Big Ideas Math: California Algebra 1

Parent Newsletter

BIG IDEAS LEARNING.

Standards

California Common Core:

A.REI.4a: Use the method of completing the square to transform any quadratic equation in x into an equation of the form $(x - p)^2 = q$ that has the same solutions. Derive the quadratic formula from this form.

A.REI.4b: Solve quadratic equations by inspection (e.g., for $x^2 = 49$), taking square roots, completing the square, the quadratic formula and factoring, as appropriate to the initial form of the equation. Recognize when the quadratic formula gives complex solutions and write them as $a \pm bi$ for real numbers a and b.

A.REI.7: Solve a simple system consisting of a linear equation and a quadratic equation in two variables algebraically and graphically.

A.REI.11: Explain why the x-coordinates of the points where the graphs of the equations $y = f(x)$ and $y = g(x)$ intersect are the solutions of the equation $f(x) = g(x)$; find the solutions approximately, e.g., using technology to graph the functions, make tables of values, or find successive approximations. Include cases where $f(x)$ and/or $g(x)$ are linear, polynomial, rational, absolute value, exponential, and logarithmic functions.

A.SSE.3b: Complete the square in a quadratic expression to reveal the maximum or minimum value of the function it defines.

F.IF.8a: Use the process of factoring and completing the square in a quadratic function to show zeros, extreme values, and symmetry of the graph, and interpret these in terms of a context.

Essential Questions

- How can you use a graph to solve a quadratic equation in one variable?
- How can you determine the number of solutions of a quadratic equation of the form $ax^2 + c = 0$?
- How can you use "completing the square" to solve a quadratic equation?
- How can you use the discriminant to determine the number of solutions of a quadratic equation?
- How can you solve a system of two equations when one is linear and the other is quadratic?

Students will...

- Solve quadratic equations by graphing.
- Solve quadratic equations by taking square roots.
- Solve quadratic equations by completing the square.
- Solve quadratic equations by the quadratic formula.
- Use discriminants to determine the number of real solutions of quadratic equations.
- Choose a method to solve quadratic equations.
- Solve systems of linear and quadratic equations.
- Solve real-life problems.

Key Terms

A **quadratic equation** is a nonlinear equation that can be written in the standard form $ax^2 + bx + c = 0$, where $a \neq 0$.

Another method for solving quadratic equations is **completing the square**. In this method, a constant c is added to the expression $x^2 + bx$ so that $x^2 + bx + c$ is a perfect square trinomial.

Quadratic formula: $x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$, where $a \neq 0$ and $b^2 - 4ac \geq 0$.

The expression $b^2 - 4ac$ in the quadratic formula is the **discriminant**.

Games

- Quadratic Quandry
- Transform Me

These are available online in the *Game Closet* at www.bigideasmath.com.

Key Ideas

Solving Quadratic Equations Using Square Roots

You can solve $x^2 = d$ by taking the square root of each side.

- When $d > 0$, $x^2 = d$ has two real solutions, $x = \pm d$.
- When $d = 0$, $x^2 = d$ has one real solution, $x = 0$.
- When $d < 0$, $x^2 = d$ has no real solutions.

Completing the Square

To complete the square for an expression of the form $x^2 + bx$, follow these steps.

Step 1: Find one-half of b, the coefficient of x.

Step 2: Square the result from Step 1.

Step 3: Add the result from Step 2 to $x^2 + bx$.

Factor the resulting expression as the square of a binomial.

$$x^2 + bx + \left(\frac{b}{2}\right)^2 = \left(x + \frac{b}{2}\right)^2$$

Quadratic Formula

The real solutions of the quadratic equation $ax^2 + bx + c = 0$

are $x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$, where $a \neq 0$ and $b^2 - 4ac \geq 0$.

Interpreting the Discriminant

- $b^2 - 4ac > 0$
  - two real solutions
  - two x-intercepts
- $b^2 - 4ac = 0$
  - one real solution
  - one x-intercept
- $b^2 - 4ac < 0$
  - no real solutions
  - no x-intercepts

Good to know.

Methods for Solving Quadratic Equations

Method	Advantages	Disadvantages
Factoring	• Straightforward when equation can be factored easily	• Some equations are not factorable.
Graphing	• Can easily see the number of solutions • Use when approximate solutions are sufficient. • Can use a graphing calculator	• May not give exact solutions
Using Square Roots	• Use to solve equations of the form $x^2 = d$.	• Can only be used for certain equations
Completing the Square	• Best used when $a = 1$ and b is even	• May involve difficult calculations
Quadratic Formula	• Can be used for *any* quadratic equations • Gives exact solutions	• Takes time to do calculations

Reference Tools

An Information Wheel can be used to organize information about a concept. Students write the concept in the middle of the "wheel." Then students write information related to the concept on the "spokes" of the wheel. This type of organizer serves as a good summary tool because any information related to a concept can be included.
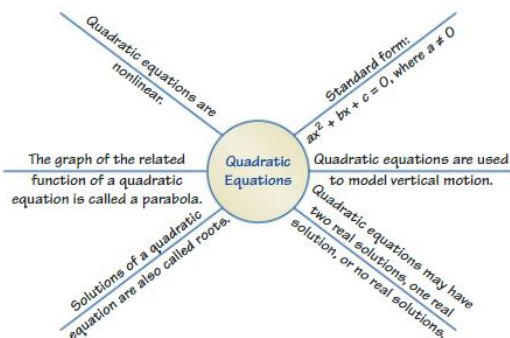
Quick Review

- The solutions of a quadratic equation are also called roots.
- A zero of a function $y = f(x)$ is an x-value for which the value of the function is zero.
- The square of a real number cannot be negative.
- Before completing the square, make sure the leading coefficient is 1.
- You can use the roots of a quadratic equation to factor the related expression.
- The solutions of a quadratic equation may be real numbers or *imaginary numbers*.

What's the Point?

Solving quadratic equations is very useful in real-life for finding what amount of goods being supplied is exactly equal to the amount of goods demanded for the economy to be in equilibrium The STEM Videos available online show ways to use mathematics in real-life situations. The Chapter 9: Wolf Population Growth STEM Video is available online at www.bigideasmath.com.