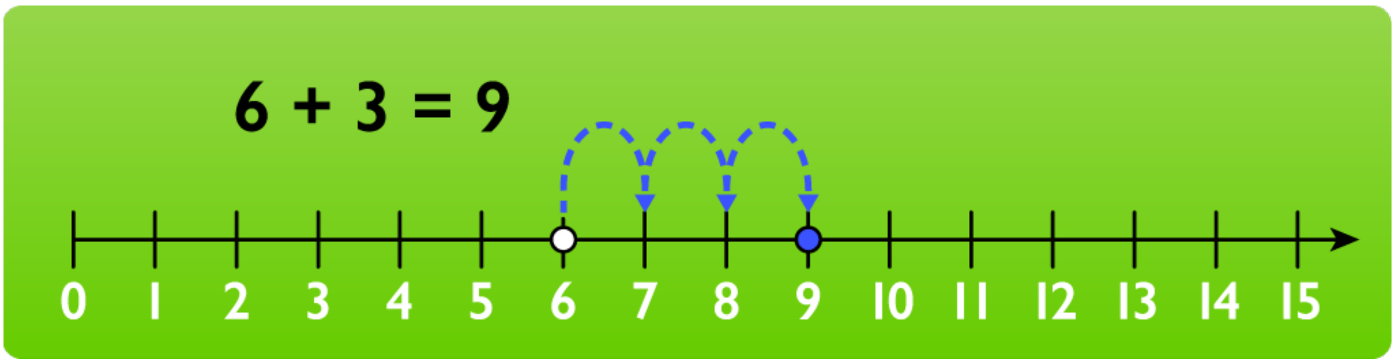
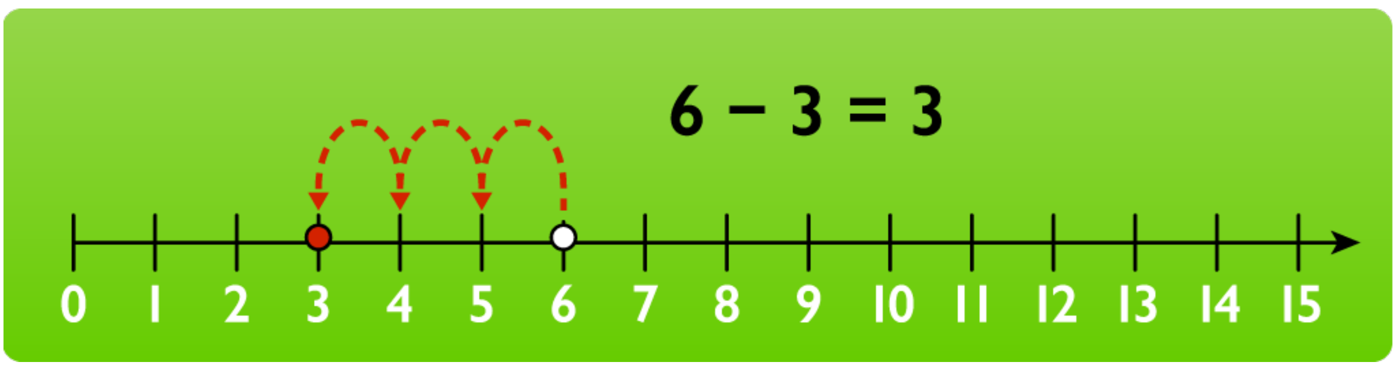
Rational Numbers: Negative Numbers

