Line in a Plane

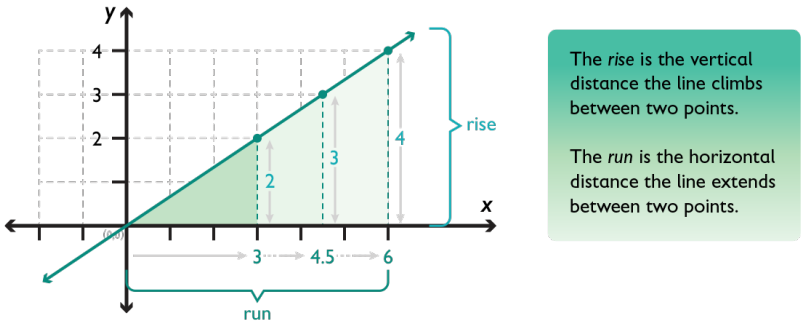
A Line in a Plane

What Is Slope?

Slope is a ratio.

Specifically, the ratio of rise to run of any portion of a straight line graph is defined as the ***slope*** of the graph.

$$slope = \frac{rise}{run}$$



You can represent the slope of a line as a fraction, a decimal, or a percent. For example, you represent the slope of the line in the previous graph in these ways:

Fraction: $\frac{2}{3}$

Decimal: ≈0.67

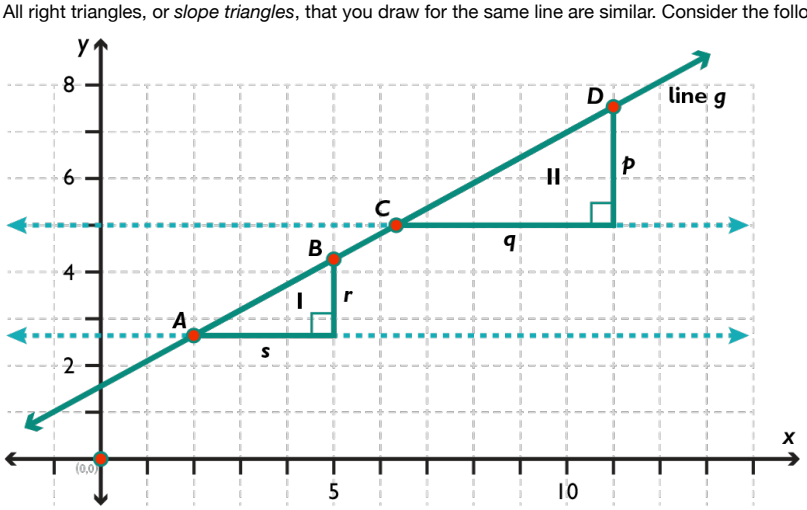
Percent: ≈67%

Slope is one way of measuring how "steep" a line is. Another way is by the angle the line makes with the horizontal axis. This angle is called the *angle of elevation*. For example, the angle of elevation of the previous graph is approximately 33.7°. The angle of elevation is not called a slope.

Slope Triangles for Any Line

You can find the slope of a line that intersects the *y*-axis at values other than 0. First, draw a right triangle under the line, and then look at the rise over the run.

All right triangles, or *slope triangles*, that you draw for the same line are similar. Consider the following diagram:



- The horizontal lines through points *A* and *C* are parallel. Angles *A* and *C* are alternate interior angles of the

transversal, line g. Therefore, angles A and C are equal.

- Two corresponding angles of Triangles I and II are equal: the right angles and angles A and C. By AA Similarity Theorem, the triangles are similar.
- Therefore, ratios of corresponding sides of Triangles I and II are equal.
- It follows that the slope ratios $p : q$ and $r : s$ are equal.

Formula for Slope

Slope is the ratio of vertical change (the rise shown on the y-axis) to horizontal change (the run shown on the x-axis).

The slope of a straight line is the same between any two points on the line. For any two points (x_1, y_1) and (x_2, y_2) on a line:

- The rise is the change in y-values between two points: $y_2 - y_1$.
- The run is the change in x-values between two points: $x_2 - x_1$.

$$\text{slope} = \frac{\text{rise}}{\text{run}} = \frac{\text{change in } y\text{-values}}{\text{change in } x\text{-values}} = \frac{y_2 - y_1}{x_2 - x_1}$$

Calculating Slope

There are several ways to calculate the slope of a line from the graph of the line on a coordinate plane.

