# LESSON I: UP, UP, AND AWAY!

### **EXERCISES**

#### **EXERCISES**

- 1. Write your wonderings about working with positive and negative numbers. Share your wonderings with a classmate.
- 2. Write a goal stating what you plan to accomplish in this unit.
- 3. Based on your previous work in math, write three things that you will do during this unit to increase your success.

For example, consider ways you will participate in classroom discussions, your study habits, how you will organize your time, what you will do when you have a question, and so on.

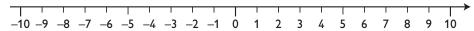
# **LESSON 2: MODELING INTEGER ADDITION**

# **EXERCISES**

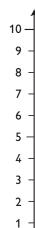
### **EXERCISES**

1. Evaluate this expression.

- A -14
- **B** -7
- **O**
- **1**4
- 2. Use this number line to show 3 + (-7).



3. Use this number line to show -4 + (-2).



0 --1 -2 -3 -4 -5 -6 -7 -8

-9 -10 -

### **LESSON 2: MODELING INTEGER ADDITION**

**EXERCISES** 

4. Find the sum.

5. Find the sum.

6. Find the sum.

$$-38 + (-56) =$$

7. Find the sum.

- 8. What is the additive inverse (opposite) of -407?
- 9. What is the additive inverse (opposite) of 81?
- 10. The temperature at 6 a.m. was -7°F. By noon, the temperature had risen 10°. At noon, what was the temperature in degrees Fahrenheit?
- 11. A 40 ft utility pole was vertically placed in a hole 6 ft deep. What is the height of the portion of the pole that is above ground?
- 12. You can use a negative number to represent a situation about debt. For example, Marcus owes his father \$7. This situation can be represented as -7.
  - a. Marcus borrows \$5 from his sister. Write an equation to represent his total debt (that is, the total amount he owes his father and his sister).
  - b. Marcus earns \$10 shoveling snow from his neighbor's driveway. He uses all of the \$10 to pay back some of his debt. Write an equation to represent the amount of his new debt.
- 13. Describe a real-world situation that can be represented by this equation.

$$-6 + 6 = 0$$

14. Describe a real-world situation that can be represented by this equation.

$$-10 + (-10) = -20$$

Challenge Problem

15. For what integer values of a and b does |a + b| = |a| + |b|?

# **LESSON 3: SUBTRACTION AS "TAKING AWAY"**

### **EXERCISES**

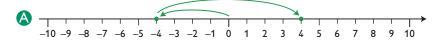
#### **EXERCISES**

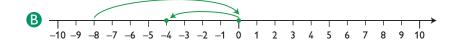
1. Evaluate this expression.

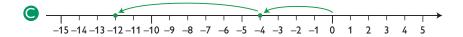
$$3 - 4$$

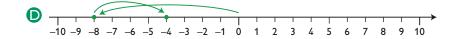
- **A** -7
- **B** -1
- **1**
- 7
- 2. Which number line shows this subtraction as taking away?

$$-4 - (-8)$$









# **LESSON 3: SUBTRACTION AS "TAKING AWAY"**

## **EXERCISES**

- 3. Use this number line to show -1 6.
  - 10 9 8 7 3 0 --1 -2 -3 -4 -5 -6 -7 -8 \_9 -10 **-**
- 4. Find the difference.

5. Find the difference.

6. Find the difference.

7. Evaluate this expression.

8. Evaluate this expression.

9. Evaluate this expression.

## **LESSON 3: SUBTRACTION AS "TAKING AWAY"**

### **EXERCISES**

- 10. The temperature was 4°F at 11 p.m. Overnight the temperature fell by 9°. What was the morning temperature in degrees Fahrenheit?
- 11. Sophie's mom's credit card statement said she owed \$75. Then the credit card company said it made a mistake and took away \$15 of this debt.
  - a. Write a subtraction expression to represent this situation
  - b. How much does Sophie's mom owe now?
- 12. A hiker is in a canyon at an elevation of -14 ft (that is, 14 ft below sea level). She descends another 37 ft. What is her elevation now?
- 13. Describe a real-world situation that can be represented by this equation.

$$-6 - 6 = -12$$

14. Describe a real-world situation that can be represented by this equation.

$$10 - 15 = -5$$

Challenge Problem

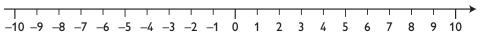
15. For what integer values of a and b does |a - b| = a - b?

