Proportional Relationships: Ratios

Ratios

A ratio is a simple idea that is used in mathematics and in everyday life. Simply put, a ***ratio*** is the comparison of two quantities by division.

Consider this example:

Suppose that 24 students and 6 teachers go on a field trip. You can compare the number of students to the number of teachers by dividing. Since 24 ÷ 6 = 4, you can say that there are 4 times as many students as teachers. You can also say there are 4 students for every teacher. A way of saying this using the term *ratio* is to say: "The ratio of students to teachers is 4 to 1."

Relative Comparisons

There is a wide variety of ways to talk about any relative comparison, so it is worth taking time to examine this language.

Think back to the example of 24 students and 6 teachers going on a field trip. You can make two different kinds of comparisons from this example.

- **A comparison of the number of students to the number of teachers**

To make this comparison, divide the number of students by the number of teachers.

246 = 41 = 4

Ways of talking about the student-teacher ratio include:

- There are 4 times as many students as teachers.

- There are 4 students for every teacher.
- There are 4 students per teacher.
- There is a 4 to 1 ratio of students to teachers.
- The ratio of students to teachers is 4:1.

- **A comparison of the number of teachers to the number of students**

To make this comparison, divide the number of teachers by the number of students.

624 = 14 = 0.25

Ways of talking about the teacher-student ratio include:

- There are 0.25 times as many teachers as students. (Note: Although this statement is mathematically correct, it would be clearer and less awkward to use the statement that says, "There are 4 times as many students as teachers.")
- There are 14 as many teachers as students.
- There is 0.25 of a teacher for every student.
- There is a 1:4 ratio of teachers to students.
- The ratio of teachers to students is 1 to 4.
- There is 1 teacher for every 4 students.

Equivalent Ratios

Two ratios are said to be ***equivalent*** if the fractions that represent them are equal. For example, the boy-girl ratio in a class with 12 boys and 15 girls is equivalent to the boy-girl ratio in a school of 284 boys and 355 girls. The ratio is 4:5 in each case.

1215=3•43•5=45

284355=71•471•5=45

The two ratios are equivalent because they reduce to the same fraction in lowest terms.

Representing Ratios in Tables

Equivalent ratios are often represented in a table. For example, consider this table:

4	8	12	16	20	24	28	32	36	40	44
5	10	15	20	25	30	35	40	45	50	55

Each pair of numbers in this table has the ratio 4:5.

Now consider the following table:

3	6	9		15	18	21	24	27		33
4		12	16	20	24		32	36	40	44

You can see that several values are missing from the table. You can find the missing numbers by using your knowledge of equivalent fractions.

Each pair of numbers in this table has the ratio 3:4. To find the first missing number in the second column, for example, you can write and solve the following equation:

34=6?

