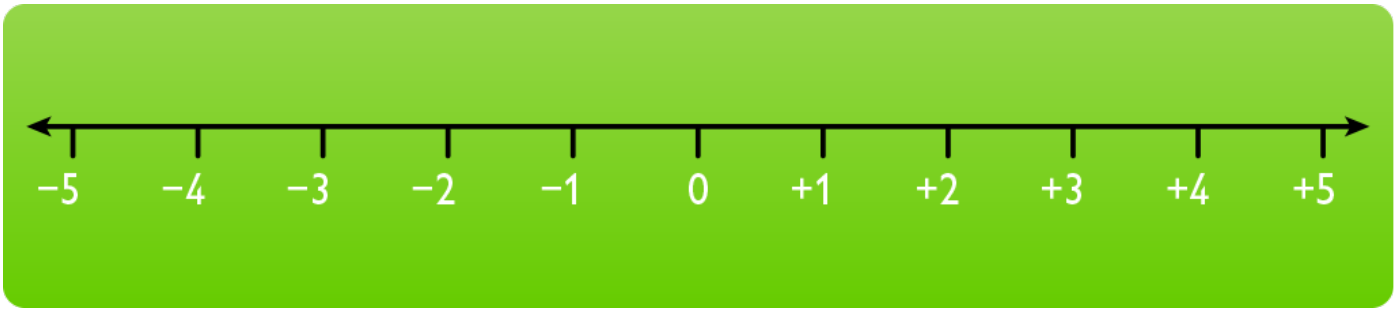
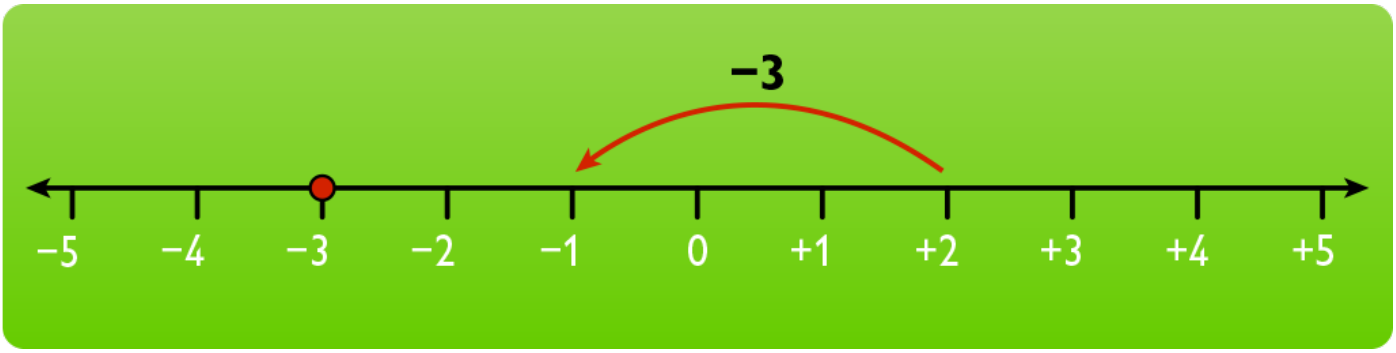
Rational Numbers: Add and Subtract

Previously you learned that on a horizontal number line, positive numbers lie to the right of 0 and negative numbers lie to the left of 0. For example, the number line below shows five positive numbers marked to the right of 0 and five negative numbers marked to the left of 0.



A number can be interpreted as a *location* on the number line or as an *arrow* from one point on the number line to another point. So, −3 is either the point −3 or an arrow of length 3 heading left. Note that the point −3 is fixed on the number line, but the arrow −3 can start at *any* point on the number line.



