Rational Numbers: Word Problems

You can use the same strategies to solve word problems that involve rational numbers that you have used in previous problems. Here is a basic framework for solving any type of word problem.

1. Read the word problem, understand the situation, and understand the questions you are asked. What is the problem situation? Identify the quantities in the situation. Name and label those quantities.
2. Represent the problem situation using some of these tools:
    - Make diagrams with labels or pictures of relationships.
    - Describe the problem situation in your own words.
    - Make tables.
    - Try out simple numbers.
    - Break the problems into smaller parts.
    - Make up equations to express relationships.
3. Explain your representations to other students. Listen to other students' explanations and understand their representations.
4. Answer questions about the problem. This may include developing equations to answer the questions posed, making any needed calculations, and finding the answer to any question asked in the problem.
5. Understand and represent the problem situation. Do not race toward an answer. The purpose of problem solving is to learn mathematics and to get better at solving new and unfamiliar problems.

Word Problems Dealing With Rational Numbers

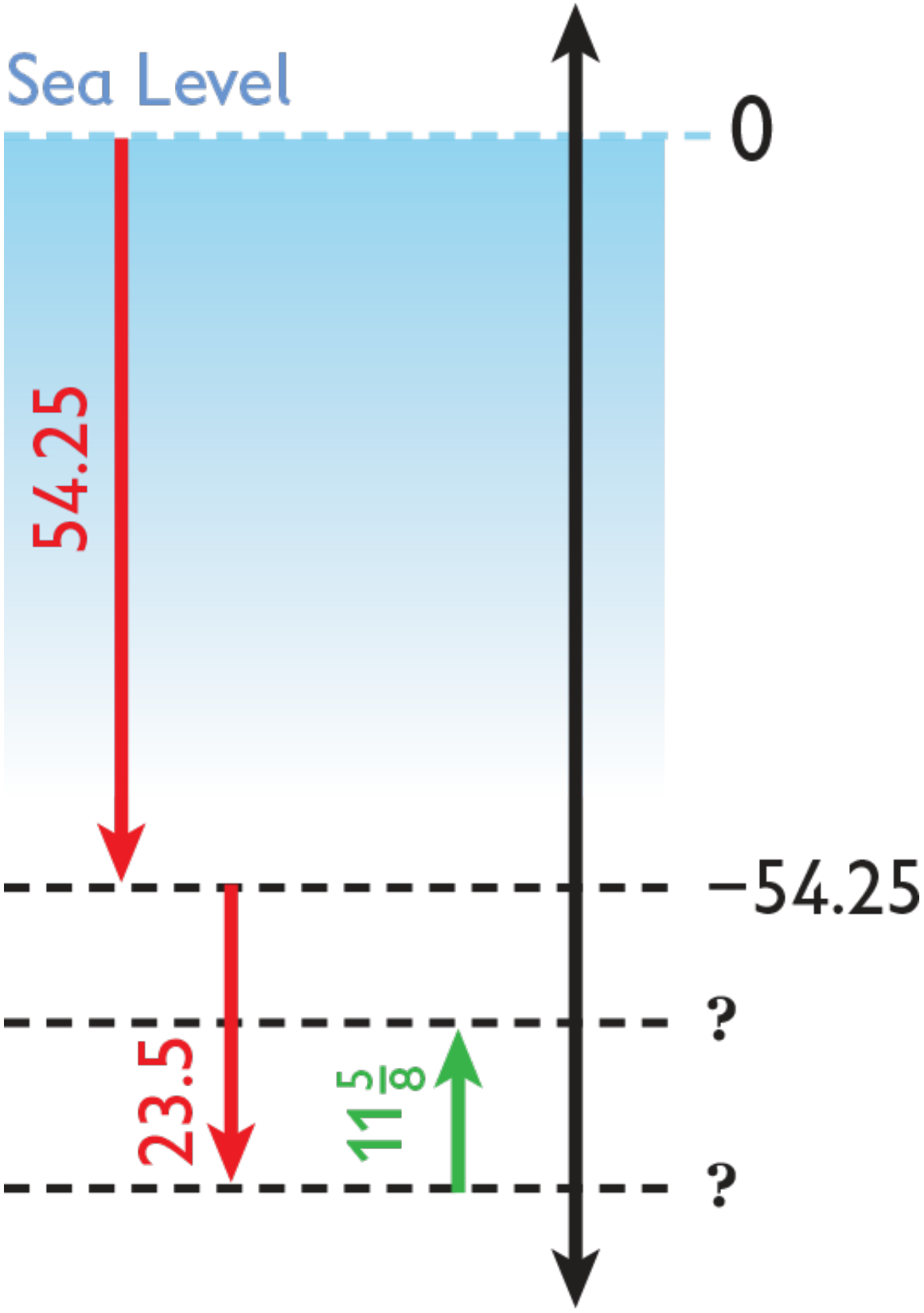
Consider the following problem:

