Rational Numbers: Multiply and Divide

Multiplying Positive and Negative Numbers

You know that when you multiply a positive number by a positive number, the result is positive. Multiplication with negative numbers yields different results.

Multiplying a Positive Number by a Negative Number

When multiplying a positive number by a negative number, the result is negative.

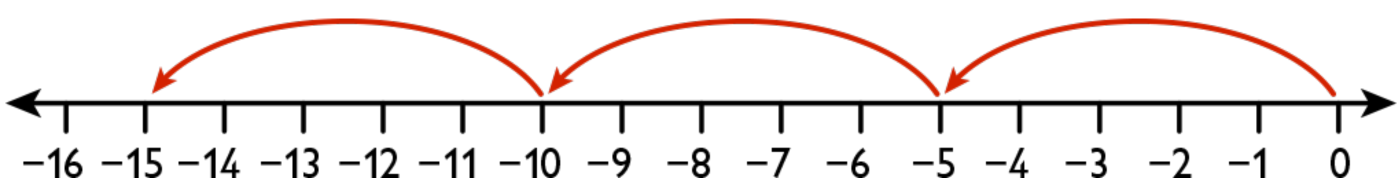
For example, imagine that a submarine submerges 5 m. Later, it submerges another 5 m. Still later, it submerges yet another 5 m. You could represent this situation with the following expression:

$3 \cdot (-5)$

This expression can be thought of as 3 groups of negative 5:

$3 \cdot (-5) = (-5) + (-5) + (-5) = -15$

The result is a negative number. This number line illustrates this process.



Multiplying a Negative Number by a Positive Number

When multiplying a negative number by a positive number, the result is negative.

For example, think about the expression (–5) • 3. Because multiplication is commutative, this expression is the same as 3 • (–5). From the previous section, you know that the result is –15.

Multiplying a Negative Number by a Negative Number

When multiplying a negative number by a negative number, the result is positive.

For example, think about the expression –3 • (–5). This expression can be thought of as the opposite of 3 • (–5). The latter expression equals a negative number, –15. Thus, –3 • (–5) = 15—a positive number.

Summary of Multiplication Rules

The previous information can be summarized in the following four simple rules:

- **positive** times **positive** equals **positive**
- **positive** times **negative** equals **negative**
- **negative** times **positive** equals **negative**
- **negative** times **negative** equals **positive**

Dividing Negative and Positive Numbers

You can divide with negative numbers, provided that the divisor is not zero. In general, if *p* and *q* are integers, then:

$-(pq)=(-p)q=p(-q)$

Using the Properties of Operations

The Inverse Property of Multiplication

Every nonzero number has a reciprocal. Multiplying a nonzero number by its reciprocal gives 1. The reciprocal of a number is called the *multiplicative inverse*. The concept of the multiplicative inverse is parallel to the concept of the additive inverse. The multiplicative inverse of a nonzero number is the number written 1 over p such that:

$p \bullet 1p = 1p \bullet p = 1$

For example, 34 and 43 are multiplicative inverses because 34 × 43 = 43 × 34 = 1.

The Commutative Property of Multiplication

$a \bullet b = b \bullet a$, where a and b can be any numbers, including negative numbers.

The Associative Property of Multiplication

$(a \bullet b) \bullet c = a \bullet (b \bullet c)$, where a, b, and c can be any numbers, including negative numbers. Applying the laws of signs for multiplying nonzero numbers:

- If just one of these three numbers is negative, then the product will be negative.
- If exactly two of these numbers are negative, then the product will be positive.
- If all three of these numbers are negative, then the product will be negative.

The Distributive Property

$a \bullet (b + c) = (a \bullet b) + (a \bullet c)$, where a, b, and c can be any numbers, including negative numbers. For example:

$$\mathbf{-5 \bullet [2 + (-3)] = (-5 \bullet 2) + [-5 \bullet (-3)]}$$

$$\mathbf{-5 \bullet (-1) = -10 + 15}$$

5 = 5

More About Rational Numbers

A rational number is any number that can be written in the form *ab*, where *a* and *b* are integers, and where *b* ≠ 0.

The expression 02 is one of the many ways of writing 0.

The expression 50 has no meaning as a number, since it is impossible to divide by 0.

As decimals, all rational numbers are either:

- Terminating decimals, such as 134 = 1.75.
- Repeating decimals, such as 311=0.272727...=0.¯27

The bar over the 27 means it repeats forever.

