

# Rational Numbers: Fraction Division

Dividing a fraction by another fraction involves the same concepts and tools that you use to divide a whole number by a fraction.

## Area Model

To help you understand the process, it is helpful to look at an area model.

You know that to find the area of a rectangle, you can use the multiplication  $A = lw$ , where  $A$  = area,  $l$  = length, and  $w$  = width. This multiplication is equivalent to this division:

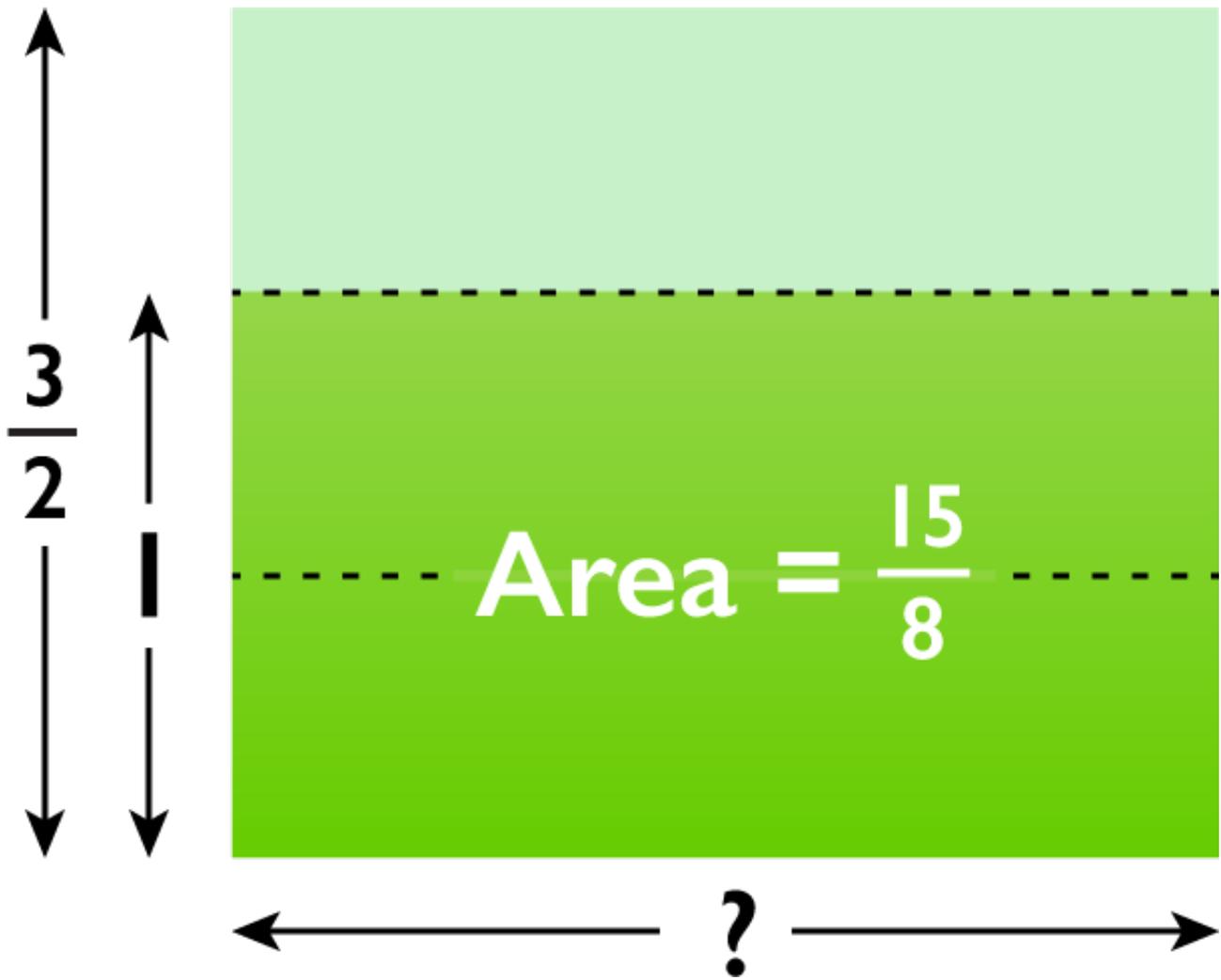
$$l = \frac{A}{w}$$

Suppose you know that the area of a large rectangle is 158 square units and the width is 32 units. Then:

$$\text{length} = \frac{\text{area}}{\text{width}} = 158 \div 32$$

To find the length, you have to divide the fraction 158 by the fraction 32.

Look at this area model:



The length of the side you want to find is marked with a question mark.

The area of the shaded rectangle is 23 the area of the large rectangle:  $158 \cdot 23$ . The width of the shaded rectangle is 1. Because the length of the shaded rectangle is the same as the length of the large rectangle, you have:

$\text{length} = \text{areawidth} = [158 \cdot 23] \div 1 = 158 \div 32$  (because 1 is the identity element)

This shows that dividing by 32 is equivalent to multiplying by 23. You can now apply this to the original problem:

$\text{length} = \text{areawidth} = 158 \div 32 = 158 \cdot 23 = 3024 = 54$

**Multiplying by a Fraction That Makes the Denominator Equal**

**to 1**

Another way to think about dividing by a fraction is to multiply the numerator and denominator by a fraction that makes the denominator equal to 1:

$$158 \div \frac{3}{2} = \frac{158}{\frac{3}{2}} = \frac{158 \cdot 2}{3 \cdot 2} = \frac{158 \cdot 2}{3 \cdot 1} = \frac{158 \cdot 2}{3}$$

So, you can see that dividing by  $\frac{3}{2}$  is the same as multiplying by  $\frac{2}{3}$ .

## **Dividing by $n$**

Another approach is to think about what dividing by  $n$  means. Dividing by  $n$  is equivalent to multiplying by the reciprocal of  $n$ . For example:

$$158 = 15 \cdot 18$$

The reciprocal of 8 is  $\frac{1}{8}$ .

This works when  $n$  is a fraction, too:

$$158 \div \frac{3}{2} = \frac{158}{\frac{3}{2}} = \frac{158 \cdot 2}{3}$$

## **General Rule for Dividing a Fraction by a Fraction**

From these examples, you can see that the general rule for dividing a fraction by a fraction is the same as dividing a whole number by a fraction: *To divide a fraction by a fraction, multiply by the reciprocal of the fraction.* You can write this rule as follows:

$$a \div \frac{b}{c} = a \cdot \frac{c}{b} = \frac{a \cdot c}{b}$$

## **Applying the Rule**

You can use the rule to solve a wide variety of problems.

For example, suppose you have 23c of juice. You want to know how many 16c servings you can get from the juice. To find this out, you need to divide:

$$23 \div 16 = ?$$

Applying the rule, you get:

$$23 \div 16 = 23 \cdot 61 = 2 \cdot 63 \cdot 1 = 123 = 4$$

Thus, from 23 c of juice, the number of 16 c servings you can get is 4.