: Clare showed $2 + x$ instead of $2 \cdot x$. She might not understand that $2x$ means 2 multiplied by $x$, or she might not understand that the tape diagram shows parts adding up to a whole.

Lesson 9

Problem 1
Solve each equation.
1. $4x = -28$
2. $x - 6 = -2$
3. $-x + 4 = -9$
4. $-3x + 7 = 1$
5. $25x + -11 = -86$

Solution
1. $-7$
2. $-8$
3. $13$
4. $2$
5. $-3$

Problem 2
Here is an equation $2x + 9 = -15$. Write three different equations that have the same solution as $2x + 9 = -15$. Show or explain how you found them.
Solution
Equations vary. Sample equations: $24 + 2x = 0$, $4x + 10 = -38$, $85 = 2x + 109$

Sample explanation:
- Start with: $2x + 9 = -15$.
- Add 20 to each side: $2x + 29 = 5$.
- Use the commutative property of addition: $29 + 2x = 5$.
- Subtract 5 from each side: $24 + 2x = 0$.

Problem 3
(from Unit 6, Lesson 3)
Select all the equations that match the diagram.

1. $x + 5 = 18$
2. $18 ÷ 3 = x + 5$
3. $3(x + 5) = 18$
4. $x + 5 = \frac{1}{3} \cdot 18$
5. $3x + 5 = 18$

Solution
B, C, D

Problem 4
(from Unit 6, Lesson 4)
Match each story to an equation.

A. A stack of nested paper cups is 8 inches tall. The first cup is 4 inches tall and each of the rest of the cups in the stack adds $\frac{1}{4}$ inch to the height of the stack.

B. A baker uses 4 cups of flour. She uses $\frac{1}{4}$ cup to flour the counters and the rest to make 8 identical muffins.

C. Elena has an 8-foot piece of ribbon. She cuts off a piece that is $\frac{1}{4}$ of a foot long and cuts the remainder into four pieces of equal length.

1. $\frac{1}{4} + 4x = 8$
2. $4 + \frac{1}{4}x = 8$
3. $8x + \frac{1}{4} = 4$

Solution
A. 2
B. 3
C. 1

Problem 5
(from Unit 6, Lesson 2)
There are 88 seats in a theater. The seating in the theater is split into 4 identical sections. Each section has 14 red seats and some blue seats.

1. Draw a tape diagram to represent the situation.
2. What unknown amounts can be found by using the diagram or reasoning about the situation?

Solution

Answers vary. Sample responses:

1. A tape diagram with 4 equal parts, each labeled \( x + 14 \), for a total of 88.

2. Each section has 22 seats, of which 8 are blue. There are 32 blue seats and 56 red seats in the theater.

Lesson 10

Problem 1
(from Unit 4, Lesson 11)
Andre wants to buy a backpack. The normal price of the backpack is $40. He notices that a store that sells the backpack is having a 30% off sale. What is the sale price of the backpack?

Solution

$28

Problem 2
(from Unit 4, Lesson 12)
On the first math exam, 16 students received an A grade. On the second math exam, 12 students received an A grade. What percentage decrease is that?

Solution

25% \((4 \div 16 = 0.25)\)

Problem 3
Solve each equation.

1. \(2(x - 3) = 14\)
2. \(-5(x - 1) = 40\)
3. \(12(x + 10) = 24\)
4. \(\frac{1}{6}(x + 6) = 11\)
5. \(\frac{5}{7}(x - 9) = 25\)

Solution

1. 10
2. -7
3. -8
4. 60
5. 44

Problem 4
Select all expressions that represent a correct solution to the equation \(6(x + 4) = 20\).

1. \((20 - 4) \div 6\)
2. \(\frac{1}{5}(20 - 4)\)
3. \(20 - 6 - 4\)
4. \(20 \div 6 - 4\)
5. \(\frac{1}{6}(20 - 24)\)
6. \((20 - 24) \div 6\)
Solution
D, E, F

Problem 5
Lin and Noah are solving the equation $7(x + 2) = 91$.

Lin starts by using the distributive property. Noah starts by dividing each side by 7.

2. What is the same and what is different about their methods?

Solution
Answers vary. Sample response:

1. Lin's solution method: $7x + 14 = 91$, $7x = 77$, $x = 11$
   Noah's solution method: $x + 2 = 13$, $x = 11$

2. Both methods involve dividing by 7, but Noah does the division first, while Lin does the division last. Also, Lin's method involves subtracting 14, while Noah's method involves subtracting 2. Both solutions are correct and valid. Noah's solution could be considered more efficient for this example, because it takes fewer steps and has equally complicated arithmetic work.

Lesson 11

Problem 1
(from Unit 5, Lesson 9)
Find the value of each variable.

1. $a \cdot 3 = -30$
2. $-9 \cdot b = 45$
3. $-89 \cdot 12 = c$
4. $d \cdot 88 = -88,000$

Solution
1. $a = -10$
2. $b = -5$
3. $c = -1.068$
4. $d = -1.000$

Problem 2
Match each equation to its solution and to the story it describes.