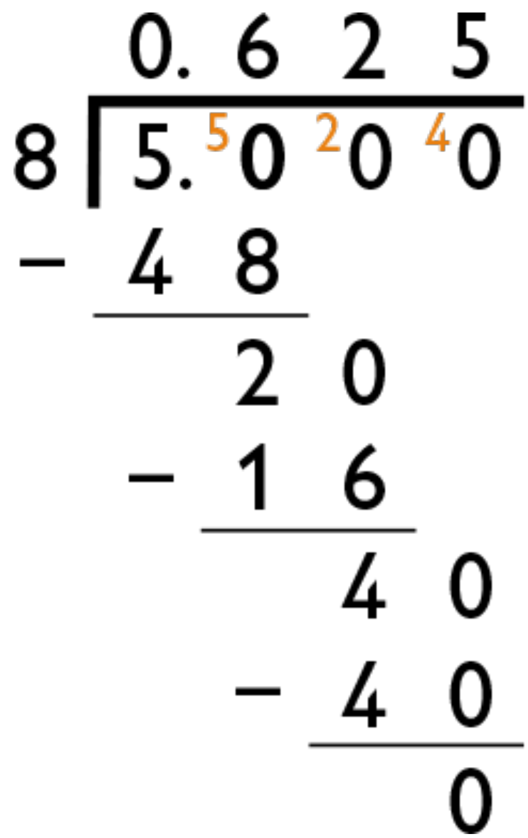
Irrational numbers are neither terminating nor repeating decimals.

Converting a Rational Number to a Decimal

You can convert a rational number to a decimal using long division—simply divide the numerator by the denominator:

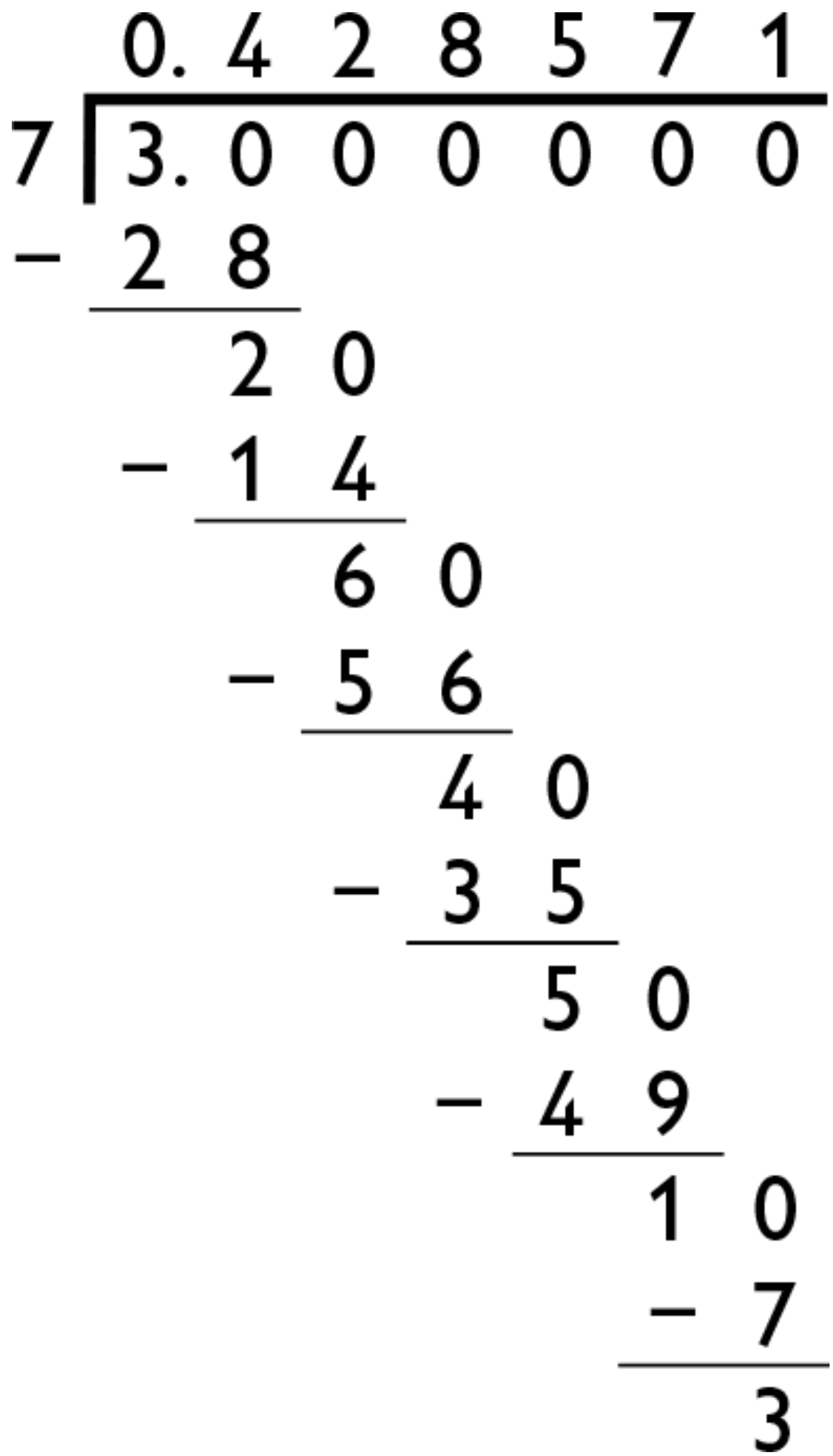
numeratordenominator or denominator)‾

For example, to convert to a decimal, write the 5 as 5.000 and perform the following long division:



Sometimes you end up with a repeating decimal.

For example, to convert to a decimal, perform the following long division:



So, $= 0.428571$

The sequence 428571 repeats over and over.