## **LESSON 4: SUBTRACTION AS DISTANCE**

#### **EXERCISES**

#### **EXERCISES**

1. a. What is the distance between -5 and 0?

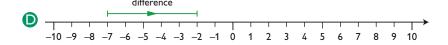


- b. The distance is \_\_\_\_ units.
- 2. Which number line shows this subtraction as the distance between two points on the number line? (-2) (-5)

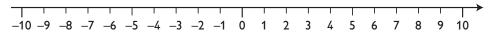








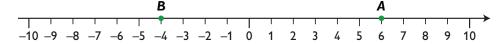
3. a. What is the distance between 5 and -7?



- b. The distance is \_\_\_\_ units.
- 4. What is the distance between -8 and -3?

The distance is \_\_\_\_ units.

5. Look at the number line.

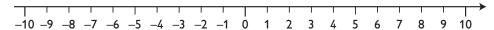


Explain how to use subtraction to find the distance between A and B.

### **LESSON 4: SUBTRACTION AS DISTANCE**

#### **EXERCISES**

6. Explain how to use subtraction to find the distance between -8 and -1.



- 7. Which expression is equivalent to the distance between -43 and 72 on a number line?
  - $\triangle$  |43 + (-72)|
  - $\bigcirc$  |72 + (-43)|
  - **(** |43 72|
  - **▶** |-43 − (72)|
- 8. What is the distance between -561 and -305?
- 9. In the state of California, the highest point is located on Mount Whitney and the lowest point is located in Death Valley. Mount Whitney has an elevation of 14,494 ft. Death Valley has an elevation of –282 ft. What is the difference in elevations between the two locations?
- 10. In Marcus's town, the school, the library, and the park are all on the same street. The school is 13 blocks west of the park, and the library is 5 blocks west of the park. What is the distance between the library and the school?
- 11. The highest and lowest recorded temperatures in the state of lowa are 118°F and -47°F. What is the difference between these two temperatures?

The difference is \_\_\_\_\_°.

Challenge Problem

12. When does |a-b| = -(a-b)? Explain.

# **LESSON 5: ADDING AND SUBTRACTING**

**EXERCISES** 

#### **EXERCISES**

1. Find the sum.

$$3\frac{1}{2} + (-3\frac{1}{2}) =$$
\_\_\_\_

2. Find the sum.

3. Find the sum.

$$4\frac{2}{3}-7\frac{3}{5}=$$
 \_\_\_\_

4. Evaluate this expression.

$$-5.03 - (-2.92) =$$
\_\_\_\_

5. Evaluate this expression.

$$\frac{4}{7} + \left(-\frac{3}{4}\right) =$$
\_\_\_\_

6. Evaluate this expression.

$$-0.5 - 0.8 =$$
\_\_\_\_

7. Which number is the correct evaluation of this expression? There may be more than one correct evaluation.

$$-\frac{5}{8}+\left(-\frac{5}{6}\right)$$

- $\triangle -\frac{35}{24}$
- **B**  $-\frac{10}{14}$
- $-\frac{10}{24}$
- $-1\frac{11}{24}$
- $-\frac{5}{24}$

### **LESSON 5: ADDING AND SUBTRACTING**

#### **EXERCISES**

- 8. Maya owes her mother \$26.75
  - a. Maya borrows another \$5.50 from her mother to buy a ticket to the school play. Write an equation to represent Maya's total debt to her mother.
  - b. Maya earned \$12 babysitting and used all of that money to repay some of her debt to her mother. Write an equation to represent Maya's new debt.



The city of Chicago has a biking and walking path along its lakefront.

- a. Jack is on the path at the Oak Street Beach. The north end of the path is 7.55 mi from him, and the south end of the path is 10.76 mi from him. What is the total length of the path?
- b. The Shedd Aquarium is 3.29 mi south of Jack. The Theater on the Lake is 1.81 mi north of him. What is the distance between the aquarium and the theater?

10. A hot air balloon starts at an altitude of 0 m. If the balloon rises  $15\frac{3}{8}$  m and then descends  $5\frac{1}{4}$  m, what is the new altitude of the balloon?

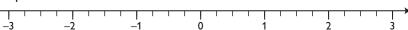
The new altitude is \_\_\_\_ m.

