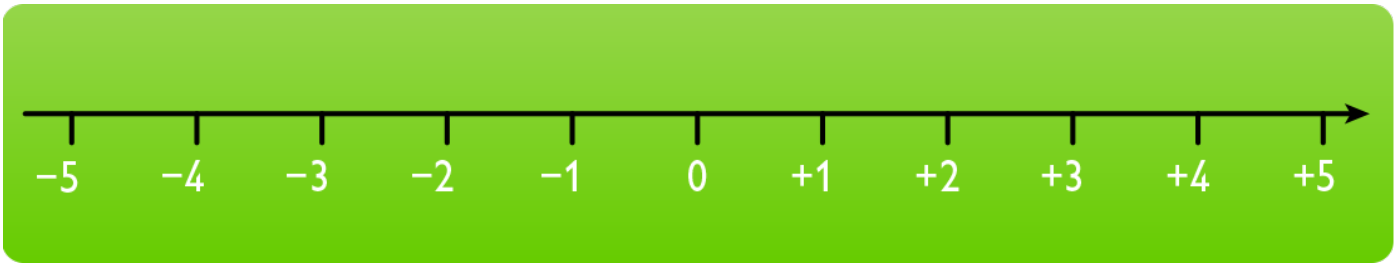
# Rational Numbers: Graphing: The Coordinate Plane

A special kind of plane used in mathematics is the *coordinate plane*, sometimes called the *Cartesian plane* after its inventor, René Descartes. It is one of the most useful tools in mathematics.

The coordinate plane makes it possible to represent relationships between numbers and variables geometrically as well as algebraically. It shows a way in which geometry corresponds to algebra.

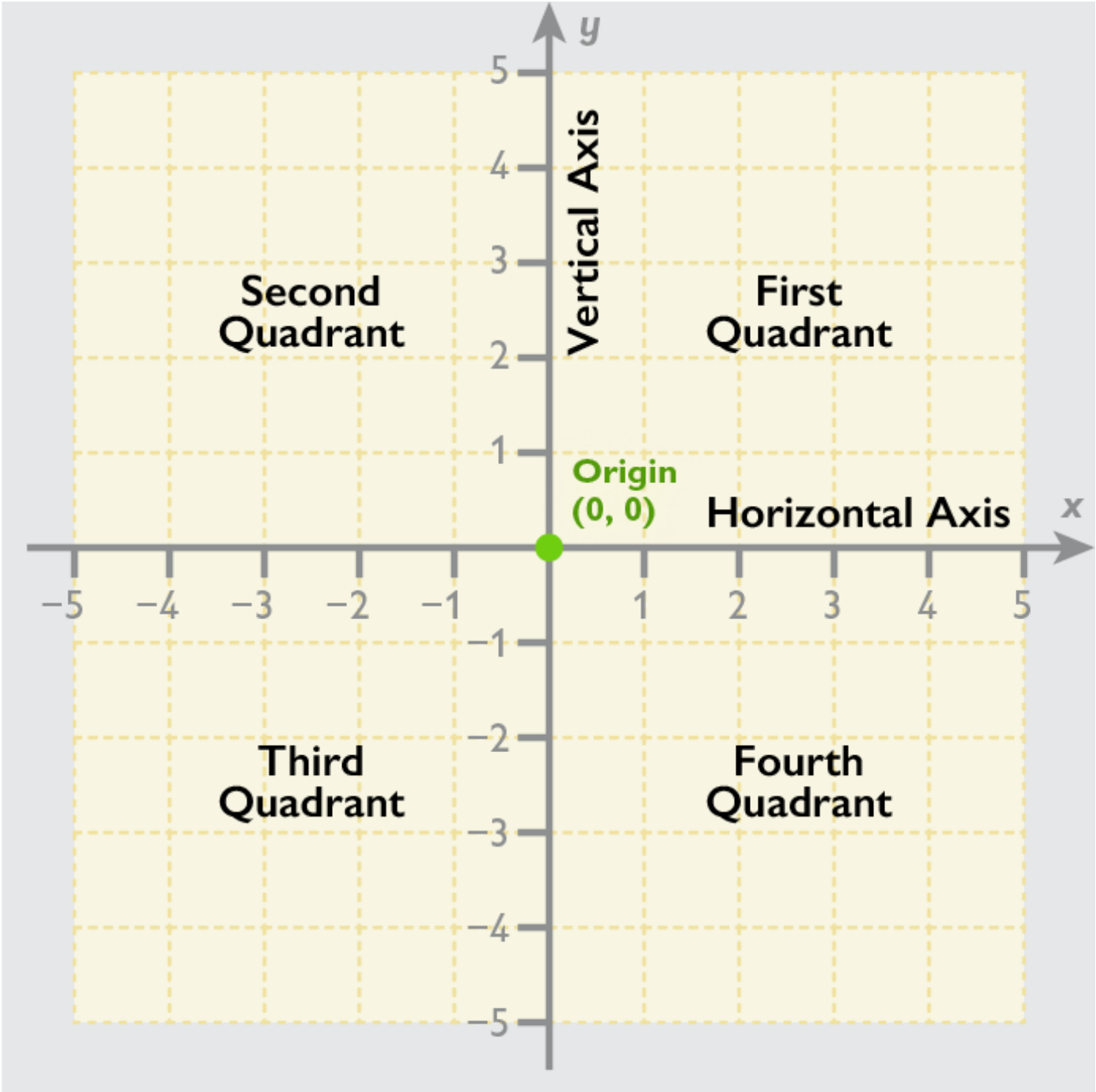
To construct a coordinate plane, draw a horizontal number line. The number line locates numbers along a single line like a ruler. Make sure you have marked 0 on your line. To the left of 0 are the negative numbers and to the right are the positive numbers.