34=2•32•4

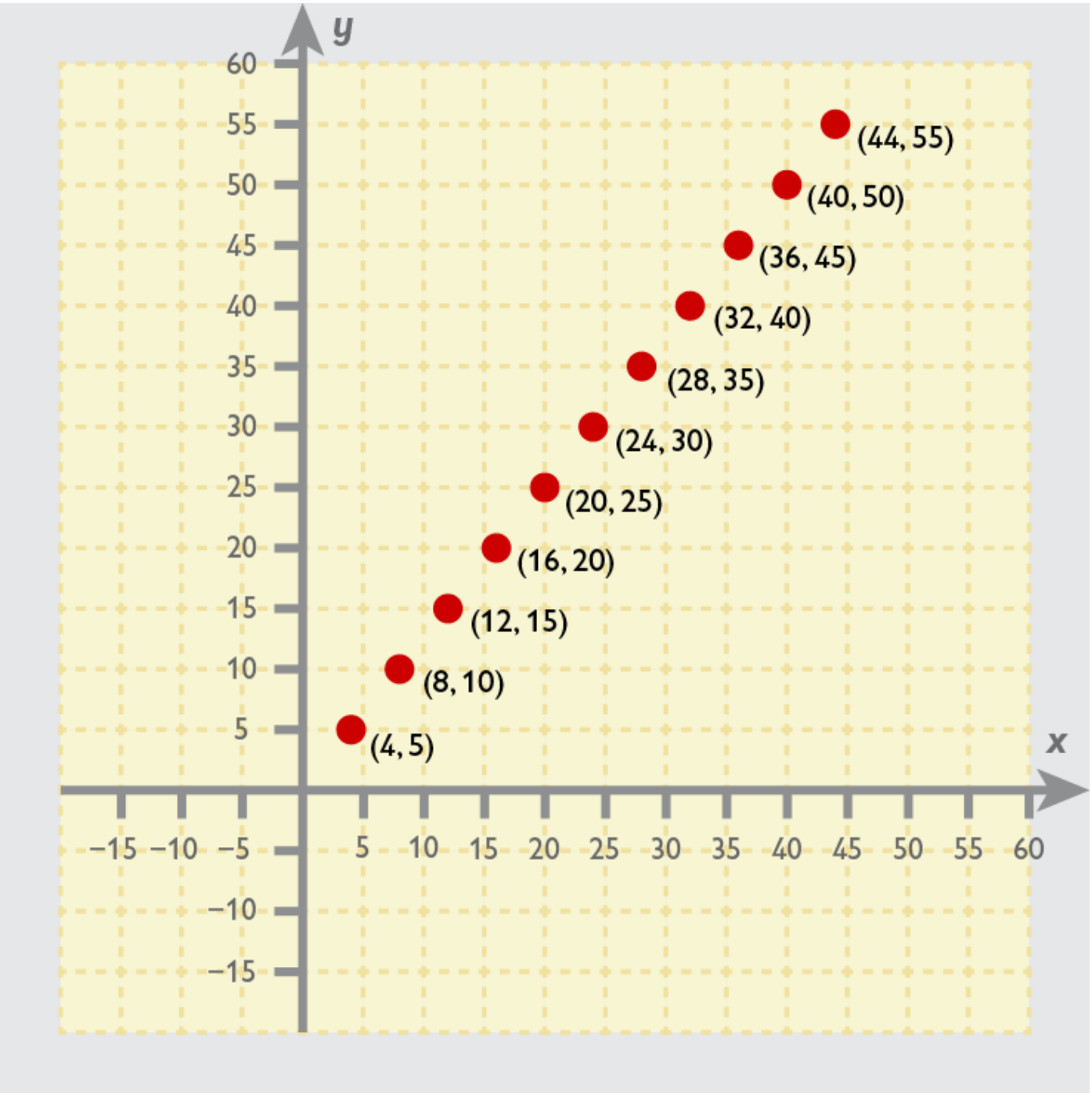
34=68

Representing Ratios in Graphs

Consider this table again, which shows the ratio 4:5.

4	8	12	16	20	24	28	32	36	40	44
5	10	15	20	25	30	35	40	45	50	55

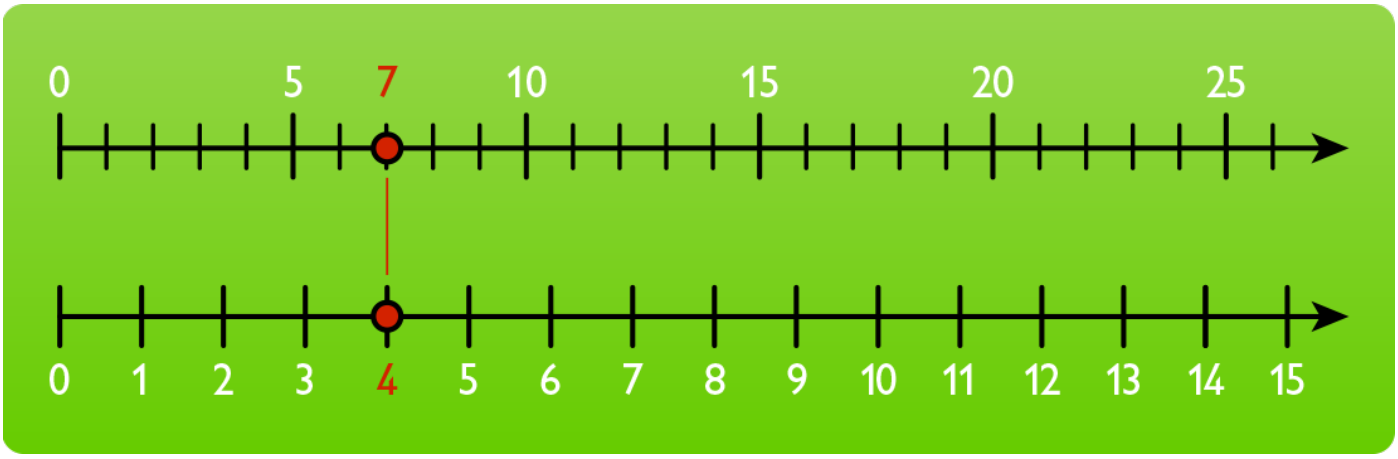
You can draw a graph that represents the ratio 4:5 using the pairs of numbers in the table. Thus, you would plot the coordinates (4, 5), (8, 10), (12, 15), and so on. The result is a straight-line graph through the origin.



This graph relates pairs of x and y that are in the same ratio.

Representing Ratios in Double Number Lines and Tape Diagrams

A useful way of expressing many equivalent ratios is to use a ***double number line***. Here is a double number line that represents many ratios, all equivalent to 7:4.



Any vertical line segment through these two number lines will pass through numbers m (upper line) and n (lower line) that are in the ratio 7:4.

Another useful way of representing ratios is using a tape diagram. Here is a tape diagram that shows a ratio of 7:4.

7	4

Specifically, there is a 7:4 ratio of the lengths of the two sections of tape. Tape diagrams help us visualize ratios in situations, such as the 7:4 ratio of girls to boys in a school of 91 girls and 52 boys:

7	4
91 girls	52 boys
143 students	
7 girls for every 4 boys	