Using a Formula

1. Choose any two points on the line, (x_1, y_1) and (x_2, y_2).
2. Then calculate the slope as $\frac{y_2 - y_1}{x_2 - x_1}$

Using a Right Triangle with Base Length of One Unit

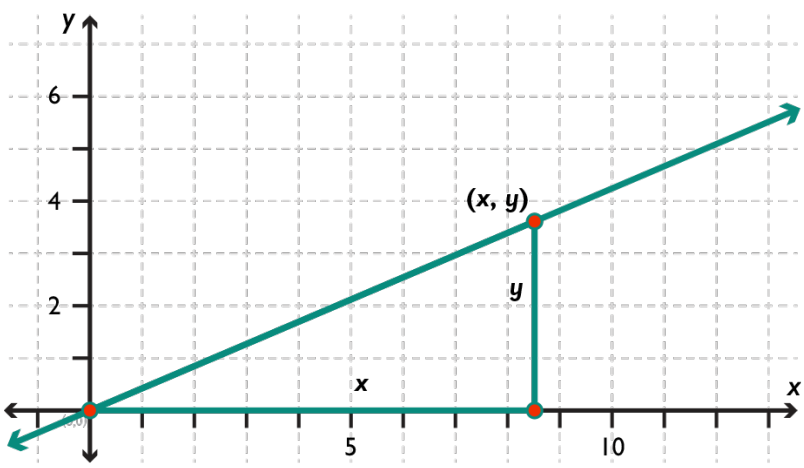
1. Choose one point on the line.
2. From your point, draw a line segment that extends one unit to the right (i.e., one step horizontally), and from there, draw a line segment that extends up or down until you reach the line. You will have drawn a right triangle with a hypotenuse that extends along the line.
3. Count the number of units you moved up or down to reach the line. That number is the slope of the line.

Using the Slope-Intercept Equation

Suppose $y = mx + b$ is the slope-intercept equation of the line. In this equation, the coefficient of x is the slope of the line: m = slope.

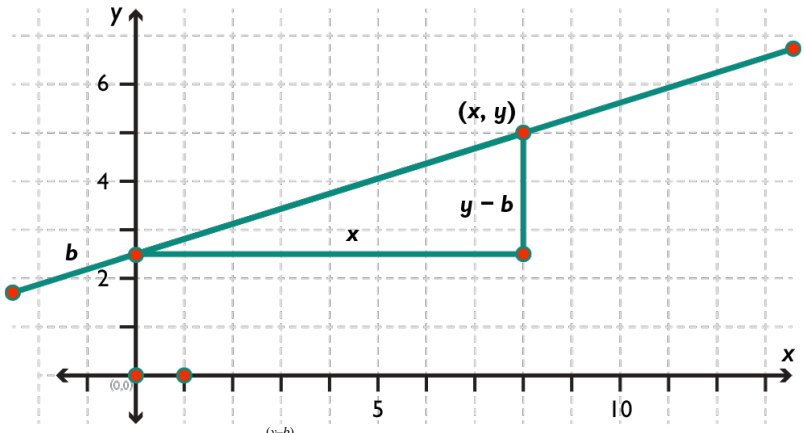
Deriving the Slope-Intercept Equation

Consider any line through the origin with slope m.



Let (x, y) be any point on this line. Since the slope is m, you have $\frac{y}{x} = m$. Therefore, $y = m \cdot x$.

Consider any line through the point $(0, b)$ with slope m.



Since the slope is m, you have $\frac{(y-b)}{x} = m$. Therefore, $y = mx + b$.

You can define every line in the coordinate plane by an equation. An equation is a statement of the relationship between x-values and y-values. For example:

$y = 2x + 1$

This equation states that y is equal to 1 more than 2 times x. The slope is 2. The y-intercept is 1.

In this equation, if $x = -3$, then $y = 2(-3) + 1 = -6 + 1 = -5$. The ordered pair $(-3, -5)$ satisfies this equation. The point with these coordinates lies on the line.

An (x, y)-table shows additional ordered pairs of x-values and y-values that satisfy the equation.

x	-3	-2	-1	0	1	2	3
y	-5	-3	-1	1	3	5	7

Each pair of (x, y)-values in this (x, y)-table gives the x-coordinate and y-coordinate of a point that lies on the line defined by the equation $y = 2x + 1$.

The slope-intercept equation of a line has the form: $y = mx + b$.

$$y = mx + b$$

slope $\quad$ y-intercept

The y-intercept of a line is the y-coordinate where the line crosses the y-axis.