Equations:
A. $5x - 7 = 3$
B. $7 = 3(5 + x)$
C. $3x + 5 = -7$
D. $\frac{1}{3}(x + 7) = 5$

Solutions:
1. $-4$
2. $\frac{4}{3}$
3. 2

4. 8

Stories:

- The temperature is -7. Since midnight the temperature tripled and then rose 5 degrees. What was temperature at midnight?
- Jada has 7 pink roses and some white roses. She gives all of them away: 5 roses to each of her 3 favorite teachers. How many white roses did she give away?
- A musical instrument company reduced the time it takes for a worker to build a guitar. Before the reduction it took 5 hours. Now in 7 hours they can build 3 guitars. By how much did they reduce the time it takes to build each guitar?
- A club puts its members into 5 groups for an activity. After 7 students have to leave early, there are only 3 students left to finish the activity. How many students were in each group?

**Solution**

A. 3, club activity story

B. 2, building guitars story

C. 1, temperature story

D. 4, roses story

**Problem 3**
The baby giraffe weighed 132 pounds at birth. He gained weight at a steady rate for the first 7 months until his weight reached 538 pounds. How much did he gain each month?

**Solution**

58 pounds. He gained $538 - 132$, or 406 pounds, over 7 months. $406 \div 7 = 58$. (Or solve $132 + 7x = 538$.)

**Problem 4**
Six teams are out on the field playing soccer. The teams all have the same number of players. The head coach asks for 2 players from each team to come help him move some equipment. Now there are 78 players on the field. Write and solve an equation whose solution is the number of players on each team.

**Solution**

$6(x - 2) = 78$ (or $6x - 12 = 78$). $x = 15$

**Problem 5**
(from Unit 4, Lesson 7)
A small town had a population of 960 people last year. The population grew to 1200 people this year. By what percentage did the population grow?

**Solution**

The town has grown by 25%.

**Problem 6**
The gas tank of a truck holds 30 gallons. The gas tank of a passenger car holds 50% less. How many gallons does it hold?

![Graph showing gas (gallons) on a scale from 0 to 150 with 0%, 50%, 100%, and 150% labels.]

**Solution**

15 gallons because 50% less than 30 is 15. (If the double number line is used, the tick marks on the top are labeled 0, 15, 30, 45.)

Lesson 12

**Problem 1**

(from Unit 4, Lesson 12)

A backpack normally costs $25 but it is on sale for $21. What percentage is the discount?

**Solution**

16%

**Problem 2**

(from Unit 5, Lesson 9)

Find each product.

1. $\frac{1}{5} \cdot (-10)$
2. $0.8 \cdot \left(\frac{2}{5}\right)$
3. $\frac{10}{6} \cdot 0.6$
4. $\left(-\frac{100}{37}\right) \cdot (-0.37)$

**Solution**

1. -4
2. 12
3. 1
4. 1

**Problem 3**

Select all expressions that show $x$ increased by 35%.

1. $1.35x$
2. $\frac{35}{100}x$
3. $x + \frac{35}{100}x$
4. $(1 + 0.35)x$
5. $\frac{100 + 35}{100}x$
6. $(100 + 35)x$

**Solution**

A, C, D, E

**Problem 4**

(from Unit 4, Lesson 11)

Complete each sentence with the word discount, deposit, or withdrawal.

1. Clare took $20 out of her bank account. She made a _____.

2. $\frac{35}{100}x$
3. $x + \frac{35}{100}x$
4. $(1 + 0.35)x$
5. $\frac{100 + 35}{100}x$
6. $(100 + 35)x$
2. Kiran used a coupon when he bought a pair of shoes. He got a _____.
3. Priya put $20 into her bank account. She made a _____.
4. Lin paid less than usual for a pack of gum because it was on sale. She got a _____.

Solution
1. withdrawal
2. discount
3. deposit
4. discount

Problem 5
Here are two stories:
- The initial freshman class at a college is 10% smaller than last year’s class. But then during the first week of classes, 20 more students enroll. There are then 830 students in the freshman class.
- A store reduces the price of a computer by $20. Then during a 10% off sale, a customer pays $830.

Here are two equations:
- $0.9x + 20 = 830$
- $0.9(x - 20) = 830$

1. Decide which equation represents each story.
2. Explain why one equation has parentheses and the other doesn’t.
3. Solve each equation, and explain what the solution means in the situation.

Solution
Answers vary. Sample responses:
1. The freshman class: $0.9x + 20 = 830$, computer: $0.9(x - 20) = 830$
2. It depends on which came first, the additive increase or decrease (parentheses needed) or the percent decrease (no parentheses needed, since the convention is to multiply before adding when there are no parentheses).
3. The freshman class: $x = 900$, computer: $x = 942.22$ (rounding to the nearest cent)

Lesson 13

Problem 1
For each inequality, find two values for $x$ that make the inequality true and two values that make it false.
1. $x + 3 > 70$
2. $x + 3 < 70$
3. $-5x < 2$
4. $5x < 2$

Solution
Answers vary. Sample response:
1. True: $x = 70$ and $x = 100$, false: $x = 0$ and $x = -10$
2. True: $x = 60$ and $x = 0$, false: $x = 70$ and $x = 100$
3. True: $x = 1$ and $x = 2$, false: $x = -1$ and $x = -2$
4. True: $x = 0$ and $x = -1$, false: $x = 1$ and $x = 100$

**Problem 2**
Here is an inequality: $-3x > 18$.

1. List some values for $x$ that would make this inequality true.

2. How are the solutions to the inequality $-3x > 18$ different from the solutions to $-3x > 18$? Explain your reasoning.