Adding Positive and Negative Numbers

You can use the number line to show the addition $a + b$, where a and b are any two numbers (positive, negative, or 0), as follows:

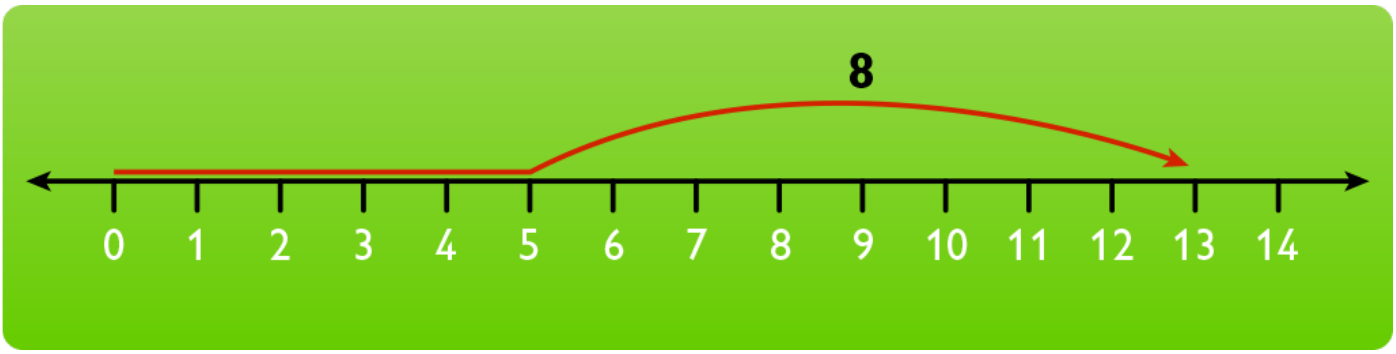
- Start at a (which may be positive, negative, or 0).
- From a, move a distance to the right if b is positive or to the left if b is negative.

- You will end up at $a + b$.

You are already familiar with using a number line to add two positive numbers.

example

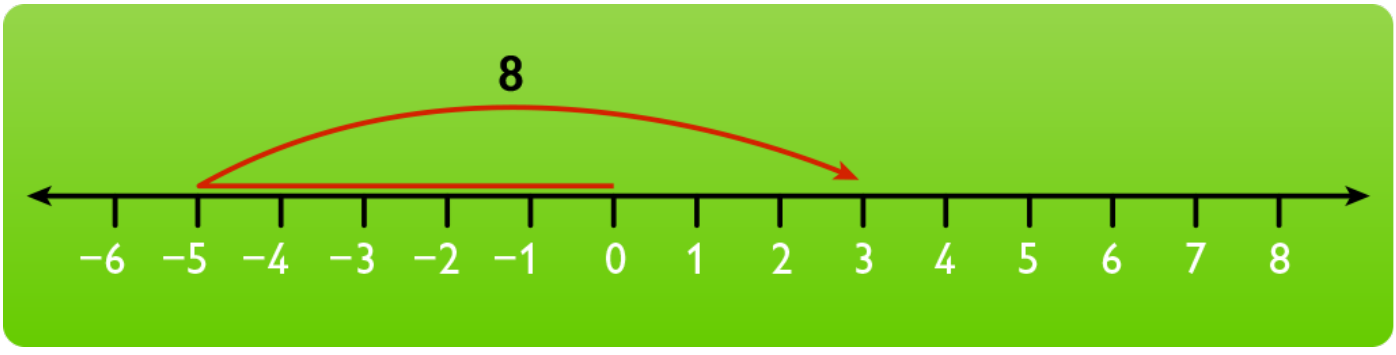
Example: 5 + 8
Start at 5. Go a distance of 8 in the positive direction.
You end up at 13.
5 + 8 = 13



You can use the number line to add a **positive number** to a **negative number**.