Next, draw a vertical number line that intersects the horizontal line at the 0 point. The vertical line and the horizontal line should be perpendicular (at right angles) to each other.

The horizontal line is called the *x-axis*. The vertical line is called the *y-axis*.

On the $y$-axis, the negative numbers are below 0 and the positive numbers are above 0. The $x$– and $y$-axes intersect at only one point: the point at which $x = 0$ and $y = 0$. This point is called the *origin* of the coordinate plane.



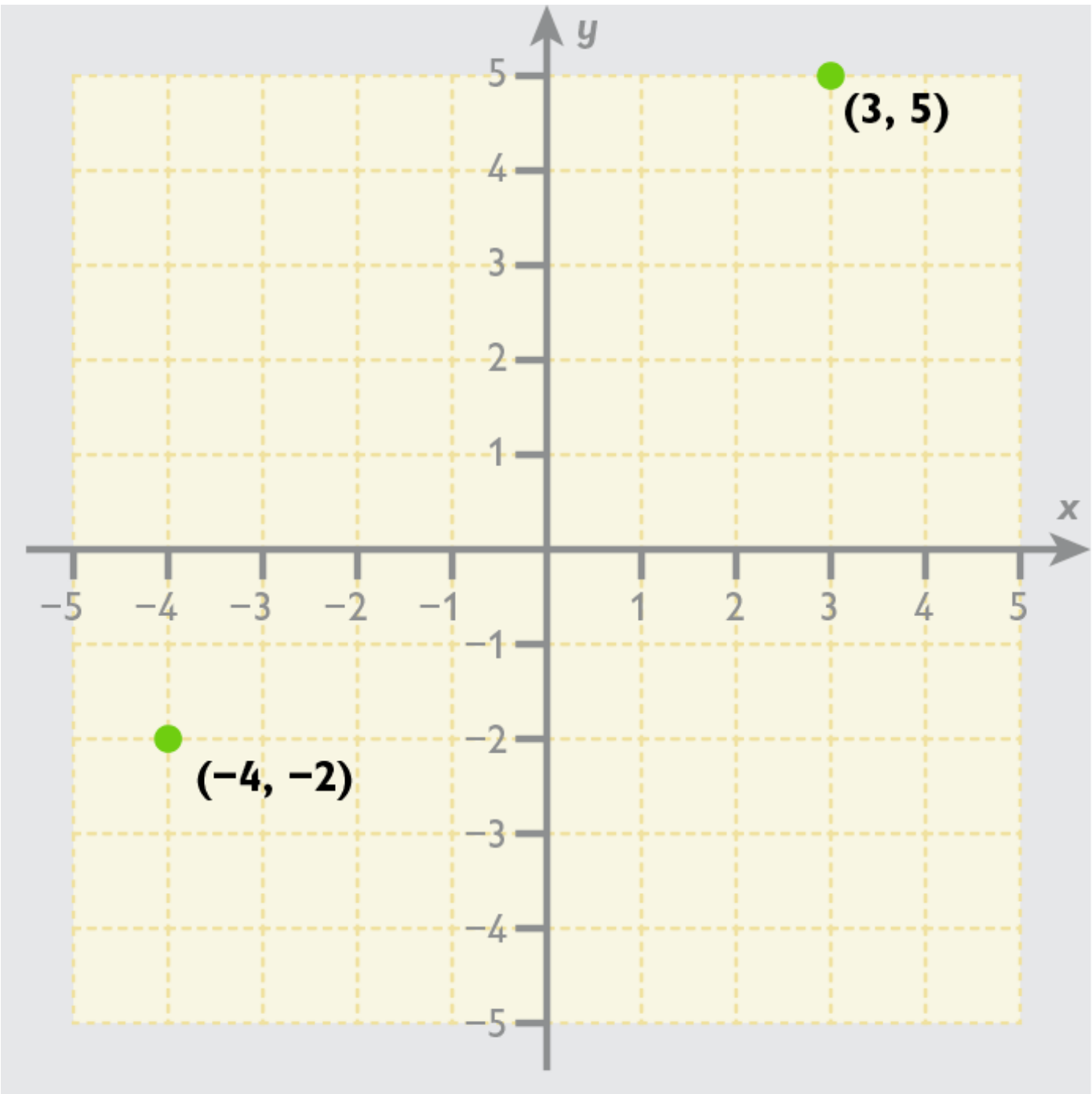

The two axes divide the plane into four parts, called *quadrants*.

- In the first quadrant, $x$ is positive and $y$ is positive.
- In the second quadrant, $x$ is negative and $y$ is positive.
- In the third quadrant, $x$ is negative and $y$ is negative.
- In the fourth quadrant, $x$ is positive and $y$ is negative.

## Plotting Points on the Coordinate Plane

The coordinate plane has two dimensions. In this case, the dimensions are named the *x-dimension* and the *y-dimension*. Every point (location) in the coordinate plane is identified by two numbers, the *x-coordinate* and the *y-coordinate* (also known as the *x-value* and the *y-value*). A point is identified by giving these two values. By convention, the $x$-coordinate is always given first, followed by a comma, a space, and then the $y$-coordinate: $(x, y)$. $(x, y)$ is called an *ordered pair*.

For example, look at this coordinate plane:



On this coordinate plane:

