Area and Perimeter: Circle

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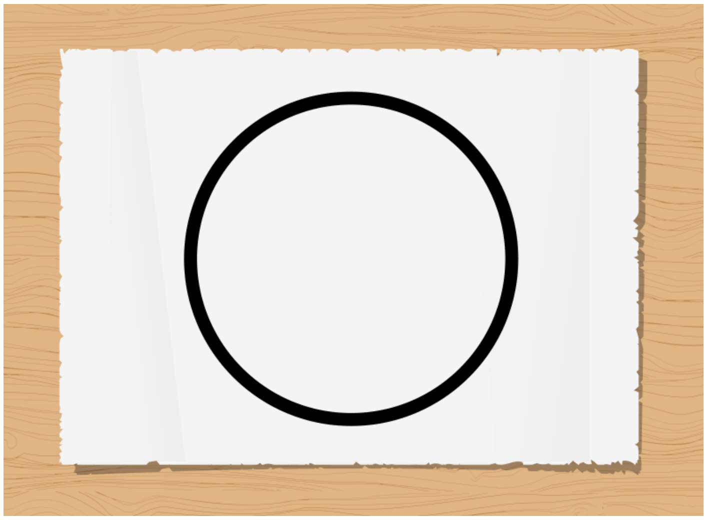
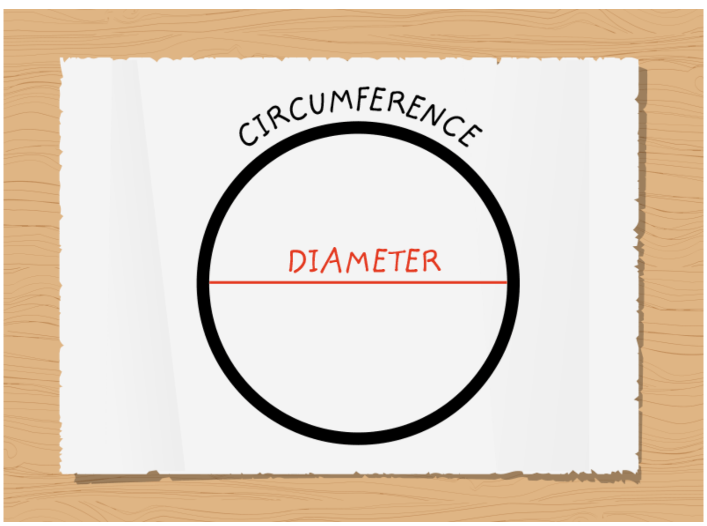
A ***circle*** is the set of points in a plane that lie a fixed distance from a given point in the plane.

Using a compass to draw a circle is a way of directly applying this definition. The "fixed distance" is the radius of the circle. A circle consists of only the curve itself. A circle together with its interior points is called a ***disc***.



Circumference and Diameter of a Circle

The ***circumference*** of a circle is the distance around the circle—it's essentially the perimeter of the circle. The ***diameter*** is the distance across the circle through its center.



The ratio of the circumference to the diameter is the same for every circle and is given by:

$\frac{C}{d} = \pi$, where $\pi \approx 3.14$

Another way to write this relationship is as the formula $C = \pi d$.

Thus, if you know the measurement for either the circumference or the diameter, you can determine the measurement of the other. For example, suppose you know that the diameter of a circle is 5 in. To find the circumference:

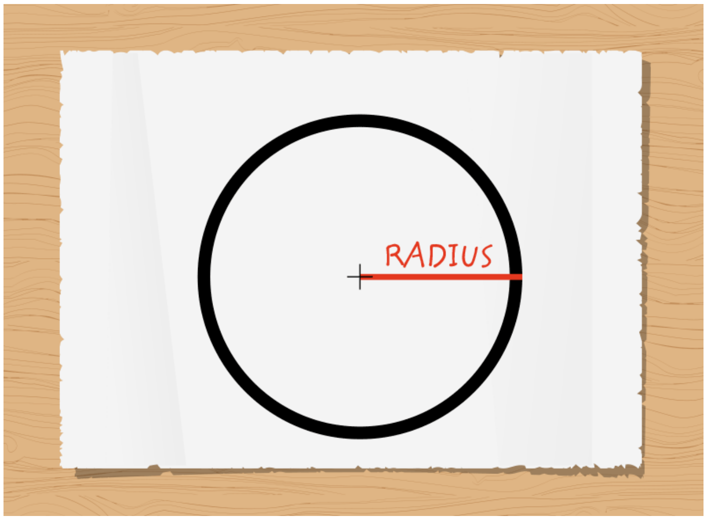
$C = \pi \cdot 5$
$C \approx 3.14 \cdot 5$
$C \approx 15.7$

The circumference is about 15.7 in.