example

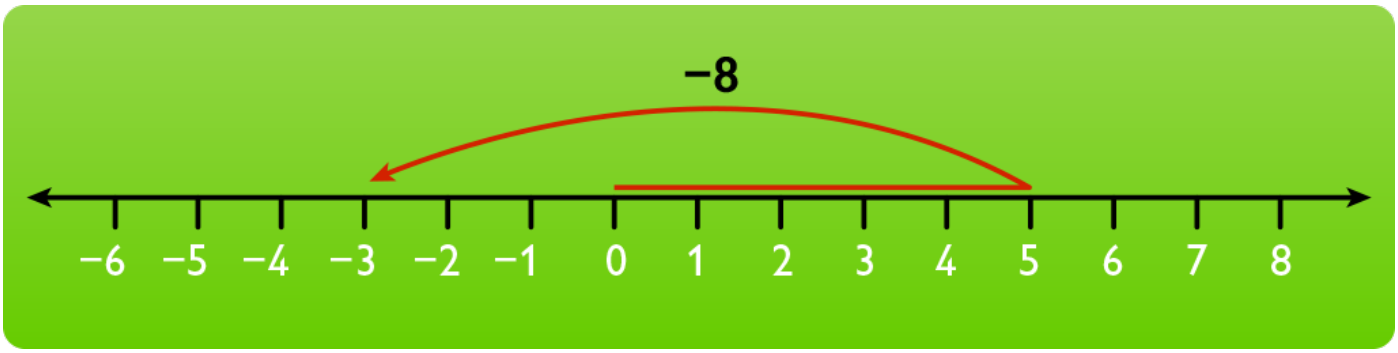
Example: −5 + 8
Start at −5. Go a distance of 8 in the positive direction.
You end up at 3.
−5 + 8 = 3



You can use the number line to add a **negative number** to a **positive number**.

example

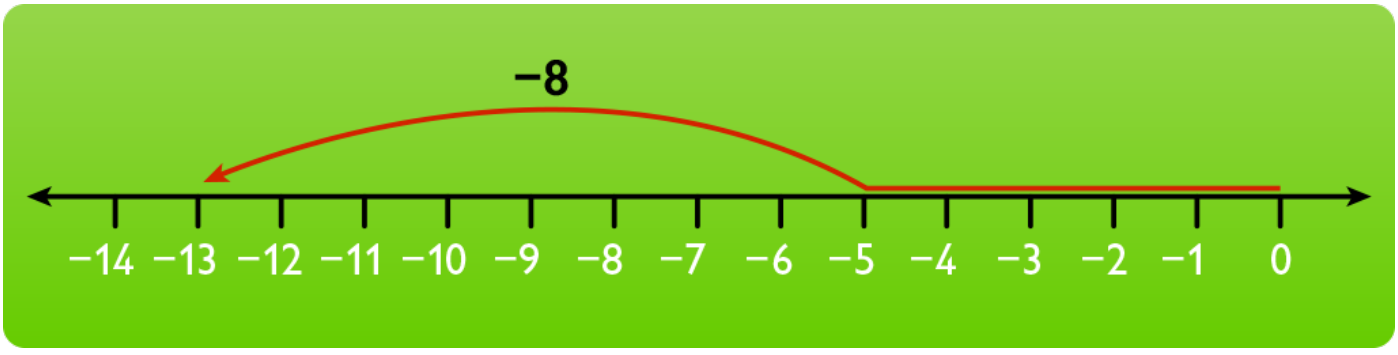
Example: 5 + (−8)
Start at 5. Then, because −8 is negative, go a distance of 8 in the negative direction.
You end up at −3.
5 + (−8) = −3



You can use the number line to add a **negative number** to a **negative number**.

example

Example: −5 + (−8)
Start at −5. Then, because −8 is negative, go a distance of 8 in the negative direction.
You end up at −13.
−5 + (−8) = −13



The Opposite of a Number (the *Additive Inverse*)

Recall that the negative sign – is used to indicate the opposite of any number. Thus, the opposite of 5 is –5, and the opposite of –5 is written –(–5), which is equal to 5. Numbers that are opposites are the same distance from 0 on the number line, but they are on opposite sides of 0.