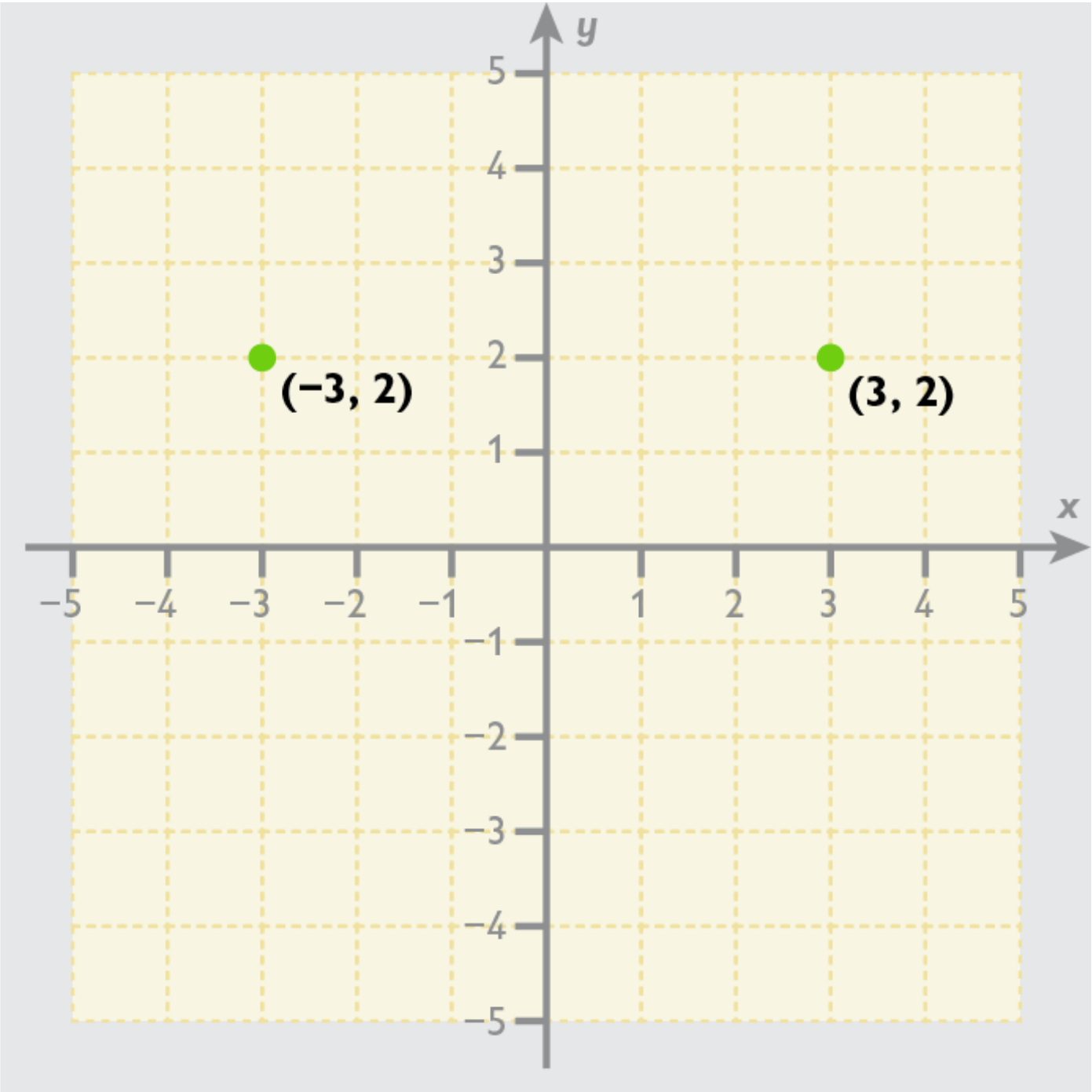
- (3, 5) is the point located at $x = 3$ and $y = 5$.
- 3 is the $x$-coordinate and 5 is the $y$-coordinate.
- (3, 5) is located in the first quadrant.
- (−4, −2) is the point located at $x = -4$ and $y = -2$.
- −4 is the $x$-coordinate and −2 is the $y$-coordinate.
- (−4, −2) is located in the third quadrant.

The graph of a point shows the relationship between its two coordinates. Since each point relates an $x$-value to a $y$-value, the most common use of coordinate graphs is showing the relationship between two quantities.

The signs of the numbers in ordered pairs indicate their location in the quadrants.