Extending the Number Line

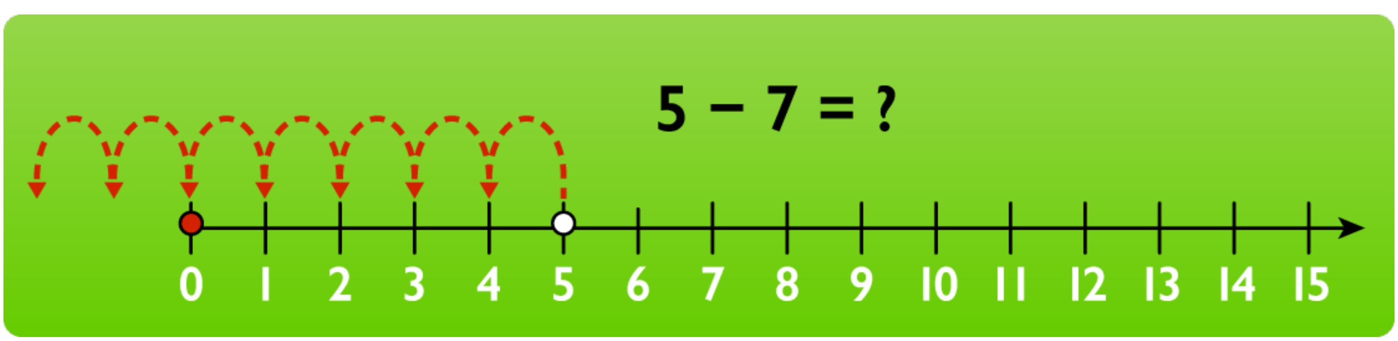
You know that to add on a horizontal number line, you move to the right. To subtract on a horizontal number line, you move to the left. To show 6 + 3 on a horizontal number line, start at 6 and move 3 units to the right to get 9.



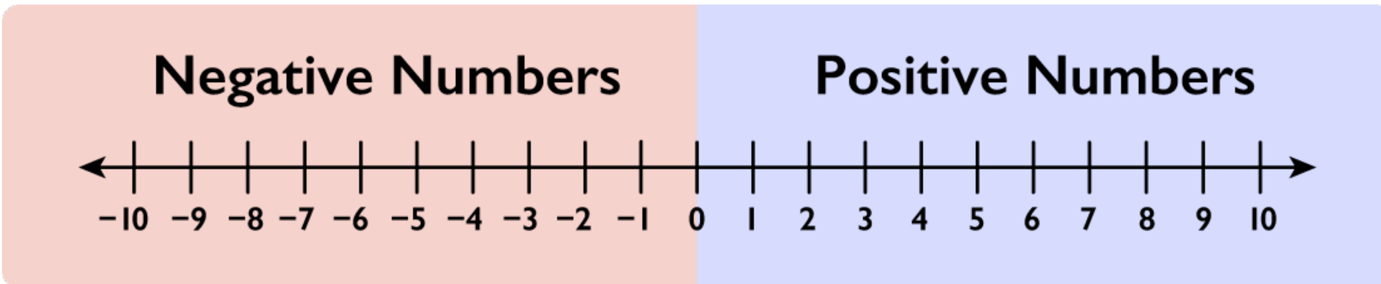
To subtract 3 from 6, start at 6 and move 3 units to the left to get 3.



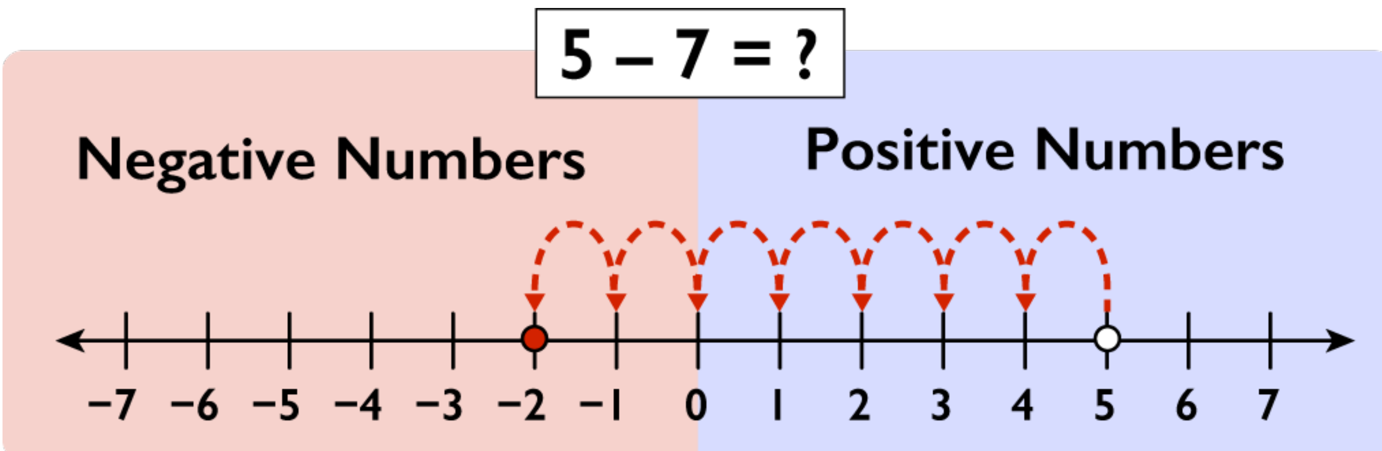
Now think about 5 – 7. At first glance, you might say that 5 – 7 = 0, because if you start at 5 on the horizontal number line and move 7 units to the left, you eventually get to 0, and that's as far as you can go.