When you combine numbers that are opposites, you get 0:

$$x + (-x) = 0$$

For example, if a helicopter flies straight up 100 ft and then flies straight down 100 ft (representing –100), it winds up at 0 ft: 100 + (–100) = 0.

The opposite of a number is also known as the *additive inverse* of the number. Specifically, the additive inverse of x is the number, written $-x$, which has the property that $x + (-x) = 0$.

Numbers that are opposites have the following properties:

- The opposite of the opposite of a number is the number itself: $-(-x) = x$
- The sum of a number and its opposite is 0: $(x) + (-x) = 0$
- Subtracting a number from 0 gives the opposite: $0 - x = -x$

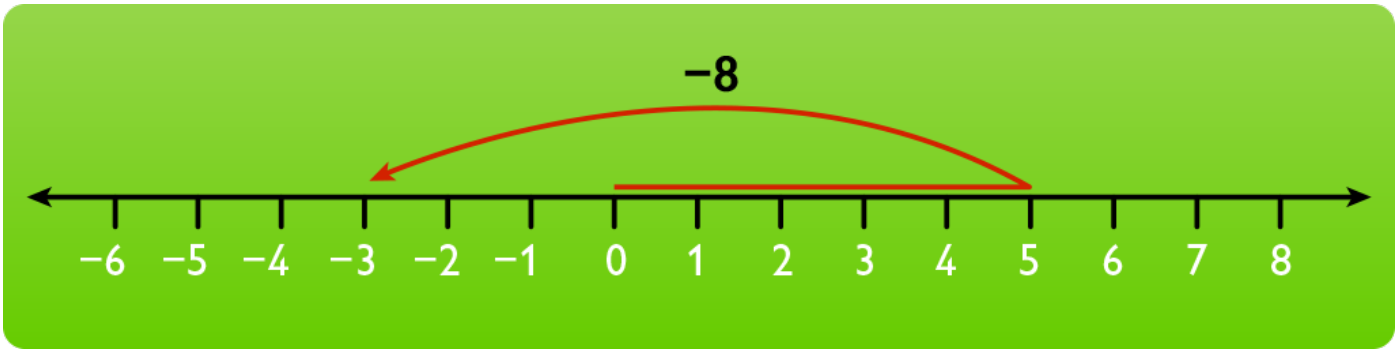
Subtracting Negative and Positive Numbers

Subtraction is simply adding the opposite, so $a - b$ means the same as $a + (-b)$. You can use a number line to show $a - b$, or $a + (-b)$, as you did for addition.

example

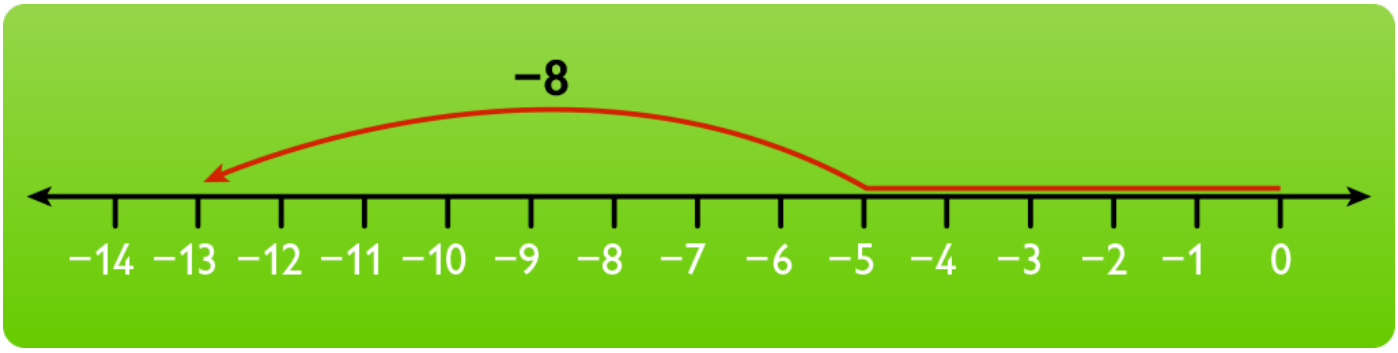
Example: 5 – 8 = 5 + (–8)
Start at 5. Then, because –8 is negative, go a distance of 8 in the negative