**Solution**
1. Any value less than -6
2. The inequalities have almost the same solutions, but the first includes -6 and the second does not.

**Problem 3**
Match each sentence with the inequality that could represent the situation.

A. Han got $2 from Clare, but still has less than $20.
B. Mai spent $2 and has less than $20.
C. If Tyler had twice the amount of money he has, he would have less than $20.
D. If Priya had half the money she has, she would have less than $20.

1. $x - 2 < 20$
2. $2x < 20$
3. $x + 2 < 20$
4. $\frac{1}{2}x < 20$

**Solution**
A. 3
B. 1
C. 2
D. 4

**Problem 4**
(from Unit 4, Lesson 12)
Here are the prices for cheese pizza at a certain pizzeria:

<table>
<thead>
<tr>
<th>pizza size</th>
<th>price in dollars</th>
</tr>
</thead>
<tbody>
<tr>
<td>small</td>
<td>11.60</td>
</tr>
<tr>
<td>medium</td>
<td>16.25</td>
</tr>
</tbody>
</table>

1. You had a coupon that made the price of a large pizza $13.00. For what percent off was the coupon?

2. Your friend purchased a medium pizza for $10.31 with a 30% off coupon. What is the price of a medium pizza without a coupon?

3. Your friend has a 15% off coupon and $10. What is the largest pizza that your friend can afford, and how much money will be left over after the purchase?
Solution
1. 20%
2. $14.73
3. Small, $0.14

Problem 5
(from Unit 6, Lesson 4)
Select all the stories that can be represented by the diagram.

1. Andre studies 7 hours this week for end-of-year exams. He spends 1 hour on English and an equal number of hours each on math, science, and history.
2. Lin spends $3 on 7 markers and a $1 pen.
3. Diego spends $1 on 7 stickers and 3 marbles.
4. Noah shares 7 grapes with 3 friends. He eats 1 and gives each friend the same number of grapes.
5. Elena spends $7 on 3 notebooks and a $1 pen.

Solution
A. D. E

Lesson 14

Problem 1
The solution to $5 - 3x > 35$ is either $x > -10$ or $-10 > x$. Which solution is correct? Explain how you know.

Solution
$x < -10$. Sample reasoning: If I try -100 in place of $x$, I get $305 > 35$, which is true. Any value of $x$ that is less than -10 makes the inequality true. $-10 > x$ refers to all values of $x$ that are less than -10.

Problem 2
The school band director determined from past experience that if they charge $t$ dollars for a ticket to the concert, they can expect attendance of $1000 - 50t$. The director used this model to figure out that the ticket price needs to be $8 or greater in order for at least 600 to attend. Do you agree with this claim? Why or why not?

Solution
No. Explanations vary. Sample response: If ticket prices are higher, fewer people will attend (this can be seen by trying some different values of $t$ in $1000 - 50t$). $8$ is the solution to $1000 - 50t = 600$, but they need to charge $8 or less if they want 600 people or more to attend.

Problem 3
(from Unit 6, Lesson 13)
Which inequality is true when the value of $x$ is -3?

1. $x - 6 < -3.5$
2. $x - 6 > 3.5$
3. $x - 6 > -3.5$
4. $x - 6 > -3.5$

Solution
C
Problem 4  
(from Unit 6, Lesson 13)  
Draw the solution set for each of the following inequalities.

1. \( x \leq 5 \)

\[ -10 \quad -9 \quad -8 \quad -7 \quad -6 \quad -5 \quad -4 \quad -3 \quad -2 \quad -1 \quad 0 \quad 1 \quad 2 \quad 3 \quad 4 \quad 5 \quad 6 \quad 7 \quad 8 \quad 9 \]

2. \( x < \frac{5}{2} \)

\[ -10 \quad -9 \quad -8 \quad -7 \quad -6 \quad -5 \quad -4 \quad -3 \quad -2 \quad -1 \quad 0 \quad 1 \quad 2 \quad 3 \quad 4 \quad 5 \quad 6 \quad 7 \quad 8 \quad 9 \]

Solution

1. 

\[ -10 \quad -9 \quad -8 \quad -7 \quad -6 \quad -5 \quad -4 \quad -3 \quad -2 \quad -1 \quad 0 \quad 1 \quad 2 \quad 3 \quad 4 \quad 5 \quad 6 \quad 7 \quad 8 \quad 9 \]

2. 

\[ -10 \quad -9 \quad -8 \quad -7 \quad -6 \quad -5 \quad -4 \quad -3 \quad -2 \quad -1 \quad 0 \quad 1 \quad 2 \quad 3 \quad 4 \quad 5 \quad 6 \quad 7 \quad 8 \quad 9 \]

Problem 5  
(from Unit 6, Lesson 3)  
Write three different equations that match the tape diagram.