What if you could go beyond 0? Would that make the subtraction 5 – 7 possible? Look at the expanded number line that includes the ***negative numbers***. All numbers to the right of 0 are positive numbers. All numbers to the left of 0 are negative numbers.



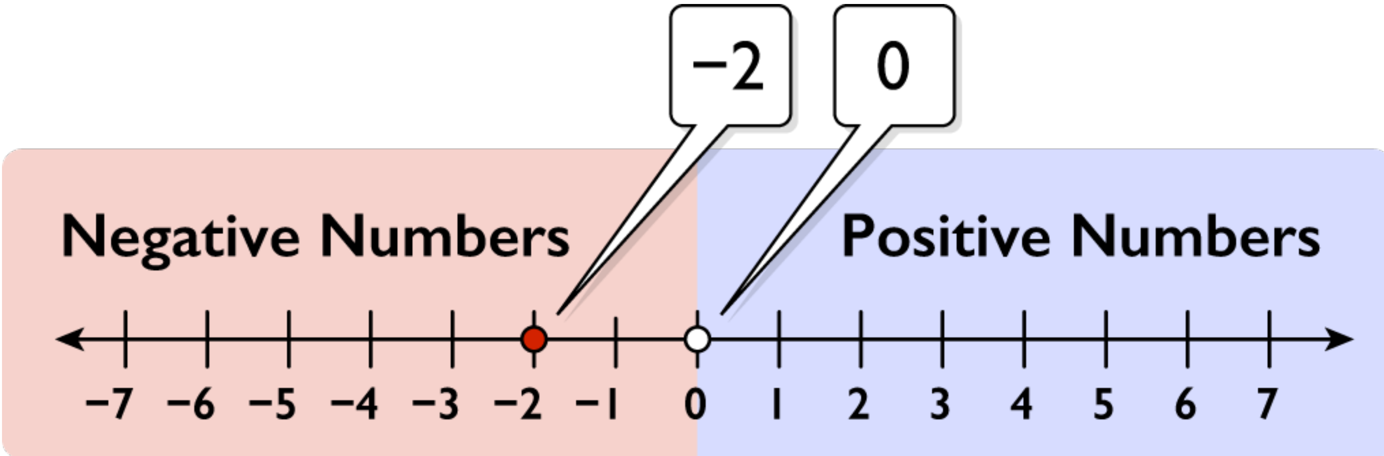
Using the expanded horizontal number line, you can subtract 7 from 5 by starting at 5 and moving 7 units to the left.



As shown, 5 – 7 is –2.

The Negative Scale

What does it mean for a number to be a negative number? Consider –2. It lies to the left of 0 on the horizontal number line. The number –2 also has a ***negative sign***. Both positive and negative numbers have signs. Positive numbers may be written with or without a positive "plus" sign (i.e., either 2 or +2); negative numbers always have a negative "minus" sign. (Sometimes –2 is read "minus two.")