# **LESSON 5: ADDING AND SUBTRACTING**

### **EXERCISES**

11. Sophie, Lucy, and Jack live along the same street. The library is also on their street. Sophie's apartment is  $2\frac{1}{2}$  blocks from the library. Lucy lives 1.25 blocks from the library but in the opposite direction from Sophie. Jack lives 0.75 block from the library and his apartment is closer to Sophie's than to Lucy's.

Represent this situation on a number line.



12. In December at the Amundsen-Scott South Pole Station, the mean daily low temperature was

-29.3°C and the mean daily high temperature was -26.5°C. What is the difference between these two temperatures?

The difference is \_\_\_\_°.

Challenge Problem

13. If a - b = -12, what is the value of b - a? Explain.

## **LESSON 6: PROPERTIES OF OPERATIONS**

#### **EXERCISES**

#### **EXERCISES**

- 1. What property tells you that  $6\frac{1}{2} + 0 = 6\frac{1}{2}$ ?
  - Additive identity property of 0
  - B Additive inverses
  - Associative property of addition
  - Commutative property of addition
- 2. What is the additive inverse of  $\frac{7}{9}$ ?
- 3. What is the additive inverse of -0.053?
- 4. Which expression is a step you could use to simplify  $\left(4\frac{5}{8}+3\frac{2}{3}\right)+6\frac{1}{3}$  using the associative property of addition?
  - $4\frac{5}{8} + (3\frac{2}{3} + 6\frac{1}{3})$
  - **B**  $4\frac{5}{8} \left(3\frac{2}{3} 6\frac{1}{3}\right)$
  - $\frac{5}{8} + 10$
- 5. Write the property from the list next to the step that it justifies in simplifying the expression.

Additive identity property of 0

Additive inverses

Associative property of addition

Commutative property of addition

(3.8 - 4.5) - 3.8 = (3.8 + [-4.5]) + (-3.8)	Subtracting is the same as adding the opposite
= (-4.5 + 3.8) + (-3.8)	
= -4.5 + (3.8 + [-3.8])	
= -4.5 + 0	
= -4.5	

## **LESSON 6: PROPERTIES OF OPERATIONS**

#### **EXERCISES**

6. a. Find the error(s) in Marcus's calculation.

- b. Calculate the correct value.
- 7. a. Find the error(s) in Lucy's calculation.

$$12 - (12 + 17) = (12 - 12) + 17$$
  
= 0 + 17  
= 17

- b. Calculate the correct value.
- 8. a. Find the error(s) in Jack's calculation.

- b. Calculate the correct value.
- 9. Evaluate the expression by combining terms.

$$\left(3\frac{2}{5}+1\frac{4}{9}\right)+\left(2\frac{3}{5}-\frac{4}{9}\right)$$

10. Evaluate the expression.

$$16 - 9 - 7 + 20 - 9$$

11. Evaluate the expression.

$$(-0.09 + 0.73) + (-1 + 0.27)$$

12. Two of these expressions are equivalent. Identify the two equivalent expressions.

$$5 - 12 + 8 - 3$$

$$3 - 8 + 12 - 5$$

$$8 - 12 + 5 - 3$$

# **LESSON 6: PROPERTIES OF OPERATIONS**

# **EXERCISES**

#### Challenge Problem

13. Use properties of operations to prove that these two expressions are equivalent. Show each step and its justification.

$$a-b+c-d$$

$$c - b + a - d$$

### **LESSON 7: PUTTING IT TOGETHER I**

#### **EXERCISES**

#### **EXERCISES**

 Read your Self Check and think about your work so far in this unit. Think about adding and subtracting with negative numbers and about the properties of operations for addition.

Write down three things you have learned during the unit.

Share your work with a classmate.

Does your classmate understand what you wrote?

2. Is there anything that still confuses you about adding and subtracting with negative numbers or about the properties of operations for addition?

Make a list of things that still confuse you.

- 3. Explain the method, approach, or steps you prefer to use when subtracting with negative numbers. Consider including an example as part of your explanation.
- 4. Use your notes from class and your thoughts about the unit to start a math vocabulary list in your Notebook.

Use a chart similar to the one shown to organize your list. Include the vocabulary word or phrase, a definition in your own words, and one or more examples. When appropriate, your example should include a diagram, a picture, or a step-by-step problem solving approach.