Solution

Answers vary. Sample responses:

1. \( 7x + 19 = 40 \)
2. \( 40 = 7x + 19 \)
3. \( 7x = 40 - 19 \)

Problem 6  
(from Unit 6, Lesson 2)  
A baker wants to reduce the amount of sugar in his cake recipes. He decides to reduce the amount used in 1 cake by \( \frac{1}{2} \) cup. He then uses \( 4\frac{1}{2} \) cups of sugar to bake 6 cakes.

\[ \frac{X}{2} \quad \frac{X}{2} \quad \frac{X}{2} \quad \frac{X}{2} \quad \frac{X}{2} \quad \frac{X}{2} \]

\[ 4\frac{1}{2} \]

1. Describe how the tape diagram represents the story.
2. How much sugar was originally in each cake recipe?

Solution

1. Answers vary. Sample response: The six equal parts of the diagram represent the 6 cakes the baker bakes. The label \( x - \frac{1}{2} \) in each part represents the amount of sugar, measured in number of cups, that the baker used in each cake. \( x \) represents the original amount of sugar.
used in each cake and $x - \frac{1}{2}$ represents the original number of cups reduced by $\frac{1}{2}$ cup. 4 1/2 is the total amount of sugar, measured in cups, used for the 6 cakes.

2. $1\frac{1}{4}$ cups

Problem 7

(from Unit 4, Lesson 12)
One year ago, Clare was 4 feet 6 inches tall. Now Clare is 4 feet 10 inches tall. By what percentage did Clare’s height increase in the last year?

Solution
About 7% (4 feet 6 inches is 54 inches and she grew 4 inches: $\frac{4}{54} \approx 0.07$)

Lesson 15

Problem 1
1. Consider the inequality $-1 \leq \frac{x}{2}$.
   a. Predict which values of $x$ will make the inequality true.

   b. Complete the table to check your prediction.

<table>
<thead>
<tr>
<th>$x$</th>
<th>-4</th>
<th>-3</th>
<th>-2</th>
<th>-1</th>
<th>0</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\frac{x}{2}$</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

2. Consider the inequality $1 \leq \frac{x}{2}$.
   a. Predict which values of $x$ will make it true.

   b. Complete the table to check your prediction.

<table>
<thead>
<tr>
<th>$x$</th>
<th>-4</th>
<th>-3</th>
<th>-2</th>
<th>-1</th>
<th>0</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\frac{x}{2}$</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Solution
1. a. $x \geq -2$

   b. 

<table>
<thead>
<tr>
<th>$x$</th>
<th>-4</th>
<th>-3</th>
<th>-2</th>
<th>-1</th>
<th>0</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\frac{x}{2}$</td>
<td>-2</td>
<td>-1.5 (or $\frac{3}{2}$)</td>
<td>-1</td>
<td>-0.5 (or $\frac{1}{2}$)</td>
<td>0</td>
<td>0.5 (or $\frac{1}{2}$)</td>
<td>1</td>
<td>1.5 (or $\frac{3}{2}$)</td>
<td>2</td>
</tr>
</tbody>
</table>

2. a. $x \leq -2$

   b. 

<table>
<thead>
<tr>
<th>$x$</th>
<th>-4</th>
<th>-3</th>
<th>-2</th>
<th>-1</th>
<th>0</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\frac{x}{2}$</td>
<td>2</td>
<td>1.5 (or $\frac{3}{2}$)</td>
<td>1</td>
<td>0.5 (or $\frac{1}{2}$)</td>
<td>0</td>
<td>-0.5 (or $\frac{1}{2}$)</td>
<td>1</td>
<td>-1.5 (or $\frac{3}{2}$)</td>
<td>-2</td>
</tr>
</tbody>
</table>

Problem 2
Diego is solving the inequality $100 - 3x \geq -50$. He solves the equation $100 - 3x = -50$ and gets $x = 50$. What is the solution to the inequality?

1. $x < 50$
2. $x \leq 50$
3. $x > 50$
4. $x \geq 50$

Solution
Problem 3
Solve the inequality \(-5(x - 1) > -40\), and graph the solution on a number line.

**Solution**
1. \(x < 9\)
2. A number line with an open circle at 9 and the arrow going to the left

Problem 4
(from Unit 6, Lesson 13)
Select **all** values of \(x\) that make the inequality \(-x + 6 \geq 10\) true.

1. -3.9
2. 4
3. -4.01
4. -4
5. 4.01
6. 3.9
7. 0
8. -7

**Solution**
C, D, H

Problem 5
(from Unit 6, Lesson 13)
Draw the solution set for each of the following inequalities.

1. \(x > 7\)

![Number line with an open circle at 8 and the arrow going to the right]

2. \(x \geq -4.2\)

![Number line with a closed circle at -4 and the arrow going to the right]

**Solution**
1. ![Number line with an open circle at 8 and the arrow going to the right]
2. ![Number line with a closed circle at -4 and the arrow going to the right]

Problem 6
(from Unit 4, Lesson 12)
The price of a pair of earrings is $22 but Priya buys them on sale for $13.20.

1. By how much was the price discounted?
2. What was the percentage of the discount?

**Solution**
1. $8.80
Lesson 16

Problem 1
Priya looks at the inequality $12 - x > 5$ and says “I subtract a number from 12 and want a result that is bigger than 5. That means that the solutions should be values of $x$ that are smaller than something.”

Do you agree with Priya? Explain your reasoning and include solutions to the inequality in your explanation.

