Surface Area and Volume

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Surface Area of 2-D Figures

Here is a brief review of some area formulas you may already know.

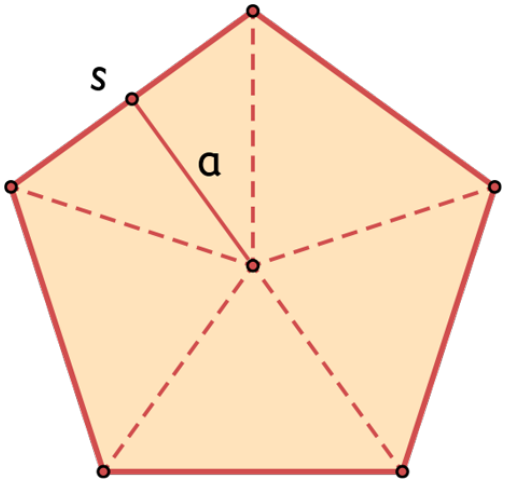
- The area of a rectangle or a parallelogram is length multiplied by width (or base multiplied by height):

$$A = lw \text{ or } A = bh$$

- The area of a triangle is the base multiplied by the height divided by 2 or one-half the product of the base and the height:

$$A = \frac{1}{2}bh$$

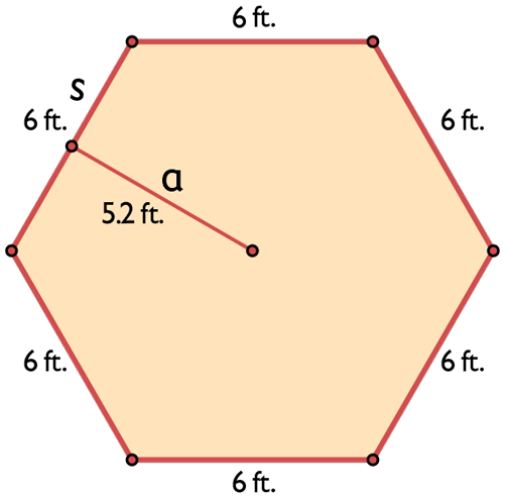
- The area of a polygon is generally found by breaking it into triangles. For example, to find the surface area of this regular pentagon, you could draw lines to create congruent triangles.



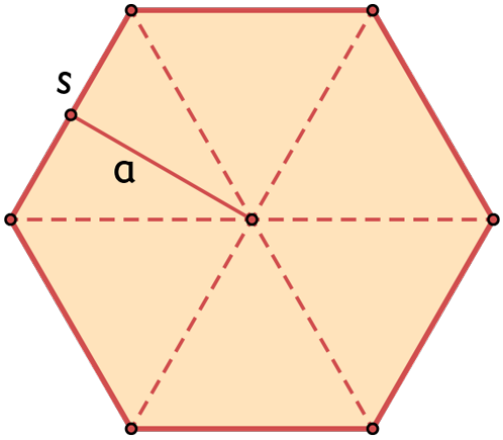
Notice the line drawn from the center of the figure to the midpoint of a side. This line is called an ***apothem***. In any regular polygon, the center is the same distance from the midpoint of any side.

Using these formulas, you can solve real-world problems. Consider this problem:

The school wants to carpet the media room shown. How many square feet of carpet will the school need?



To find the area of this regular hexagon, you can draw 6 congruent triangles:



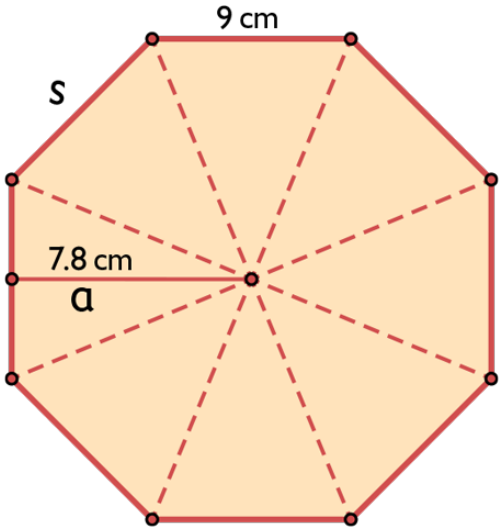
The base of each triangle is 6 ft, and the height of each triangle is about 5.2 ft. Using the formula for the area of a triangle, you have:

$$A = \frac{1}{2} \cdot 6 \cdot 5.2 \approx 15.6$$

$$\text{Area of hexagon: } 6 \cdot 15.6 \approx 93.6 \text{ sq ft}$$

Thus, the school will need about 93.6 sq ft of carpet.