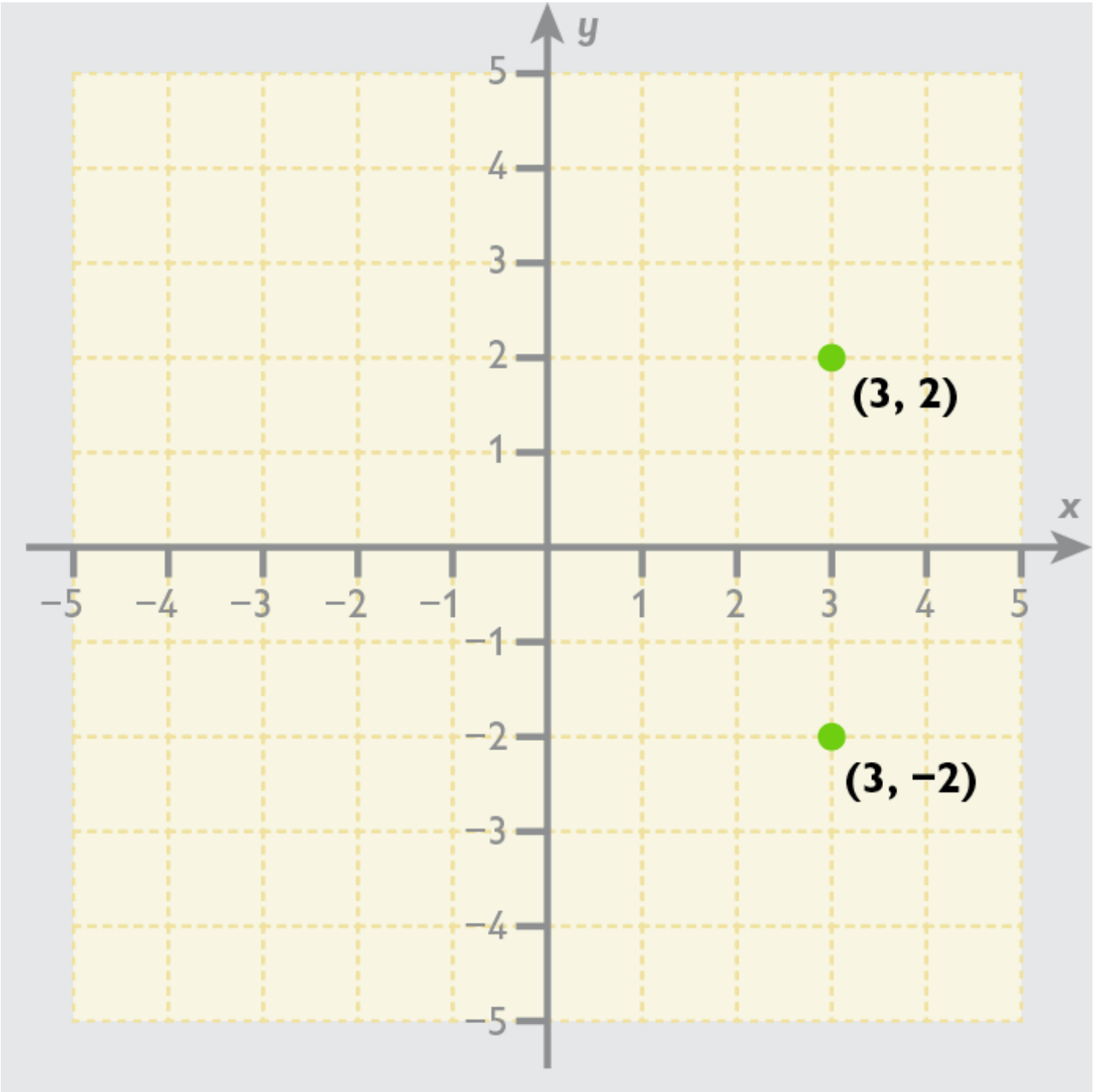
If the $x$-coordinates in two coordinate pairs differ only by the sign, the two points are reflections of each other over the $y$-axis.

For example, (3, 2) and (−3, 2) reflect over the $y$-axis:




If the $y$-coordinates in two coordinate pairs differ only by the sign, the two points are reflections of each other over the $x$-axis.

For example, (3, −2) and (3, 2) reflect over the $x$-axis:




In other words,

- $(m, n)$ and $(-m, n)$ are reflections of each other across the $y$-axis.
- $(m, n)$ and $(m, -n)$ are reflections of each other across the $x$-axis.
- $(m, n)$ and $(-m, -n)$ are reflections of each other across the origin (0, 0).