**Solution**
Yes, Priya is correct. Explanations vary. Sample response: Try subtracting different numbers from 12. For example, $12 - 3$ is larger than $12 - 8$ because subtracting 3 is subtracting less. When $x = 7$, the inequality is not true anymore, but for anything smaller than 7, it is still true. The solution to the inequality is $x < 7$.

Problem 2
When a store had sold $\frac{2}{3}$ of the shirts that were on display, they brought out another 30 from the stockroom. The store likes to keep at least 150 shirts on display. The manager wrote the inequality $\frac{2}{3}x + 30 \geq 150$ to describe the situation.

1. Explain what $\frac{2}{3}$ means in the inequality.

2. Solve the inequality.

3. Explain what the solution means in the situation.

**Solution**
Answers vary. Sample responses:

1. Since $\frac{2}{3}$ of the original shirts were sold, $\frac{1}{3}$ of the original shirts remain on display.

2. $\frac{2}{3}x + 30 \geq 150, \frac{2}{3}x \geq 120, x \geq 200$

3. There were 200 or more shirts originally on display. At least 120 were left when they brought out 30 more.

Problem 3
(from Unit 6, Lesson 13)
You know $x$ is a number less than 4. Select all the inequalities that **must** be true.

1. $x < 2$
2. $x + 6 < 10$
3. $5x < 20$
4. $x - 2 > 2$
5. $x < 8$

**Solution**
B, C, E

Problem 4
(from Unit 6, Lesson 13)
Here is an unbalanced hanger.

1. If you knew each circle weighed 6 grams, what would that tell you about the weight of each triangle? Explain your reasoning.
2. If you knew each triangle weighed 3 grams, what would that tell you about the weight of each circle? Explain your reasoning.

Solution
1. The triangles would weigh more than 4 grams each. The 3 triangles weigh more than 2 circles. The 2 circles weigh 12 grams, so that means each triangle would weigh more than 4 grams.

2. The circles would weigh less than 4.5 grams each. The 3 triangles weigh 9 grams and this is more than 2 circles. So each circle weighs less than 4.5 grams.

Problem 5
(from Unit 4, Lesson 12)
At a skateboard shop:

1. The price tag on a shirt says $12.58. Sales tax is 7.5% of the price. How much will you pay for the shirt?

2. The store buys a helmet for $19.00 and sells it for $31.50. What percentage was the markup?

3. The shop pays workers $14.25 per hour plus 5.5% commission. If someone works 18 hours and sells $250 worth of merchandise, what is the total amount of their paycheck for this pay period? Explain or show your reasoning.

Solution
1. $13.52

2. 65.8% or 66%

3. $270.25, because 18 \cdot (14.25) + (0.055) \cdot 250 = 270.25.

Lesson 17

Problem 1
28 students travel on a field trip. They bring a van that can seat 12 students. Elena and Kiran’s teacher asks parents to drive cars that seat 3 children each to transport the rest of the students.

Elena wonders if she should use the inequality $12 + 3n > 28$ or $12 + 3n \geq 28$ to figure out how many cars are needed. Kiran doesn’t think it matters in this case. Do you agree with Kiran? Explain your reasoning.

Solution
Sample explanation: Yes, it doesn’t matter. In this case $n$ represents a number of cars, so only whole number values of $n$ make sense for the situation, and there can’t be fractions of cars. $12 + 3n = 28$ has the solution $n = \frac{16}{3}$, so the number of cars needed is 6.

Problem 2
1. In the cafeteria, there is one large 10-seat table and many smaller 4-seat tables. There are enough tables to fit 200 students. Write an inequality whose solution is the possible number of 4-seat tables in the cafeteria.

2. 5 barrels catch rainwater in the schoolyard. Four barrels are the same size, and the fifth barrel holds 10 liters of water. Combined, the 5 barrels can hold at least 200 liters of water. Write an inequality whose solution is the possible size of each of the 4 barrels.

3. How are these two problems similar? How are they different?

Solution
1. $10 + 4n \geq 200$

2. $10 + 4n \geq 200$

3. Solutions to the first inequality must be whole numbers greater or equal to 47.5 because a solution represents a number of tables. Solutions to the second inequality can be any number greater or equal to 47.5 because a solution represents the volume of a bucket, which can be a whole number or not.

Problem 3
Solve each equation.

1. \(5(n - 4) = -60\)
2. \(-3t + 8 = 25\)
3. \(7p - 8 = -22\)
4. \(\frac{2}{5}(j + 40) = 4\)
5. \(4(w + 1) = -6\)

**Solution**

1. \(n = -8\)
2. \(t = -11\)
3. \(p = -2\)
4. \(j = -50\)
5. \(w = \frac{-19}{4}\) (or equivalent)

**Problem 4**

(from Unit 6, Lesson 13)
Select all the inequalities that have the same graph as \(x < 4\).

1. \(x < 2\)
2. \(x + 6 < 10\)
3. \(5x < 20\)
4. \(x - 2 > 2\)
5. \(x < 8\)

**Solution**

B, C

**Problem 5**

(from Unit 4, Lesson 12)
A 200 pound person weighs 33 pounds on the moon.