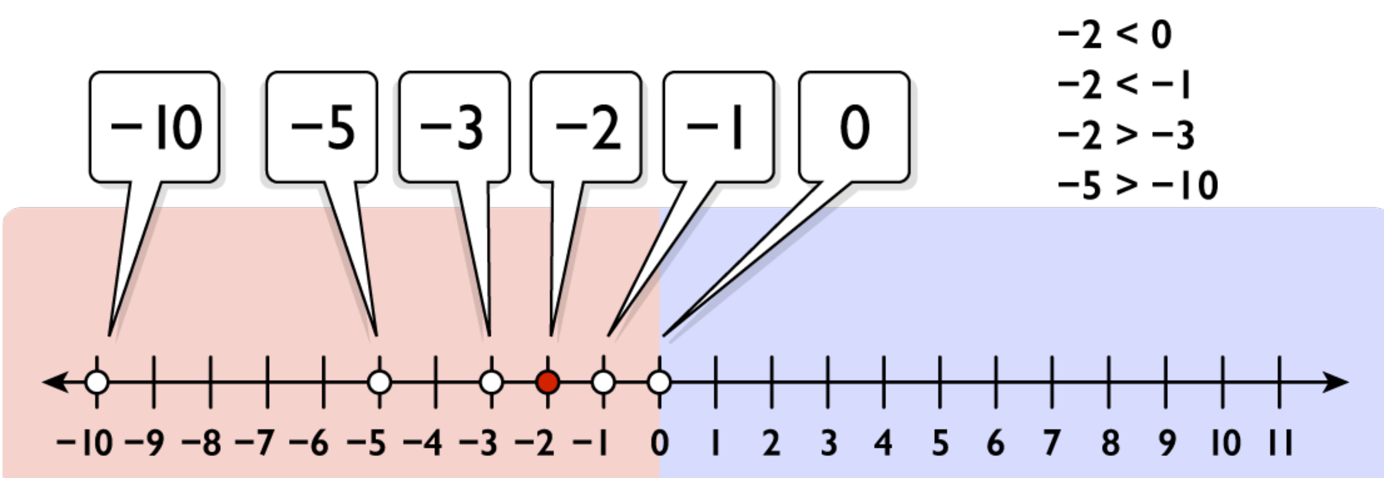


The number –2 is less than 0. Consider:

- For any two numbers on a horizontal number line, the number on the left is always *less than* the number on the right.
- For any two numbers on a horizontal number line, the number on the right is always *greater than* the number on the left.

The number –2 is less than 0 because it lies to the left of 0 on the number line. Because it lies to the right of –2 on the number line, 0 is greater than –2.

Is 0 a positive or a negative number? It is defined as a special number that is neither positive nor negative. It is the only number that takes neither a positive nor a negative sign.



You can see that –2 is to the left of –1, so it is less than –1. Also, –2 is greater than –3 because it is to the right of –3. In general, as you move to the left on the horizontal number line, the negative values get less and less. The value of

–10 is less than –5, while –100 is much less than either quantity.

Why Are Negative Numbers Necessary?

Negative numbers are necessary in mathematics for several reasons. You have already seen that to perform the subtraction 5 – 7, you need negative numbers. Without negative numbers, we would have a number system in which not every subtraction problem could be solved. With negative numbers, every subtraction problem has a solution, and thus our number system is ***closed*** under subtraction. (A set of numbers has ***closure*** under an operation if performing that operation on numbers of the set always produces a number of the same set.)

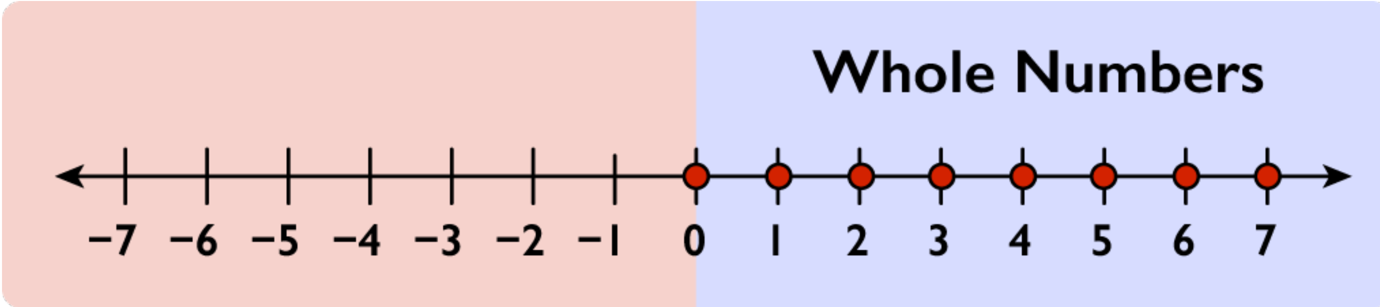
Having negative numbers is also important in many real-world applications. For example, think about temperature. Having negative numbers provides for numbers on both sides of a reference number 0, so that temperatures below 0 can be represented and evaluated.

