Functions: Linear

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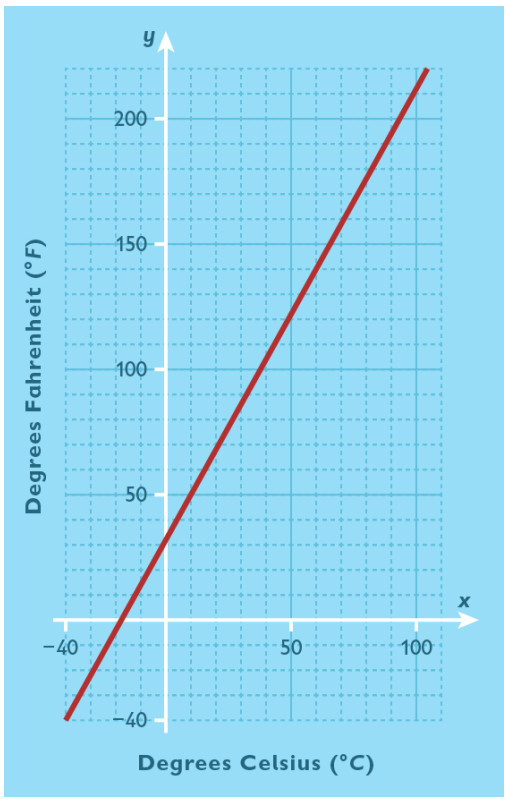
Linear Functions

A function with a defining formula (equation) of the form $y = mx + b$ is a ***linear function***. The graph of a linear function is a straight line with slope m. If the input x of such a function is increased by 1, the output y is increased by m.

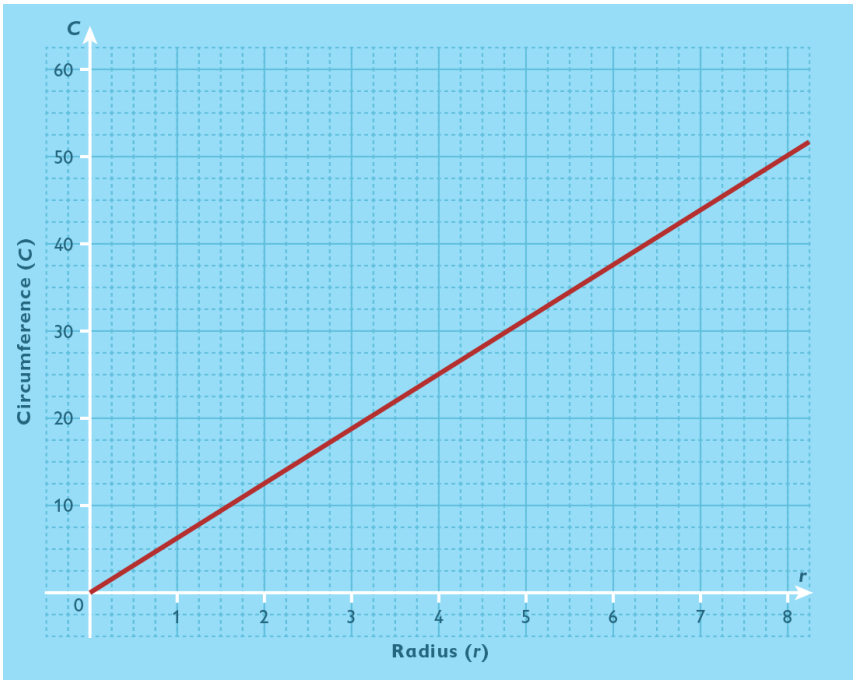
In any function, varying the input causes the output to *co-vary*. For example, consider this function:

$$F = \frac{9}{5}C + 32$$

Since $\frac{9}{5} \cdot (10) = 18$, if you increase any value of C by 10, then F is increased by 18. If you decrease any value of C by 10, then F is decreased by 18, and so on. In general, when you *vary* the input C, the output F *co-varies* in kind.