Word or Phrase	Definition	Examples
integer	A whole number	Can be positive, negative, or zero
		1 –5,411 0
		256 –8

Add these words to your vocabulary list.

- integer
- additive inverse
- additive identity property of 0
- · associative property of addition
- commutative property of addition
- 5. Complete any exercises that you have not finished from earlier lessons in this unit.

# **LESSON II: MULTIPLYING INTEGERS**

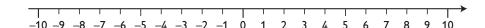
## **EXERCISES**

#### **EXERCISES**

1. Evaluate this expression.

2. Use the number line to show the expression.

$$4(-2)$$



3. a. Complete the table by finding each of the products.

4 • -9 =	
3 • –9 =	
2 • -9 =	
1 • –9 =	
0 • -9 =	

- b. Describe the pattern you see in the equations.
- 4. Extend the pattern you described in the previous exercise to complete this table.

0 • -9 =	0
<b>-1 • −9 =</b>	
<b>-2 • −9 =</b>	
_3 • _9 =	

## **LESSON II: MULTIPLYING INTEGERS**

#### **EXERCISES**

5. Use a pattern—like the ones used in the previous two exercises—to show how to find the product of this equation.

6. Evaluate this expression.

7. Evaluate this expression.

8. Evaluate this expression.

$$8(-3)$$

9. Evaluate this expression.

$$-15(-20)$$

10. Evaluate this expression.

$$(-2)(-2)(-2)$$

11. Evaluate this expression.

$$(-3) \cdot 3 \cdot (-3)$$

12. Without actually finding the product, determine whether these equations have a positive or negative product.

$$(-10) \cdot (-26) \cdot 5 \cdot (-6) =$$

$$(-17) \cdot (-42) \cdot (-19) \cdot 38 =$$
\_\_\_\_

$$(-8) \cdot (-1) \cdot (-9) \cdot (-2) =$$
\_\_\_\_

13. The temperature at midnight is 0°F. The temperature drops 2° every hour for the next 7 hours. Write an equation that shows the temperature at 7 a.m.

## **LESSON II: MULTIPLYING INTEGERS**

#### **EXERCISES**

- 14. When playing her favorite math app, Lucy loses 5 points every time she makes a mistake on an equation.
  - What does the expression 20 (-5) represent in this situation?
  - A Lucy's total score after playing for 25 minutes
  - B The number of points Lucy loses if she makes 20 mistakes
  - The number of points Lucy loses if she correctly solves 5 equations
  - Lucy's total score if she makes 5 mistakes
- 15. Karen spends \$9 of her savings each week to go to the movies.
  - a. What does the expression 6(-9) represent in this situation?
  - b. What does the expression -2(-9) represent in this situation?

#### Challenge Problem

- 16. Consider the expression a(b + 5), where a and b are integers.
  - a. When will the value of the expression be positive?
  - b. When will the value of the expression be negative?

### **LESSON 12: PROVING RULES FOR MULTIPLYING**

#### **EXERCISES**

#### **EXERCISES**

- 1. What is the product of this equation?  $-1 \cdot \left(\frac{1}{2}\right) =$ 
  - **A** -1
  - **B**  $-\frac{1}{2}$
  - $\frac{1}{2}$
  - 1
- 2. According to the multiplicative identity property of 1, what is the product of this equation?

3. The following steps show that this equality is true.

$$-\frac{1}{9} \cdot -3 = \frac{1}{9} \cdot 3$$

Write the property from the list next to the step that it justifies.

Associative property of multiplication

Commutative property of multiplication

Multiplicative identity property of 1

Multiplicative inverses

Multiplicative identity property of 0

$$-\frac{1}{9} \cdot -3 = \left(-1 \cdot \frac{1}{9}\right) \cdot (-1 \cdot 3)$$

$$= -1 \cdot \left(\frac{1}{9} \cdot -1\right) \cdot 3$$

$$= -1 \cdot \left(-1 \cdot \frac{1}{9}\right) \cdot 3$$

$$= (-1 \cdot -1) \cdot \left(\frac{1}{9} \cdot 3\right)$$

$$= 1 \cdot \left(\frac{1}{9} \cdot 3\right)$$

$$= \frac{1}{9} \cdot 3$$

## **LESSON 12: PROVING RULES FOR MULTIPLYING**

**EXERCISES** 

4. Which property tells you that this equality is true?

$$-6 \cdot \left(-\frac{1}{6}\right) \cdot \left(-\frac{5}{4}\right) = 1 \cdot \left(-\frac{5}{4}\right)$$

- A Associative property of multiplication
- **B** Commutative property of multiplication
- Oistributive property
- Multiplicative identity property of 1
- 5. Evaluate this expression.