direction.
You end up at −3.
5 − 8 = −3



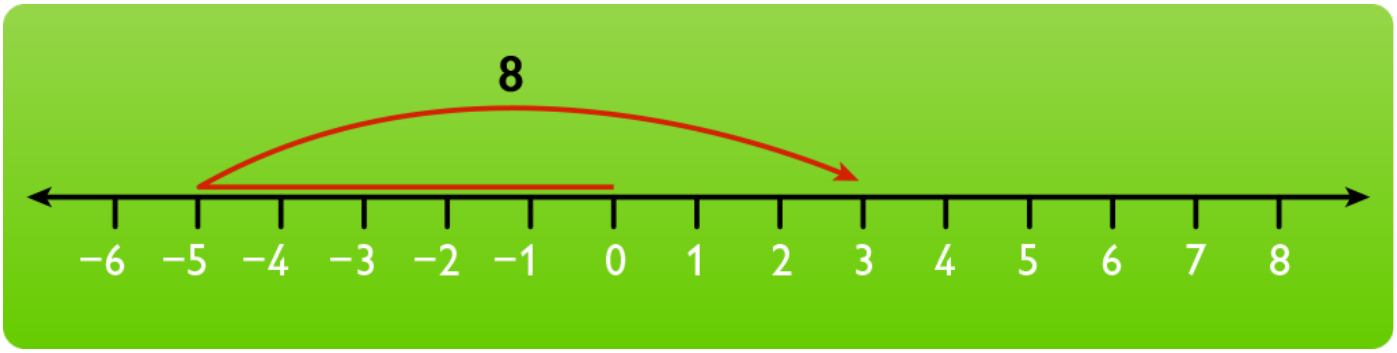
example

Example: −5 − 8 or −5 + (−8)
Start at −5. Then, because −8 is negative, go a distance of 8 in the negative direction.
You end up at −13.
−5 − 8 = −13



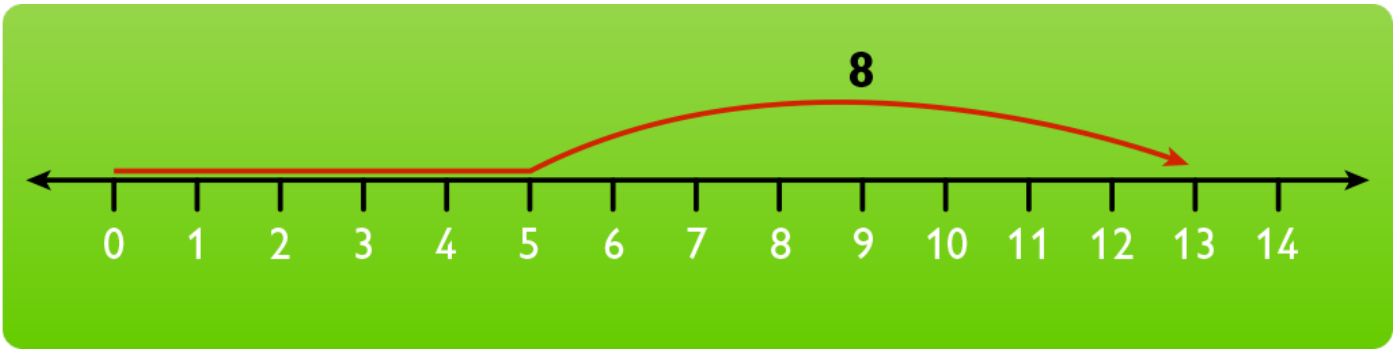
example

Example: −5 − (−8) = −5 + 8
Start at −5. Go a distance of 8 in the positive direction.
You end up at 3.
−5 − (−8) = 3



example

Example: 5 – (–8) = 5 + 8
Start at 5. Go a distance of 8 in the positive direction.
You end up at 13.
5 – (–8) = 13