The ***radius*** of a circle is a line that goes from the center of the circle to any point on its circumference. The radius is half the length of the diameter: $r = \frac{1}{2}d$ or $2r = d$.

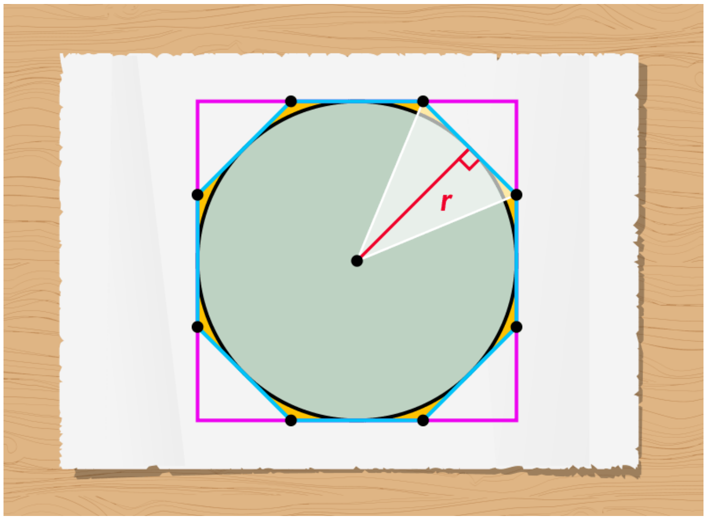
You can write the formula for circumference in terms of the radius as follows:

$$C = 2\pi r$$



Area of a Circle

Remember that you can find the area of a regular polygon by multiplying the apothem by half the perimeter. If you think of a circle as a regular polygon with many sides, you can come up with the correct formula for the area of a circle.



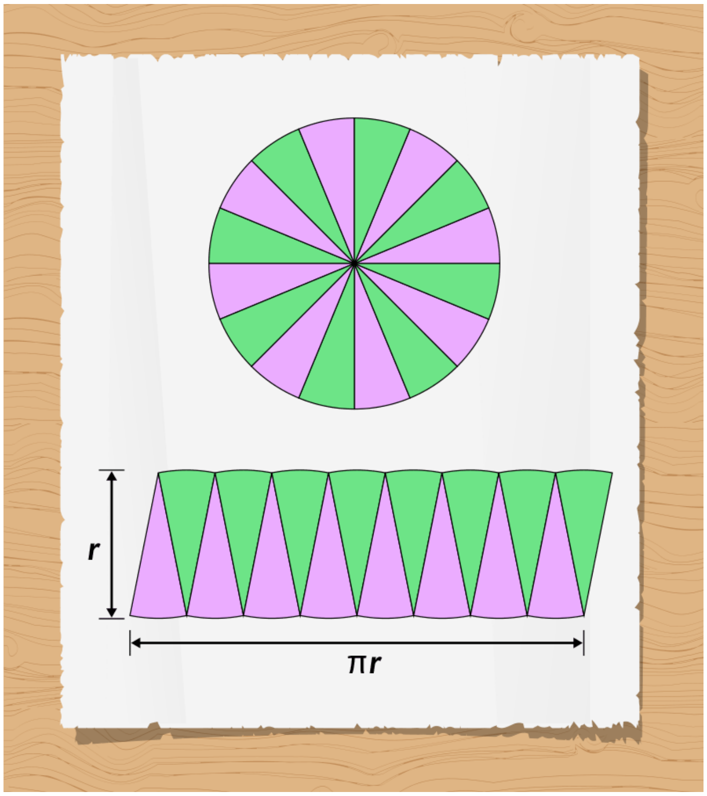
For a circle, the apothem is the radius, and the perimeter is the circumference. Thus, you can follow this series of steps to derive the formula for the area of a circle:

$$A = a\frac{a}{2} ---> A = r\frac{C}{2} ---> A = r\frac{\pi d}{2} ---> A = r\frac{\pi 2r}{2} ---> A = r\pi r = \pi r^2$$

So the formula for the area of a circle is $A = \pi r^2$.

It is interesting to look at another way that you might derive this area formula from the formula for the circumference of a circle.

The argument is based on dividing a circle into many equal sectors. Shown here is a circle with 16 sectors:



Rearrange the sectors in a row, cut the leftmost sector in two, put half of it on the right end of the row, and then enclose the row in a rectangle of height h and length l. Although $h > r$, $h \approx r$. Similarly, although $l < \pi r$, $l \approx \pi r$. This last approximation comes from the number π as it appears in $C = 2\pi r$.

When the circle is divided into many sectors, a very good approximation to the area of the rectangle is $hl \approx r(\pi r) = \pi r^2$.

This argument gives an informal proof of the formula $A = \pi r^2$. Moreover, it shows that the constant π in the *length* formula $C = (2\pi) \cdot r$ and the constant π in the *area* formula $A = \pi r^2$ are, in fact, the same number!