There is an easier way to find the area of a regular polygon. You can multiply the apothem by half the perimeter. Consider this regular octagon:



Half the perimeter is:

$$\frac{1}{2}(8 \cdot 9) = \frac{1}{2}(72) = 36$$

Multiplying this by the apothem gives:

$$36 \cdot 7.8 \approx 280.8$$

Thus, the area of the octagon is about 280.8 sq cm.

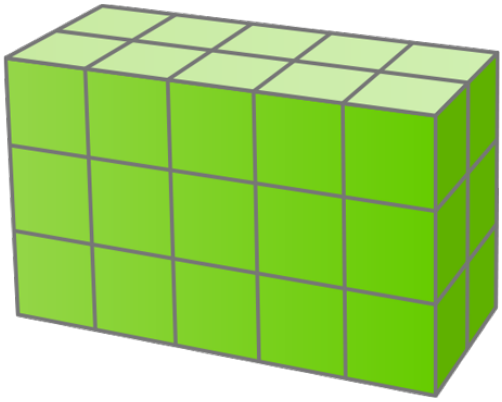
Volume

Volume measures how much space a figure takes up. It is measured in cubic units, such as cubic centimeters.

There is a simple formula for the volume of a rectangular prism:

$$v = l \cdot w \cdot h$$

To illustrate why this formula makes sense, consider the following prism.



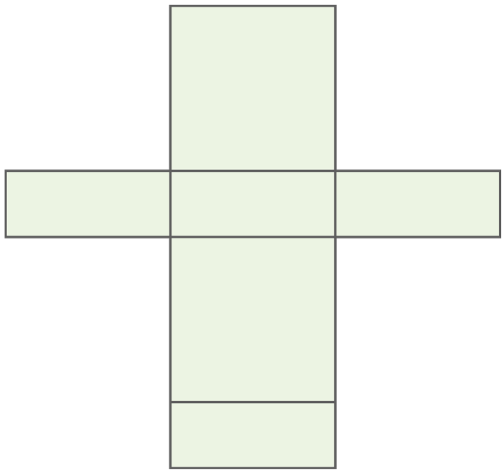
Note that the area of the base is the same as the volume of the first layer of the prism. This is the same as the number of 1 cm cubes in the bottom layer. The area of the base indicates the volume of one layer of the prism, while the height tells the number of layers.

This prism has a rectangular base, but the shape of the base doesn't matter. What matters is that the area of the base can be found. So the volume of a right prism is the area of the base ($B = lw$) multiplied by the height:

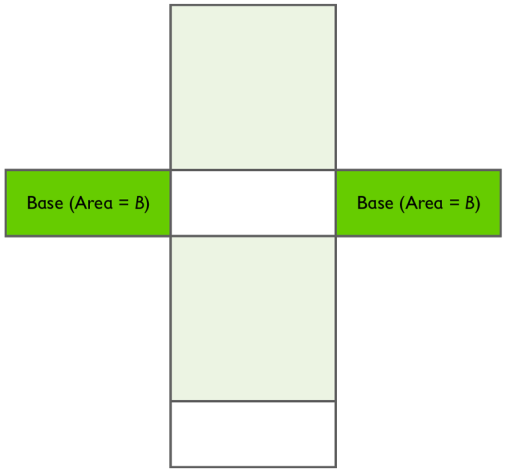
$$V = Bh$$

Surface Area of 3-D Figures

You can find the surface area of a rectangular prism by constructing a net for the prism. The surface area is the area of the net.



However, there is another way to consider the surface area of a prism. The top and bottom faces of a prism are the bases. The remaining faces are the lateral faces. Notice that if the lateral faces are opened out flat, they form one large rectangle. This rectangle is called the *lateral surface* of the prism.



Using this idea, the surface of any right prism with base area B and height h can be found using this formula:

$$SA = 2B + (\text{perimeter of the base})h$$