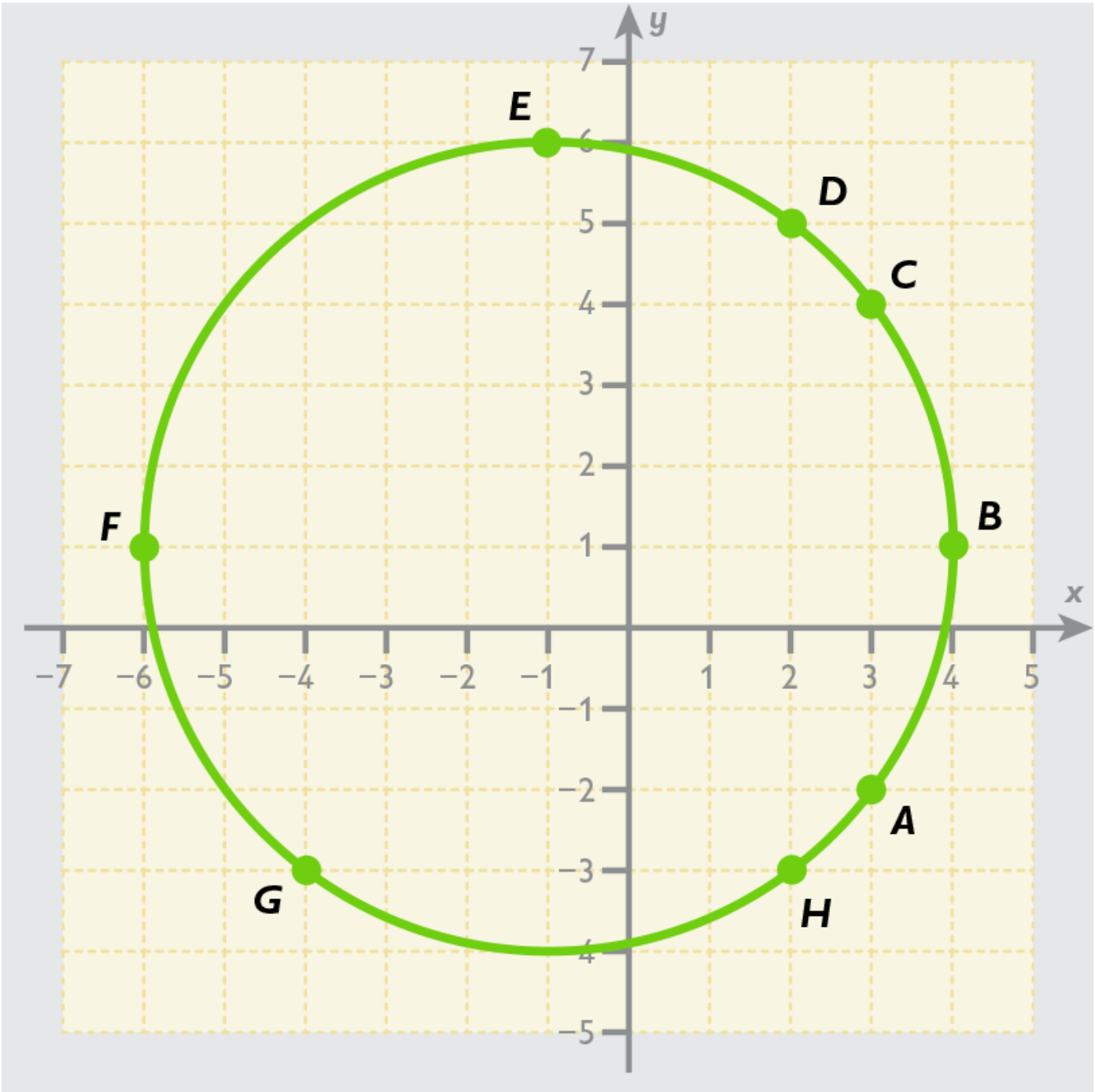
## Plotting Points From a Table

If you have a table that relates two variables, you can plot those variables on the coordinate plane.

For example, the following table gives corresponding values for variables $x$ and $y$.

| Points | A | B | C | D | E | F | G | H |
|---|---|---|---|---|---|---|---|---|
| $x$ | +3 | +4 | +3 | +2 | −1 | −6 | −4 | +2 |
| $y$ | −2 | +1 | +4 | +5 | +6 | +1 | −3 | −3 |

When you plot these points, the graph reveals the pattern in the relationship: all the points lie on a circle.