1. How much did the person’s weight decrease?
2. By what percentage did the person’s weight decrease?

**Solution**

1. 167 pounds
2. About 84% \((167 \div 200 = 0.835)\)

**Lesson 18**

**Problem 1**

For each expression, write an equivalent expression that uses only addition.

1. \(20 - 9 + 8 - 7\)
2. \(4x - 7y - 5z + 6\)
3. \(-3x - 8y - 4 - \frac{8}{7z}\)

**Solution**
1. $20 + .9 + 8 + -7$
2. $4x + -7y + -5z + 6$
3. $-3x + -8y + -4 + -\frac{8}{7}z$

**Problem 2**

Use the distributive property to write an expression that is equivalent to each expression. If you get stuck, draw boxes to help organize your work.

1. $9(4x - 3y - \frac{3}{4})$
2. $-2(-6x + 3y - 1)$
3. $\frac{1}{2}(20y - 4x - 13)$
4. $8(x - \frac{1}{2})$
5. $-8(-x - \frac{1}{2}y + \frac{3}{4})$

**Solution**

1. $36x - 27y - 6$
2. $12x - 6y + 2$
3. $4y - \frac{4}{5}x - \frac{13}{5}$
4. $-8x - 4$
5. $8x + 6y - 28$

**Problem 3**

Kiran wrote the expression $x - 10$ for this number puzzle: “Pick a number, add -2, and multiply by 5.”

Lin thinks Kiran made a mistake.

1. How can she convince Kiran he made a mistake?
2. What would be a correct expression for this number puzzle?

**Solution**

1. Answers vary. Sample response: for $x = 1$ the number puzzle should result in $-5$. But Kiran’s expression gives $1 - 10 = -9$.
2. $(x - 2) \cdot 5$ (or $5x - 10$)

**Problem 4**

(from Unit 2, Lesson 7)

The output from a coal power plant is shown in the table:

<table>
<thead>
<tr>
<th>energy in megawatts</th>
<th>number of days</th>
</tr>
</thead>
<tbody>
<tr>
<td>1,200</td>
<td>2.4</td>
</tr>
<tr>
<td>1,800</td>
<td>3.6</td>
</tr>
<tr>
<td>4,000</td>
<td>8</td>
</tr>
<tr>
<td>10,000</td>
<td>20</td>
</tr>
</tbody>
</table>

Similarly, the output from a solar power plant is shown in the table:
Based on the tables, is the energy output in proportion to the number of days for either plant? If so, write an equation showing the relationship. If not, explain your reasoning.

**Solution**

The coal power plant could be a proportional relationship. Its equation would be $E = 500 \cdot d$ where $E$ is the energy output in megawatts and $d$ is the number of days. The solar power plant would not be a proportional relationship since the ratio between the number of days and the energy output is not constant.

### Lesson 19

#### Problem 1

1. Expand to write an equivalent expression: $\frac{1}{4}(-8x + 12y)$

2. Factor to write an equivalent expression: $36a - 16$

**Solution**

1. $2x - 3y$

2. $4(9a - 4)$ (or $2(18a - 8)$)

#### Problem 2

Lin missed math class on the day they worked on expanding and factoring. Kiran is helping Lin catch up.

1. Lin understands that expanding is using the distributive property, but she doesn’t understand what factoring is or why it works. How can Kiran explain factoring to Lin?

2. Lin asks Kiran how the diagrams with boxes help with factoring. What should Kiran tell Lin about the boxes?

3. Lin asks Kiran to help her factor the expression $-4xy - 12xz + 20xw$. How can Kiran use this example to Lin understand factoring?

**Solution**

1. Answers vary. Sample response: Factoring is the distributive property in the other direction. Instead of expanding a product to a sum of terms, factoring takes a sum of terms and makes it into a product by looking for common factors in the terms that can be written outside the parentheses.

2. Answers vary. Sample response: The expression in each box is the product of (the expression to the left of the big rectangle) and (the expression above the box), just as the area of a rectangle is length times width. Together, the boxes form a long rectangle, so it is still true that (the expression to the left of the box) times (the expression above the long rectangle) equals the sum of all the terms in the boxes. If you want to factor an expression, look for a common factor in each box, and place it to the left of the rectangle. To decide what to write above each box, think, “What times that common factor equals what is in the box?”

3. Answers vary and should describe the box or steps. Sample response: First, find the common factor, which is $4x$. Write "$4x(\ldots)\"$. We are going to decide what needs to go in the parentheses to make an expression equivalent to $-4xy - 12xz + 20xw$. To get $-4xy$, we need to multiply by $-y$. Using similar reasoning, we can fill in the rest: $4x(-y - 3z + 5w)$.

#### Problem 3

Complete the equation with numbers that makes the expression on the right side of the equal sign equivalent to the expression on the left side.

$75a + 25b = \_\_\_\_\_ (\_\_\_\_ + \_\_\_)$

**Solution**

$25(3a + b)$
Problem 4
(from Unit 4, Lesson 3)
Elena makes her favorite shade of purple paint by mixing 3 cups of blue paint, 1 \( \frac{1}{2} \) cups of red paint, and \( \frac{1}{2} \) of a cup of white paint. Elena has \( \frac{2}{3} \) of a cup of white paint.

1. Assuming she has enough red paint and blue paint, how much purple paint can Elena make?
2. How much blue paint and red paint will Elena need to use with the \( \frac{2}{3} \) of a cup of white paint?