6. Evaluate this expression.

$$-\frac{3}{4} \cdot -\frac{2}{3}$$

7. Evaluate this expression.

$$-1.2 \cdot 0.09$$

8. Evaluate this expression.

9. Evaluate this expression.

$$\frac{1}{2} \cdot -\frac{1}{3} \cdot \frac{1}{4} \cdot -\frac{1}{5}$$

10. Maya begins a hike at an elevation of 0 ft. Each half hour, her elevation decreases by 13 ft. What is her elevation after 2 hr?

Her elevation is \_\_\_\_ ft.

Challenge Problem

11. Explain why the product of a positive number and a negative number is negative.

## **LESSON 13: DIVIDING**

### **EXERCISES**

#### **EXERCISES**

1. Evaluate this expression.

\_1 ÷ 1

2. Lucy's book is due at the library today, but she has not finished reading it. She will owe the library 12¢ for each day her book is late. If she does not return the book, after how many days will she owe the library 72¢?

Write a division expression to represent this situation.

3. Evaluate this expression.

 $34 \div -17$ 

4. Evaluate this expression.

 $-42 \div -6$ 

5. Consider this division problem.

\_9 ÷ 18 = \_\_\_

- a. Rewrite the equation as a multiplication problem.
- b. Solve the equation.
- 6. Explain how you can use the rules for finding the sign of a product to find the sign of a quotient.
- 7. Evaluate this expression.

$$-\frac{5}{12} \div \frac{5}{6}$$

- 8. Divide -0.4 by -0.02. What is the solution?
  - 0.008
  - **B** 0.05
  - **2**
  - **D** 20

# **LESSON 13: DIVIDING**

#### **EXERCISES**

9. Evaluate this expression.

- 10. Is it possible for two numbers to have a product that is positive and a quotient that is negative? Explain.
- 11. Sophie descended into a cave from an elevation of -8 ft to an elevation of -72 ft in 16 min. On average, how much did her elevation change each minute?

#### Challenge Problem

- 12. Find numbers a and b that satisfy the given criteria. If it is impossible to find numbers a and b for the given criteria, explain why.
  - a.  $a \cdot b > a \div b$  and a and b are both positive.
  - b.  $a \cdot b < a \div b$  and a and b are both positive.
  - c.  $a \cdot b > a \div b$  and a and b are both negative.
  - d.  $a \cdot b < a \div b$  and a and b are both negative.
  - e.  $a \cdot b > a \div b$ , a is negative and b is positive.
  - f.  $a \cdot b < a \div b$ , a is negative and b is positive.

# LESSON 14: THE DISTRIBUTIVE PROPERTY

#### **EXERCISES**

#### **EXERCISES**

- 1. Which expression is equivalent to 3 5.7 + 3 4.3?
  - **A** 3 5.7 4.3
  - **B** 3 5.7 + 4.3
  - $\bigcirc$  5.7(3 + 4.3)
  - $\bigcirc$  3(5.7 + 4.3)
- 2. Which property tells you that this equality is true?

$$-1(-3 + 7) = (-1)(-3) + (-1)(7)$$
?

- Associative property of multiplication
- B Multiplication property of equality
- Multiplicative identity property of 1
- Distributive property
- 3. A frozen fruit bar costs \$0.88. Jack wants to buy a frozen fruit bar for himself and his 4 cousins. Explain how he can use the distributive property to calculate the total cost.
- 4. Find the product. Use the distributive property to simplify the calculation.