Whole Numbers and Integers

You should already be familiar with ***whole numbers***. The whole numbers are the positive numbers to the right of 0 on a horizontal number line that don't have a fractional or decimal part. Consider these examples:

- Whole numbers: 2, 3, 5, and 17
- *Not* whole numbers: 2.4, 623, 48.7, –1, and –7

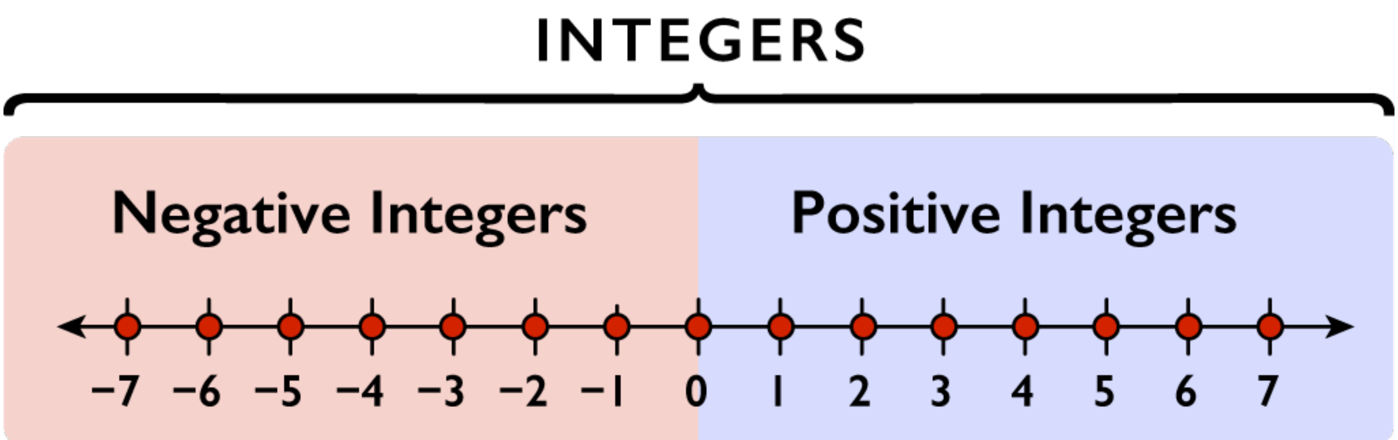
So, 2, 3, 5, and 17 are whole numbers because they do not have a fractional or decimal part. The numbers 2.4, 623, and 48.7 are *not* whole numbers because they include a fractional or decimal part. The numbers –1 and –7 are *not* whole numbers for a different reason—because they are negative.



Integers include all whole numbers and all negative numbers that don't have a fractional or a decimal part. The number 0 is also an integer. Consider these examples:

- Integers: +8, 5, –3, –44, and 1,000
- *Not* integers: 535, –2.8, and –3245

Integers include +8, 5, –3, –44, and 1,000 because they do not have a fractional or a decimal part. The numbers 535, –2.8, and –3245 are *not* integers because they have fractional or decimal parts.



Notice that all whole numbers are integers, but not all integers are whole numbers. For example, 13 is a whole number *and* an integer. On the other hand, –7 is an integer, but it is *not* a whole number because it is not positive.

Rational Numbers

Rational numbers are numbers that include a fractional or a decimal part, such as 4.71, –14, 712, 32, and –845. Notice that the whole number 32 and the negative integer –14 are both rational numbers. These numbers are considered rational because they can be expressed as fractions or decimals.