Solution
1. \( \frac{20}{3} \) cups. One batch of purple paint makes 5 cups. Elena can make \( \frac{2}{3} + \frac{1}{2} = \frac{4}{3} \) batches so that's \( \frac{20}{3} \) cups.
2. 4 cups of blue paint and 2 cups of red paint.

Problem 5
(from Unit 6, Lesson 9)
Solve each equation.

1. \( \frac{1}{8} d - 4 = \frac{-3}{8} \)
2. \( \frac{1}{2} m + 5 = 16 \)
3. \( 10b + .45 = -43 \)
4. \( -8(y - 1.25) = 4 \)
5. \( 3.2(x + 10) = 32 \)

Solution
1. \( d = -29 \)
2. \( m = -44 \)
3. \( b = \frac{1}{5} \) (or equivalent)
4. \( y = 0.75 \) (or equivalent)
5. \( x = 0 \)

Problem 6
(from Unit 6, Lesson 13)
Select all the inequalities that have the same solutions as 
\(-4x < 20\).

1. \( -x < 5 \)
2. \( 4x > -20 \)
3. \( 4x < -20 \)
4. \( x < -5 \)
5. \( x > 5 \)
6. \( x > -5 \)

Solution
A, B, F

Lesson 20

Problem 1
Andre says that \( 10x + 6 \) and \( 5x + 11 \) are equivalent because they both equal 16 when \( x \) is 1. Do you agree with Andre? Explain your reasoning.

Solution
No, equivalent expressions are equal for any value of their variable. When \( x \) is 0, they are not equal.

**Problem 2**
Select **all** expressions that can be subtracted from \( 9x \) to result in the expression \( 3x + 5 \).

1. \( 5 + 6x \)
2. \( 5 - 6x \)
3. \( 6x + 5 \)
4. \( 6x - 5 \)
5. \( 6x + 5 \)

**Solution**
A, D

**Problem 3**
Select **all** the statements that are true for any value of \( x \).

1. \( 7x + (2x + 7) = 9x + 7 \)
2. \( 7x + (2x - 1) = 9x + 1 \)
3. \( 3x + (10 - 3x) = 10 \)
4. \( 5x - (8 - 6x) = -x - 8 \)
5. \( 4x - (2x + 8) = 2x - 8 \)
6. \( 6x - (2x - 4) = 4x + 4 \)

**Solution**
A, C, E, F

**Problem 4**
(from Unit 6, Lesson 13)
For each situation, would you describe it with \( x < 25 \), \( x > 25 \), \( x \leq 25 \), or \( x \geq 25 \)?

1. The library is having a party for any student who read at least 25 books over the summer. Priya read \( x \) books and was invited to the party.

2. Kiran read \( x \) books over the summer but was not invited to the party.

3.

4. 

**Solution**
1. \( x \geq 25 \)
2. \( x < 25 \)
3. \( x \leq 25 \)
4. \( x > 25 \)

**Problem 5**
(from Unit 2, Lesson 9)
Consider the problem: A water bucket is being filled with water from a water faucet at a constant rate. When will the bucket be full? What information would you need to be able to solve the problem?

**Solution**
Answers vary. Possible response:

1. How big is the bucket?

2. What is the rate of water flow?

3. How high is the bucket?

4. How high is the water in the bucket after 1 minute?

Lesson 21

Problem 1
Noah says that $9x - 2x + 4x$ is equivalent to $3x$ because the subtraction sign tells us to subtract everything that comes after $9x$. Elena says that $9x - 2x + 4x$ is equivalent to $11x$ because the subtraction only applies to $2x$. Do you agree with either of these claims? Explain your reasoning.

Solution
Elena is correct. Rewriting addition as subtraction gives us $9x + -2x + 4x$, which shows that the subtraction symbol in front of the $2x$ applies only to the $2x$ and not to the terms that come after it.

Problem 2
Identify the error in generating an expression equivalent to $4 + 2x - \frac{1}{2}(10 - 4x)$. Then correct the error.

- $4 + 2x + \frac{1}{2}(10 + .4x)$
- $4 + 2x + -5 + 2x$
- $4 + 2x + -5 + 2x$
- $-1$

Solution
The error is in the last step. The second $2x$ was subtracted instead of being added. This would be correct if there were parentheses around $5 + 2x$. The last step should be $4x - 1$.

Problem 3
Select all expressions that are equivalent to $5x - 15 - 20x + 10$.

1. $5x - (15 + 20x) + 10$
2. $5x + -15 + -20x + 10$
3. $5(x - 3 - 4x + 2)$
4. $-5(-x + 3 + 4x + -2)$
5. $-15x - 5$
6. $-5(3x + 1)$

Solution
A, B, C, D, E, F

Problem 4
(from Unit 6, Lesson 14)
The school marching band has a budget of up to $750 to cover 15 new uniforms and competition fees that total $300. How much can they spend for one uniform?

1. Write an inequality to represent this situation.
2. Solve the inequality and describe what it means in the situation.

Solution
1. $15x + 300 \leq 750$
2. $x \leq 30$. They can spend at most $30 on each uniform.
Problem 5
(from Unit 6, Lesson 16)
Solve the inequality that represents each story. Then interpret what the solution means in the story.