5. Find the product. Use the distributive property to simplify the calculation.

$$4(5\frac{2}{9})$$

6. Find the product. Use the distributive property to simplify the calculation.

7. Use the properties of addition and multiplication to simplify this expression.

$$\frac{3}{4} \cdot \frac{1}{9} \cdot 4 + \left(5 \cdot -\frac{1}{12} \cdot \frac{4}{5}\right)$$

## LESSON 14: THE DISTRIBUTIVE PROPERTY

### **EXERCISES**

8. If you used the distributive property to simplify this expression, what would be a step in your simplification?

$$24(-2 + 0.5)$$

- A -48 + 12
- **B** 24(-2.5)
- $\bigcirc$  (-2 + 0.5) 24
- 22 + 24.5
- 9. Use the properties of addition and multiplication to simplify this expression.

$$(5.4-6.3) \div 9$$

10. Use the properties of addition and multiplication to simplify this expression.

- 11. Which expression has the same value as 12 (6 9)?
  - $\triangle$  12 6 9
  - $\bigcirc$  12 6 + 9
  - $\bigcirc$  12 + 6 9
  - $\bigcirc$  12 (9 6)

#### Challenge Problem

- 12. Decide which of the following expressions are equivalent.
  - a. a(b-c)
  - b. (ab) c
  - c. ab-c
  - **d**. (ab) (ac)
  - e. (ab a)c
  - f. ab ac
  - g. (b-c)a
  - h. b (ca)
  - i. b − ca

## **LESSON 15: RATIONAL NUMBERS**

#### **EXERCISES**

#### **EXERCISES**

- Show that 2 is a rational number by writing it as a ratio of integers.
- Which fraction represents this number as a ratio of integers?

0.03

- 3. Write this number as a ratio of integers.

\_9

4. Write this number as a ratio of integers.

28%

5. Write this number as a ratio of integers.

4.7

6. Write this number as a ratio of integers.

0.4444

7. Write this repeating decimal as a fraction  $\frac{a}{b}$ , where a and b are both integers.

 $0.\overline{3}$ 

8. Write this fraction as a terminating or repeating decimal. For a repeating decimal, make sure to indicate which digit or digits repeat.

### **LESSON 15: RATIONAL NUMBERS**

### **EXERCISES**

9. Write this fraction as a terminating or repeating decimal. For a repeating decimal, make sure to indicate which digit or digits repeat.

 $\frac{2}{9}$ 

 Write this fraction as a terminating or repeating decimal. For a repeating decimal, make sure to indicate which digit or digits repeat.

 $1\frac{3}{20}$ 

 Write this fraction as a terminating or repeating decimal. For a repeating decimal, make sure to indicate which digit or digits repeat.

5 12

12. a. Write each of these fractions as a decimal.

 $\frac{1}{9} =$ 

 $\frac{1}{99} =$ 

 $\frac{1}{999} =$ 

1 9,999 =

- b. Describe the pattern you see in your results.
- c. Predict the decimal form of  $\frac{1}{99.999}$ . Check your prediction by dividing.

#### Challenge Problem

- 13. a. Use a calculator to find the decimal equivalents of all the unit fractions from  $\frac{1}{2}$  to  $\frac{1}{12}$ —in other words, the unit fractions  $\frac{1}{2}$ ,  $\frac{1}{3}$ ,  $\frac{1}{4}$ , and so on.
  - b. What do you think determines whether the decimal form of a unit fraction will terminate or repeat? Test your idea with other unit fractions.
     (Hint: Consider the factors of the denominators.)

### **LESSON 16: PUTTING IT TOGETHER 2**

#### **EXERCISES**

#### **EXERCISES**

1. Read your work on the Self Check task and think about your other work in this unit.

Write down the three things you found the most interesting or surprising about what you learned.

Share your work with a classmate.

2. Make a list of anything that still confuses you about the topics covered in this unit.

For example, what uncertainties do you still have about the properties of operations for adding, subtracting, multiplying, or dividing with negative numbers; about inequalities with negative fractions and decimals; about how to convert decimals to fractions; or about how to convert fractions to decimals?

Make a plan for understanding the things that still confuse you. Who will you ask for help and what help will you ask for?

3. Use your notes from class and your thoughts about the unit to add to your math vocabulary list in your Notebook.

Include the vocabulary word or phrase, a definition in your own words, and one or more examples. When appropriate, your example should include a diagram, a picture, or a step-by-step problem solving approach.

Word or Phrase	Definition	Examples
integer	A whole number	Can be positive, negative, or zero
		1 –5,411 0
		256 –8

Add these words to your vocabulary list.

- associative property of multiplication
- commutative property of multiplication
- distributive property
- multiplicative identity property of 1
- multiplicative inverses
- multiplicative property of zero
- rational number
- 4. Complete any exercises that you have not finished from earlier lessons in this unit.