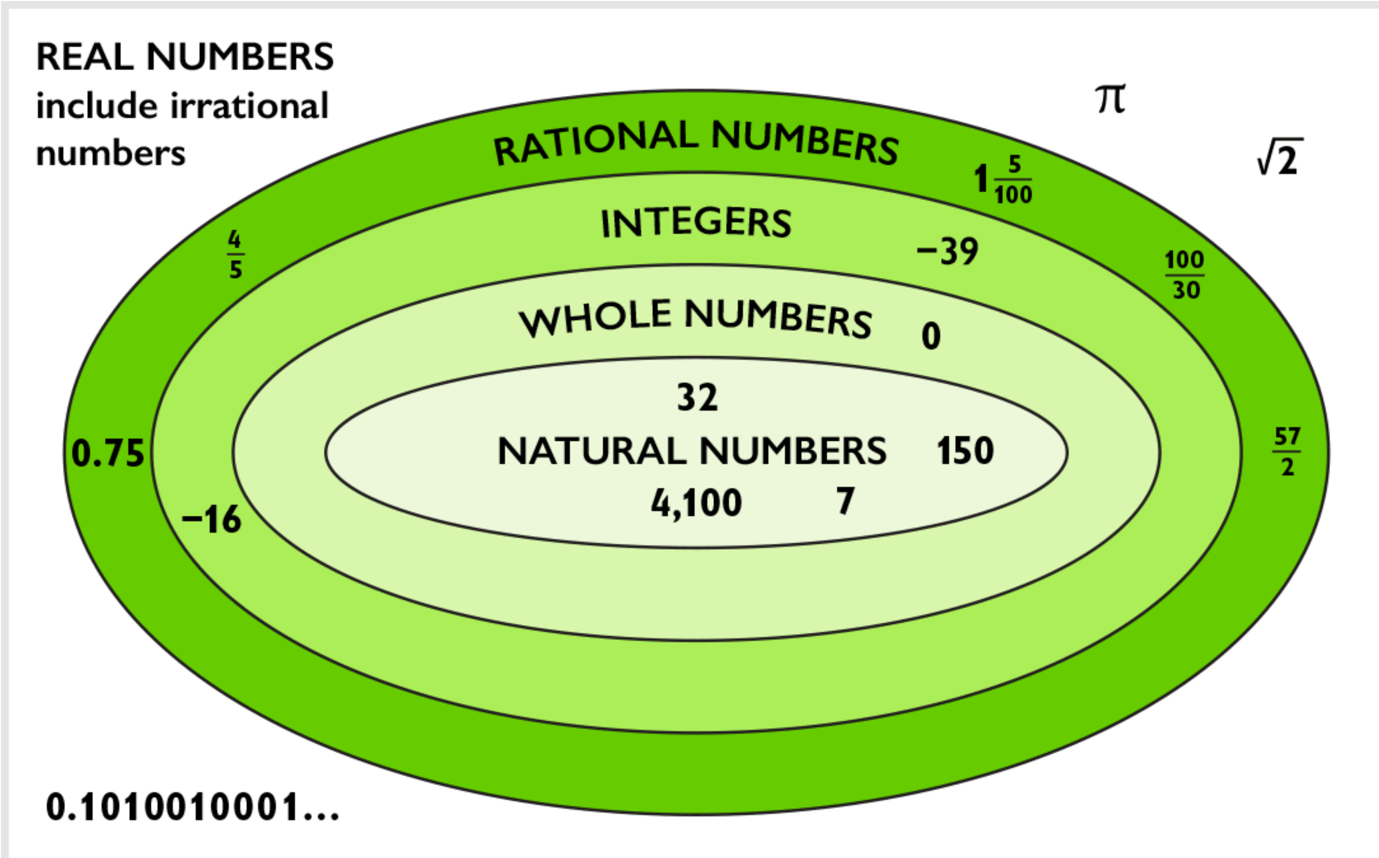
32=32·0100 or 32.00

–14=–14·0100 or –14.00

Having rational numbers in our number system is important; without them, not all division problems would have a solution. *With* rational numbers, all division problems have a solution, and thus our number system is ***closed*** under division.

Any number that can be expressed as a fraction (an integer divided by an integer) is a rational number. You might think that all numbers are rational since they can all be expressed as fractions. That is not the case. Some numbers, like π or 5‾√, cannot be expressed as fractions, so they are called ***irrational numbers***. (You will learn more about irrational numbers in later sections.)

