

Proportional Relationships: Rates and Units

A ratio where the units being compared are different is sometimes called a *rate* or a *unit rate*. Here are some examples of rates:

- Miles per hour (a rate of speed)
- People per square mile (a measure of population density)
- Dollars per hour (rate of pay)

As in all ratios, calculating rate involves comparing two quantities by division. Consider this example:

Suppose you travel 20 mi in 4 hr. You can compare these two quantities by dividing the distance by the time:

$$20 \text{ mi} / 4 \text{ hr} = 5 \text{ mi} / 1 \text{ hr} = 5 \text{ miles per hour or } 5 \text{ mi/hr or } 5 \text{ mph}$$

The rate 5 mph is a rate of *speed*.

The same two quantities could be compared by division with the numerator and the denominator interchanged:

$$4 \text{ hr} / 20 \text{ mi} = 0.2 \text{ hr} / 1 \text{ mi} = 0.2 \text{ hour per mile or } 0.2 \text{ hr/mi}$$

Since 0.2 hr is 12 min, this rate is equivalent to 12 min per mile. Runners often measure how fast they are going in minutes per mile rather than miles per hour.

Now consider another example:

Suppose you walk 12 mi in 14 hr. Just as with whole numbers or decimals, you can compare two fractions by dividing:

$$12 \div \frac{1}{4} = 12 \cdot 4 = 48 \text{ mi/hr}$$

Remember that when dividing a fraction by a fraction, you convert the second fraction into its reciprocal and then multiply.