Rational Numbers: Operations with Positive Numbers

Adding and Subtracting Decimals

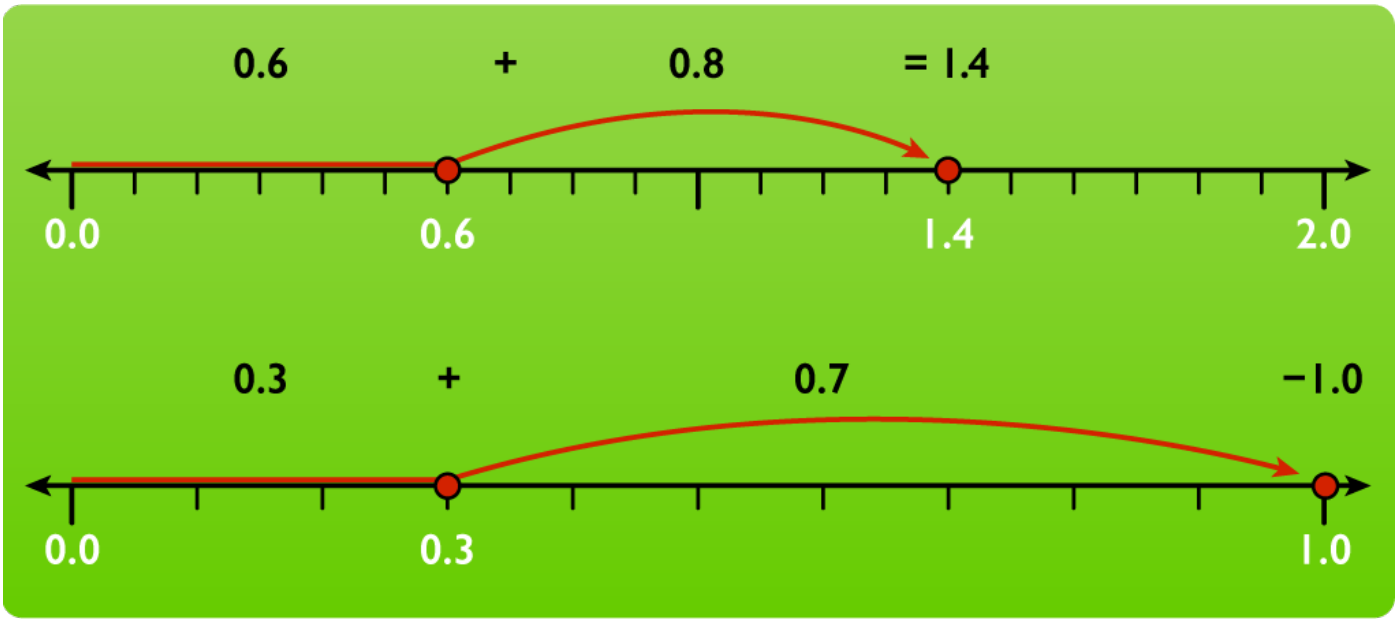
All the concepts of addition and subtraction apply to decimals, because decimal numbers are a continuation of the base-10 number system. The rules of arithmetic work for addition and subtraction of decimals, just as they do for whole numbers.

- The addition facts can be used.

For example, consider these equations:

0.6 + 0.8 = 1.4 and 0.3 + 0.7 = 1.0

You know the addition facts 6 + 8 = 14 and 3 + 7 = 10. The number line also shows the additions:



- Adding decimals in any order gives the same total. As with all numbers, adding decimals is commutative. For example: 0.6 + 0.1 = 0.1 + 0.6 = 0.7
- Addition and subtraction of decimals are inverse operations. For

example, since 0.4 + 0.7 = 1.1, then 1.1 – 0.7 = 0.4 and 1.1 – 0.4 = 0.7.

- Adding 0 to a decimal number does not change its value. For example, since 0 + 0.4 = 0.4, then 0.0 + 0.4 = 0.4.
- You add and subtract decimals the same way that you add and subtract whole numbers. Just as with whole numbers, you need to make sure the places—in this case the decimal places—are lined up correctly.