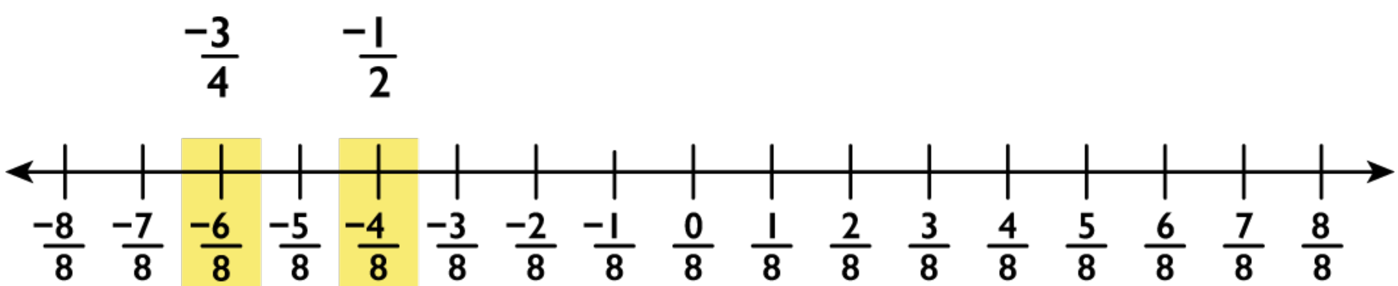
All of these number types are included in the set of ***real numbers***. Real numbers include the entire number line—both positive and negative whole numbers, integers, rational numbers, and irrational numbers.



Every number qualifies as a real number. However, some numbers fit only some of the categories, as you can see from the diagram.

Comparing Rational Numbers

Which number is greater, −12 or −34? To find out, use a number line. On the horizontal number line, −34 lies to the left of −12, so −12 is greater.

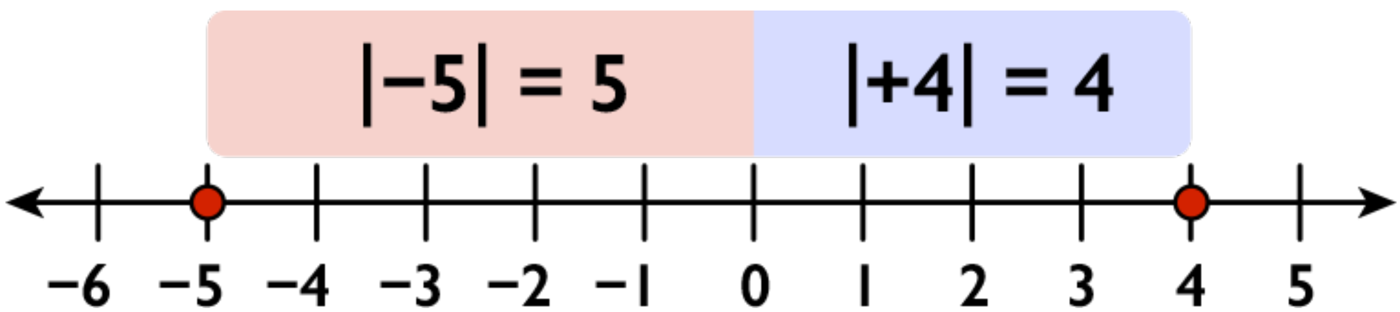


Negative decimal numbers work in the same way. The number –2.4 is less than –2.3 because –2.4 lies to the left of –2.3 on the horizontal number line. The number –12.03 is greater than –12.3 because –12.03 lies to the right of –12.3 on the horizontal number line. Remember:

- For any two numbers on a horizontal number line, the number on the left is always *less than* the number on the right.
- For any two numbers on a horizontal number line, the number on the right is always *greater than* the number on the left.

Absolute Value

You can measure the distance of a number from 0 on a number line. The distance from –5 to 0 is 5 units. The distance from 4 to 0 is 4 units.