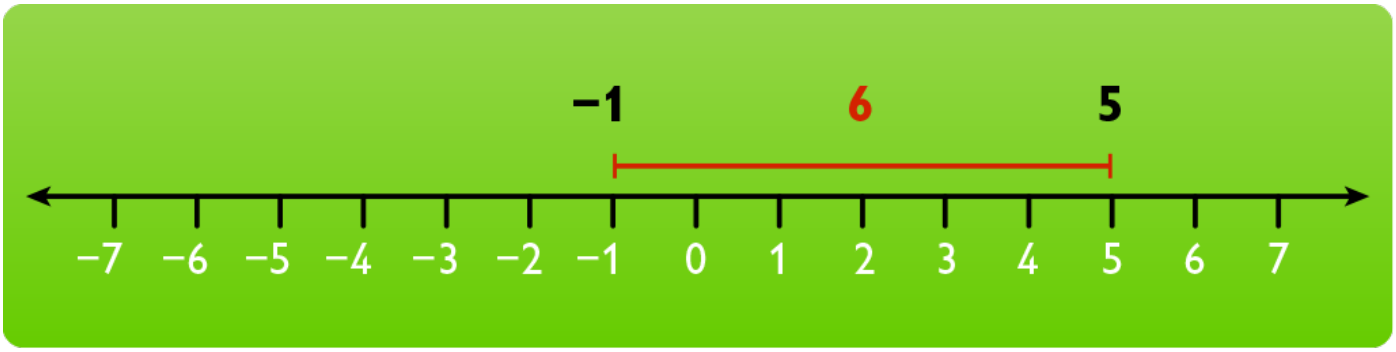


Difference on a Number Line

Given any two numbers a and b, where $b > a$, the segment on the number line from a to b has a length equal to the difference $b - a$.

example

Example: $a = -1$ and $b = 5$ or $5 - (-1)$
The segment on the number line between –1 and 5 has a length equal to the difference
5 – (–1). So, 5 – (–1) = 6, and the length between –1 and 5 is equal to 6 units.



Using the Properties of Operations to Add and Subtract

You can use the properties of operations to help you add and subtract rational numbers.

The Commutative Property of Addition

$a + b = b + a$ is true for all numbers, a and b.

For example:

$-5 + 2 = -3$
$2 + (-5) = -3$
so, $-5 + 2 = 2 + (-5)$

However, the commutative property does not apply to subtraction. In general, $a - b$ does not equal $b - a$. For example, $-5 - 2 = -7$, but $2 - (-5) = 7$. These two equations are different, showing that the commutative property does not apply to the operation of subtraction.

The Associative Property of Addition

$(a + b) + c = a + (b + c)$ is true for all numbers a, b, and c. This property allows you to add a group of numbers in any order.

For example, consider these equations:

$5 + (-3) + (-6)$ and $5 + [-3 + (-6)]$

You can add the first two numbers, and then add that sum to the third number:

$[5 + (-3)] + (-6) = 2 + (-6) = -4$

Or, you can add the second and third numbers, and then add that sum to the first number:

$5 + [-3 + (-6)] = 5 + (-9) = -4$

In each case, the result is the same, illustrating the associative property of addition.

However, the associative property does not apply to subtraction. In general, $(a - b) - c \neq a - (b - c)$. For example, if you regroup the expression $[5 - (-3)] - (-6)$ to get $5 - [-3 - (-6)]$, you do not get the same results:

$[5 - (-3)] - (-6) = 8 - (-6) = 8 + 6 = 14$

$5 - [-3 - (-6)] = 5 - (-3 + 6) = 5 - 3 = 2$

The two results are different, showing that the associative property does not apply to the operation of subtraction.