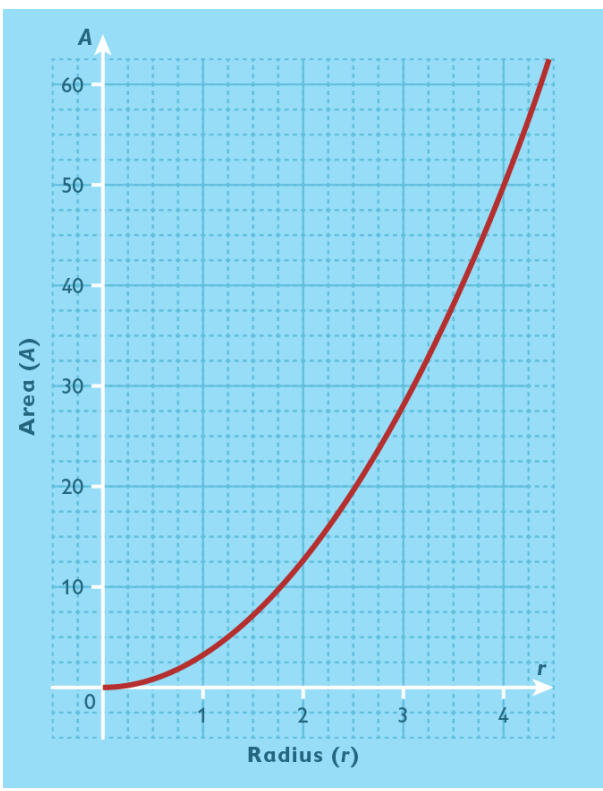


Another example is the linear function $C = 2\pi r$. This function gives the circumference C of a circle as a function of the radius r.



Nonlinear Functions

Not all functions are linear. For example, the equation $A = \pi r^2$ represents a ***nonlinear function*** that describes the relationship between the area A of a circle and its radius r. You can tell that the function is nonlinear because it includes a square term (r^2). In addition, the graph of a nonlinear function is not a straight line.



The $y = mx$ Family of Functions

The function $i = 12f$ represents the relationship between two variables, i and f, with 12 as a constant.

- Variable i is a length measured in inches.
- Variable f is the same length measured in feet.
- The 12 is the conversion rate, a constant with the unit inches per foot.

This function is one of a family of functions that are in the form $y = mx$, where:

- x and y are the variables.
- m is a constant, the constant rate.
- The unit for m is "unit of y per unit of x."

Here are some other members of this $y = mx$ family:

- $d = 5t$, where d is the number of miles traveled and t is the number of hours.
  The constant rate of 5 is the constant speed of 5 miles per hour.
- $c = 8.80w$, where c is the cost of meat in dollars and w is its weight in pounds.
  The constant rate of 8.80 is the unit cost in dollars per pound.
- $d = \frac{5}{4}e$, where d is the amount in U.S. dollars and e is the amount in European euros (at one time).
  The constant rate of exchange at that time was $\frac{5}{4}$ dollars per euro.

Every function in the family $y = mx$ of functions represents a *proportional relationship* between the variables y and x, where m is the constant of proportionality.

The $y = mx + b$ Family of Functions

Another common form for functions is $y = mx + b$. The relationship between Fahrenheit and Celsius temperatures uses this form:

$F = \frac{9}{5}C + 32$

This function is one of a family of functions that take the form $y = mx + b$, where:

- x and y are the variables.
- m is a constant, the constant rate of change.
- The unit for m is "unit of y per unit of x."
- The b is another constant with the same unit as y.

Another example of this $y = mx + n$ family of functions is $d = 5t + 2$, where:

- d is the number of miles traveled since the clock started.
- t is the number of hours since the clock started.
- The constant rate of 5 is the constant speed of 5 miles per hour.
- 2 miles have already been traveled when the clock started at $t = 0$.