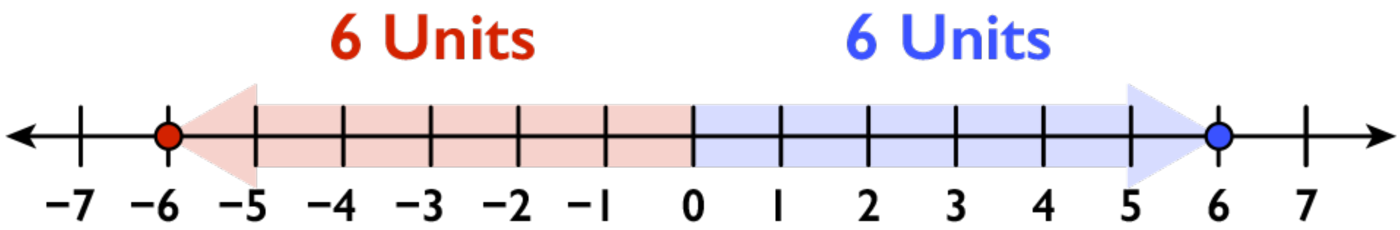


This distance is called ***absolute value*,** and it is shown by placing straight brackets around a number. The absolute value of –5 is written |–5|, and it is equal to a positive number, +5. The absolute value of +4 is written |+4|, and it is equal to a positive number, +4. Absolute value is always positive. It shows the distance from 0 that a number represents. Consider these examples:

|−3|=+3	||+423||=+423
||−6.4||=+6.4	||−59||=+59
|−0.0048|=+0.0048	|−n|=+n

A Number and its Opposite

A number and its opposite are two numbers that are the same distance from 0 on a number line but on opposite sides of 0.



The numbers –6 and +6 are opposite numbers. They are both 6 units from 0 and are on opposite sides of 0 on the number line.

Every number has an opposite number.

Opposite of 12: –12	Opposite of –812:812
Opposite of –36.4: 36.4	Opposite of 0.047: –0.047

A number and its opposite have the same absolute value.

|–6| = |+6| = +6

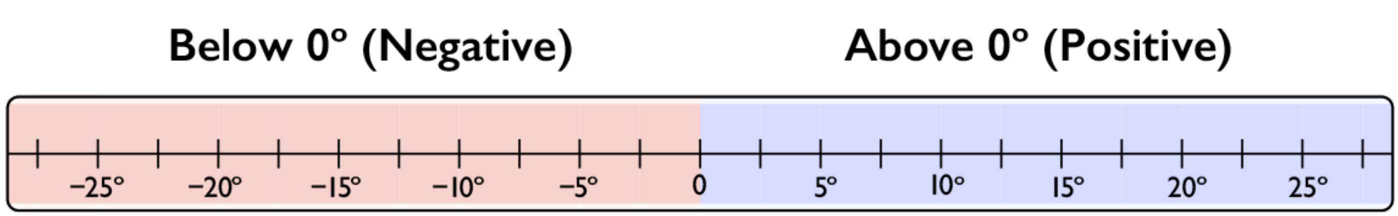
|–12| = |+12| = +12

|–36.4| = |+36.4| = +36.4

||−812||=||+812||=+812

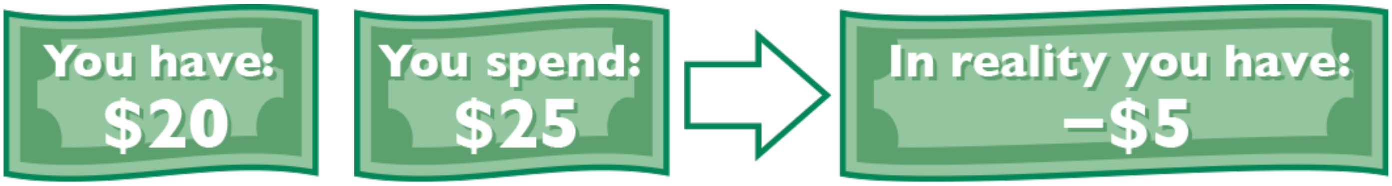
Negative Numbers in the Real World

Negative numbers are useful for expressing a variety of different quantities. You are familiar with the temperature scales, such as Celsius. Temperatures above freezing on the Celsius scale are positive. Temperatures below freezing, or "below zero," are negative.



Another example of the use of negative numbers is a bill you can't pay.

Suppose you are buying dinner, and you have \$20. Unfortunately, your bill comes to \$25. One way of describing your finances is that you have –\$5. That is, you owe the restaurant \$5 that you don't have.



Still another way to use negative numbers is to describe the actions of a moving object. A bird flies upward +40 ft in the air. Then the bird dives below the surface of the water to –20 ft to catch a fish.