For example, 1.89 + 0.37 can be calculated by regrouping using the standard method for addition:

	ones	tenths	hundredths
	1	1	
	1.	8	9
+	0.	3	7
	2.	2	6

The expression 4.56 – 1.99 can be calculated by regrouping using the standard method for subtraction:

	ones	tenths	hundredths
	3	14	
	~~4~~.	~~5~~	16
−	1.	9	9
	2.	5	7

These examples show addition and subtraction of decimals with the same number of digits and with the decimal point in the same place. To add or subtract decimals that don't have the same number of digits, you need to line up the digits according to their value. You also must answer this crucial question: "Where should I place the decimal point?" There are three important steps when adding or subtracting decimals:

- Line up the digits according to place value.
- Line up the decimal points.
- "Carry" or regroup numbers from one place/column to the next, just as you do when adding or subtracting whole numbers.

Sometimes, both numbers have the same number of decimal places.

For example, consider the addition and subtraction problems that follow:

In the addition problem, the sum of 8 and 9 in the tenths column is 17. The 17 must be regrouped as 1 one and 7 tenths. In the subtraction problem, one of the 3 ones must be regrouped as 10 tenths to give 18 – 9 = 9 in the tenths column.

Sometimes, the two numbers have different amounts of decimal places. By adding zeros at the end of one of the numbers, you can get both numbers to have the same amount of decimal places.

For example, if you have 23.84 and 0.494, you can write the 23.84 as 23.840. Adding the zero will help you line up the digits, but will not change the value:

Multiplying Decimals

You can multiply decimals using almost the same methods that you use to multiply whole numbers. The only extra step is to locate the position of the decimal point in the answer.

Consider this multiplication:

49.32 • 6.85

Start with an estimate. For this example, a good estimate is 50 • 7 = 350.

Complete the multiplication as with whole numbers; ignore the decimal points. Do not worry about lining up the decimal points!

			5	1	1	
			7	2	1	
			4	1	1	
			4	9	. 3	2
×				6	. 8	5
		2	4	6	6	0
	3	9	4	5	6	0
2	9	5	9	2	0	0
3	3	7	8	4	2	0

The result is 4,932 • 685 = 3,378,420—a number that has the same digits as the answer you want.

The important next step is to determine where to put the decimal point. One way to do this is to use the estimate. Since the estimate was 350, the exact answer must be 337.8420.

Another way to determine where to put the decimal point is to convert the decimals to fractions. For example:

49.32 has two decimal places: 49.32 = 4,932100

6.85 has two decimal places: 6.85 = 685100

49.32 • 6.85 = 4,932100·685100=4,932·685100·100=3,378,42010,000

Divide 3,378,420 by 10,000.

The result has four digits to the right of the decimal point because there are four zeros in 10,000. The answer is 337.8420.

Check that this answer is close to the original estimate of 350. Yes, it is.

In general:

- Multiply decimals using the same methods that you use to multiply whole numbers. Ignore the decimal points, and do not line up the decimal points.
- When you are done multiplying, count the total number of decimal places in each of the numbers multiplied, move left that many places from the right end, and put the decimal point in the product.

For example, in 49.32 • 6.85, the number 49.32 has two decimal places and 6.85 has two decimal places. In the product, the total number of decimal places you move to the left is four: 3,378,420 becomes 337.8420.

Dividing Decimals

You can divide a decimal by a whole number divisor using the long division method.

Consider this expression:

43.446 ÷ 6

The divisor is a whole number. First, estimate: 43.446 ÷ 6 ≈ 42 ÷ 6 = 7.

Now use the long division method. Carefully line up the numbers so that the

decimal point in the quotient is in the same position as the decimal point in the dividend.

		7	.2	4	1
6	4	3	.4	4	6
	4	2			
		1	4		
		1	2		
			2	4	
			2	4	
					6
					6
					0

The answer, 7.241, is close to the estimate of 7.

Dividing a decimal by another decimal requires an extra step. Consider this expression:

49.32 ÷ 6.85

The divisor is a decimal. Make the divisor a whole number by multiplying the dividend and divisor by 100:

49.32 ÷ 6.85 = 4,932 ÷ 685

Then use the standard method for division:

				0	0	7	.2
6	8	5	4	9	3	2	.0
		−	4	7	9	5	
				1	3	7	0
			−	1	3	7	0
							0

49.32 ÷ 6.85 = 7.2

A general rule is: *Multiply the divisor and the dividend by the same power of 10 that's large enough to make the divisor a whole number. Then divide by the long division method.*