A scuba diver dove 54.25 ft under the surface of the ocean. Then he dove down 23.5 ft more. Then he went up 1158 ft. At what depth below sea level is

the diver now?

This problem involves several types of rational numbers: decimals, fractions, and negative numbers.

A good way to start working this problem is to draw a picture. You might draw something like this:



If you think of this diagram as a vertical number line, then you would use negative numbers to represent depths below sea level.

Look at the diagram. You can see that the first depth of the diver is −54.25 ft.

The diver then goes down an additional 23.5 ft. Moving down on a vertical number line represents subtraction, so to find the next depth, you need to subtract 23.5 ft from −54.25 ft:

$-54.25 - 23.5 = -54.25 + (-23.5) = -77.75$ ft

Finally, the diver goes up 1158 ft. Because you have been working with decimal numbers, it will be easiest to complete the problem if you convert 1158 to a decimal: 1158 = 11.625.

Finding the next depth represents a movement *up* the number line, so it represents an addition. Now you have:

$-77.75 + 11.625 = -(77.75 - 11.625)$ (using the distributive property)

$= -66.125$ ft

Thus, the final position of the diver is 66.125 ft below sea level.

Word Problems Dealing With Proportional Relationships

Many problems describe situations in which one quantity is proportional to another. The situations may seem very different, but they all use formulas like $p = kq$, although the letters used for the quantities may be different and the constant of proportionality, k, may be replaced by a given number.

The following are examples of some common types of problems you will encounter when dealing with proportional relationships.

1. The quantities p and q are both counts of things.
    - Chairs/tables: k is the number of chairs at each table. Different types of tables have different values of k, but $k = 4$ for a common type of table. The formula you could use to solve problems of this type is $c = 4t$.

- Wheels/cars: k is the number of wheels on a car. Different types of cars have different values of k, but $k = 4$ for most cars.
- Students/teachers: k is the student to teacher ratio.

2. The quantities p and q are both lengths, and k expresses a size relationship, called the ***scale factor*** or ***similarity ratio***.

    - Similar triangles: corresponding sides are proportional. k is the ratio of similarity.
    - Maps or floor plans: k is the scale factor.
    - Enlargement or reduction: k is a photocopier setting (often given as a percentage).

3. The quantities p and q are both lengths, and k expresses a shape relationship.

    - Steepness of ramps, stairs, roads, or roofs: k is the slope, and it is expressed in at least two different ways (e.g., a 6% grade on a road; a 1 to 5 slope of a roof).
    - Slope: in a simple case, where vertical and horizontal scales are the same, the mathematical slope of the graph is the same as the geometric slope. However, you have to be careful because if vertical and horizontal scales are not the same, the mathematical slope is different from the geometric slope.

4. The quantities p and q are both monetary values.

    - Cost/sales tax: k is the sales tax rate, usually expressed as a percentage.
    - Cost/discount: k is the percent taken off in a sale (e.g., every item 20% off).

5. The quantities p and q are any quantities of the same type, and k is the ***conversion factor***.

- Centimeters/inch: $c = 2.54i$, where i is a length measured in inches and c is the same length measured in centimeters. Notice that the value of c will always be 2.54 times the corresponding value of i. This is because centimeters are shorter than inches. A stick that is 2 in. long is 5.08 cm long.
- Minutes/hour: $m = 60h$, where h is a time interval in hours and m is the same time interval in minutes. There are always 60 min in 1 hr.