1. For every $9 that Elena earns, she gives $x dollars to charity. This happens 7 times this month. Elena wants to be sure she keeps at least $42 from this month’s earnings. $7(9 - x) \geq 42$

2. Lin buys a candle that is 9 inches tall and burns down $x$ inches per minute. She wants to let the candle burn for 7 minutes until it is less than 6 inches tall. $9 - 7x < 6$

Solution
1. $x \leq 3$. Elena can give $3 or less to charity for every $9 she earns.
2. $x > \frac{3}{7}$. The candle needs to burn down more than $\frac{3}{7}$ inch each minute.

Problem 6
(from Unit 4, Lesson 3)
A certain shade of blue paint is made by mixing $1\frac{1}{2}$ quarts of blue paint with 5 quarts of white paint. If you need a total of 16.25 gallons of this shade of blue paint, how much of each color should you mix?

Solution
You should mix $3\frac{1}{4}$ quarts of blue paint with $12\frac{1}{2}$ quarts of white paint.

Lesson 22

Problem 1
Jada says, “I can tell that $\frac{2}{3}(x + 5) + 4(x + 5) - \frac{10}{3}(x + 5)$ equals 0 just by looking at it.” Is Jada correct? Explain how you know.

Solution
Yes. Explanations vary. Sample response: Factor out $x + 5$: $(x + 5)(\frac{2}{3} + 4 - \frac{10}{3}) = (x + 5)(\frac{11}{3}) = (x + 5)(0) = 0$.

Problem 2
In each row, decide whether the expression in column A is equivalent to the expression in column B. If they are not equivalent, show how to change one expression to make them equivalent.

A
1. $3x - 2x + 0.5x$
2. $3(x + 4) - 2(x + 4)$
3. $6(x + 4) - 2(x + 5)$
4. $3(x + 4) - 2(x + 4) + 0.5(x + 4)$

B
1. $1.5x$
2. $x + 3$
3. $2(2x + 7)$
4. $1.5$

Solution
1. Equivalent
2. Not equivalent. Answers vary. Sample responses: Change the column B entry to \( x + 4 \), change the column A entry to
\[ 3(x + 4) - 2(x + 4) - 1 \]
3. Equivalent
4. Not equivalent. Answers vary. Sample response: Change the column B entry to \( 1.5(x + 4) \).

**Problem 3**
(from Unit 6, Lesson 20)
For each situation, write an expression for the new balance using as few terms as possible.

1. A checking account has a balance of -$126.89. A customer makes two deposits, one \( 3\frac{1}{2} \) times the other, and then withdraws $25.

2. A checking account has a balance of $350. A customer makes two withdrawals, one $50 more than the other. Then he makes a deposit of $75.

**Solution**
1. \(-$126.89 + x + 3.5x - 25 = -151.89 + 4.5x\)
2. \(350 - x - (x + 50) + 75 = 375 - 2x\)

**Problem 4**
(from Unit 6, Lesson 21)
Tyler is using the distributive property on the expression \( 9 - 4(5x - 6) \). Here is his work:

\[ \begin{align*}
\text{\#1} & \quad 9 - 4(5x - 6) \\
\text{\#2} & \quad 9 + (-4)(5x + -6) \\
\text{\#3} & \quad 9 + -20x + -6 \\
\text{\#4} & \quad 3 - 20x
\end{align*} \]

Mai thinks Tyler’s answer is incorrect. She says, “If expressions are equivalent then they are equal for any value of the variable. Why don’t you try to substitute the same value for \( x \) in all the equations and see where they are not equal?”

1. Find the step where Tyler made an error.
2. Explain what he did wrong.
3. Correct Tyler’s work.

**Solution**
Answers vary. Sample response:
1. Try 1:
   \[ \begin{align*}
   \text{\#1} & \quad 9 - 4(5 \cdot 1 - 6) = 9 - 4(-1) = 9 + 4 = 13 \\
   \text{\#2} & \quad 9 + (-4)(5 \cdot 1 - 6) = 9 + (-4)(-1) = 9 + 4 = 13 \\
   \text{\#3} & \quad 9 + (-20)(-1) + -6 = 9 + -20 + -6 = -17 \\
   \text{\#4} & \quad 3 - 20(1) = 3 - 20 = -17
   \end{align*} \]
   The value of the expression switched in the third step, so that’s where the error is.
2. Tyler forgot to distribute \(-4\) to the \(-6\) term in the parentheses.
3. Starting at step 3:
   \[ \begin{align*}
   9 + -20x + 24 \\
   33 - 20x
   \end{align*} \]
Problem 5
(from Unit 6, Lesson 13)

1. If $(11 + x)$ is positive, but $(4 + x)$ is negative, what is one number that $x$ could be?

2. If $(-3 + y)$ is positive, but $(-9 + y)$ is negative, what is one number that $y$ could be?

3. If $(-5 + z)$ is positive, but $(-6 + z)$ is negative, what is one number that $z$ could be?

Solution

1. Answers vary. $x$ can be any number in between -11 and -4.

2. Answers vary. $y$ can be any number in between 3 and 9.

3. Answers vary. $z$ can be any number in between 5 and 6, for example, 5 $\frac{1}{2}$.