Proportional Relationships: Multistep Percent

There is an underlying proportional relationship in many real-world applications of percents. Thus, you can use proportional relationships to solve a wide variety of multistep percent problems. Here are some common examples.

Sales Tax

In many states, there is a sales tax on certain types of purchases. Such a rate is always stated as a percent.

Suppose the sales tax rate is 6%. Then in addition to the stated price p, you have to pay a sales tax t amounting to 6% of p. It follows that the sales tax is given by this formula:

sales tax: $\boldsymbol{t = 0.06 \bullet p}$

The total cost c is the stated price plus the sales tax. It follows that the total cost is given by this formula:

total cost: $\boldsymbol{c = p + t}$

By combining these two formulas, the total cost can be expressed in terms of p:

total cost: $\boldsymbol{c = p + (0.06 \bullet p) = 1p + 0.06\,p = (1 + 0.06)\,p = 1.06 \bullet p}$

There are two proportional relationships in this situation:

$\boldsymbol{t = 0.06 \bullet p}$

The sales tax t is proportional to the stated price p. The constant of proportionality is the sales tax rate 0.06.

$\boldsymbol{c = 1.06 \cdot p}$

The total cost is also proportional to the stated price *p*. In this case, the constant of proportionality is 1 plus the sales tax rate: 1 + 0.06 = 1.06.

Each of the two proportional relationships gives you useful information. The first relationship tells you the amount of sales tax you are paying. The second relationship tells you the total cost.

Increases

It is common to talk about *increases* in terms of a percent. For example, suppose the manager of a store increases all prices by 5%. This situation can be expressed by this formula:

increase: $\boldsymbol{i = 0.05 \cdot p}$ where *i* represents the increase amount (how much more the item will cost) and *p* represents the original price

However, the amount you are most interested in is not the increase amount (*i*) itself, but rather the new cost of an item. It follows that the new cost is given by this formula:

new cost: $\boldsymbol{c = p + i = p + (0.05 \cdot p) = 1.05 \cdot p}$ where *c* represents the new cost

Both the increase amount *i* and the new cost *c* are proportional to the original price *p*, but the important number is 1.05 in the cost formula. This number is the *growth factor*, which relates the original price to the new price.

Because the growth factor is found by adding 1 to the percent increase, it is always a number greater than 1.

Decreases

It is also common to talk about *decreases* in terms of a percent. For example, suppose the manager of a store decides to have a storewide sale and decreases prices by 5%. This situation can be expressed by this formula:

decrease: $\boldsymbol{d = 0.05 \bullet p}$ where *d* represents the decrease amount (how much less the item will cost) and *p* represents the original price

However, the amount you are most interested in is not the decrease amount (*d*) itself, but rather the new cost of an item. It follows that the new cost is given by this formula:

new cost: $\boldsymbol{c = p - d = p - (0.05 \bullet p) = 0.95 \bullet p}$ where *c* represents the new cost

In this case, the important number is 0.95. This number is the *decay factor*, which relates the original price to the new price.

Because the decay factor is found by subtracting the percent increase from 1, it is always a number less than 1.