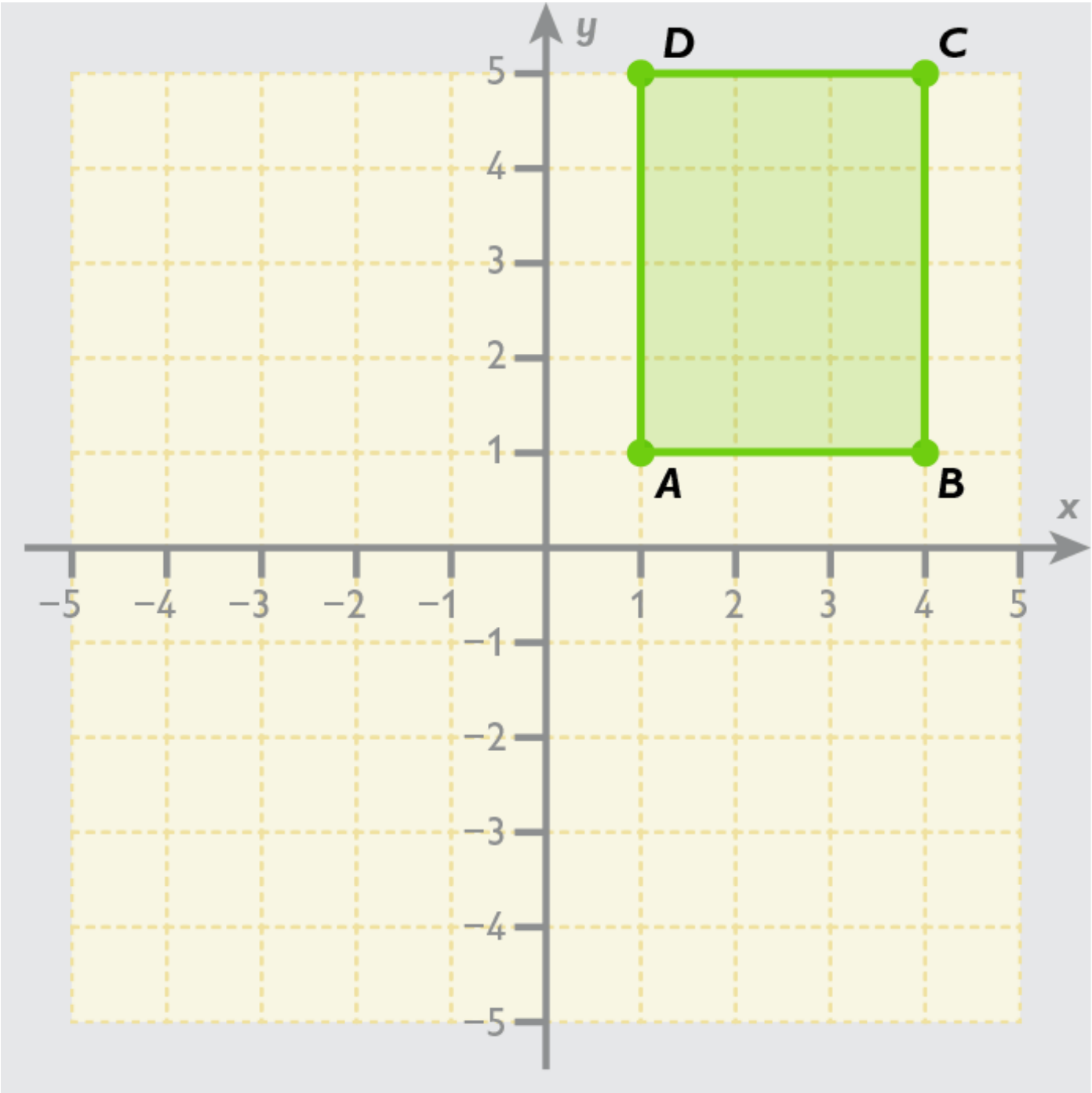

## Plotting Figures on the Coordinate Plane

In the previous example, the given coordinates created a figure on the coordinate plane—a circle. You can also draw polygons on the coordinate plane given coordinates for the vertices.

Suppose you were given the following coordinates of points *A*, *B*, *C*, and *D*:

*A* (1, 1) *B* (4, 1) *C* (4, 5) *D* (1, 5)

You could plot these four points on the coordinate plane to create a rectangle.




You can also find the distance between two points of the rectangle. Look at points *A* and *B*. The coordinates for these points are (1, 1) and (4, 1). Because the *y*-coordinates are the same, you can find the distance between the two points by finding the difference between the *x*-coordinates: $1 - 4 = -3$. Because distance must be positive, you take the absolute value of the result and get 3: $|-3| = 3$. Thus, the distance between points *A* and *B* is 3 units.

You can find the distance between the remaining points in this way:

- Distance between points $B$ and $C$: The coordinates are (4, 1) and (4, 5). The $x$-coordinates are the same, so you take the distance between the $y$-coordinates: $1 - 5 = -4$. Using the absolute value of the result, you get 4 units.
- Distance between points $C$ and $D$: Coordinates are (4, 5) and (1, 5). The $y$-coordinates are the same, so you take the distance between the $x$-coordinates: $4 - 1 = 3$. The distance is 3 units.
- Distance between points $D$ and $A$: Coordinates are (1, 5) and (1, 1). The $x$-coordinates are the same, so you take the distance between the $y$-coordinates: $5 - 1 = 4$. The distance is 4 units.

These results make sense when you think about the properties of a rectangle: opposite sides have the same length. The lengths from point $A$ to $B$ and from point $C$ to $D$, which create opposite sides, are the same: 3 units. The lengths from point $B$ to $C$ and from point $D$ to $A$, which create opposite sides, are the same: 4 units.

Note that you can only find distance between two points using this method if either the $x$-coordinates or the $y$-coordinates of the two points are the same. If both are different, you need to use a different method, which you will learn later.