MATH GRADE 7 UNIT 2

# PROPORTIONAL RELATIONSHIPS

## ANSWERS FOR EXERCISES

# LESSON 2: PROPORTIONAL RELATIONSHIPS

ANSWERS

## ANSWERS

7.RP.1 **1.** (B) 3.5 mi

7.RP.2 **2.** 1 : 8.2

7.RP.2.a **3.** (A)

| $x$ | 0 | 5 | 4 | 16 |
|---|---|---|---|---|
| $y$ | 0 | 2.5 | 2 | 8 |

7.RP.2.a **4.** (D)

| $x$ | 0 | 1 | 8 | 20 |
|---|---|---|---|---|
| $y$ | 0 | 25 | 200 | 500 |

7.RP.2 **5.**

| **Smoothies** | 1 | 2 | 3 | 5 | 10 | 50 |
|---|---|---|---|---|---|---|
| **Strawberries** | 2.5 | 5 | 7.5 | 12.5 | 25 | 125 |

7.RP.2 **6.**

| $x$ | 0 | 1 | 2 | $2.\overline{6}$ | 5 | 6 |
|---|---|---|---|---|---|---|
| $y$ | 0 | 3 | 6 | 8 | 15 | 18 |

7.RP.1 **7.** The height of the real chairs is 3 ft, which is 36 in. The model chairs are 4 in. So, the scale used to make the model chairs is $\frac{4}{36} = \frac{1}{9}$.

### Challenge Problem

7.RP.2
7.RP.2.a **8.** a.

| **Side** | 1 | 2 | 2.5 | 3 | 100 | 1,000 |
|---|---|---|---|---|---|---|
| **Perimeter** | 4 | 8 | 10 | 12 | 400 | 4,000 |

b. It is a proportional relationship because the perimeter is 4 times the side length, which is a ratio of 4 : 1.

$p = 4s$ or $s = \frac{1}{4}p$

# LESSON 3: CONSTANT OF PROPORTIONALITY

ANSWERS

## ANSWERS

7.RP.2.b **1.** The constant of proportionality is 2.3

7.RP.2.b **2.** $k = 2.4$

7.RP.2.b **3.** $k = \$0.25$

7.RP.2.b **4.** (C) $k$ and $\frac{1}{k}$

7.RP.2.b **5.**

| **x** | 0 | 2 | 4 | 5 | 8 | 10 |
|---|---|---|---|---|---|---|
| **y** | 0 | 10.4 | 20.8 | 26 | 41.6 | 52 |

7.RP.2.b **6.** For the formula $y = kx$: $k = \frac{3}{16}$

To find this constant of proportionality, I evaluated the equation as follows.

$$\frac{x}{y} = \frac{16}{3}$$

$$\cancel{y} \bullet \frac{x}{\cancel{y}} = \frac{16}{3} \bullet y$$

$$x = \frac{16}{3} y$$

$$\frac{3}{16} \bullet x = \frac{\cancel{16}}{\cancel{3}} y \bullet \frac{\cancel{3}}{\cancel{16}}$$

$$\frac{3}{16} x = y$$

$$y = \frac{3}{16} x$$

For the formula $x = \frac{1}{k} y$: $k = \frac{16}{3}$

To find this constant of proportionality, I evaluated the equation as follows.

$$\frac{x}{y} = \frac{16}{3}$$

$$\cancel{y} \bullet \frac{x}{\cancel{y}} = \frac{16}{3} \bullet y$$

$$x = \frac{16}{3} y$$

# LESSON 3: CONSTANT OF PROPORTIONALITY ANSWERS

7.RP.2.a
7.RP.2.b
7.RP.2.c

7. a.

| **Actual** (m) | 5 | 25 | 50 | 73 |
|---|---|---|---|---|
| **Scale Model**(m) | 0.05 | 0.25 | 0.5 | 0.73 |

b. $\frac{73}{0.73} = 100$ or $\frac{0.73}{73} = \frac{1}{100}$

c. $y = 100x$ or $y = \frac{1}{100}x$

## Challenge Problem

7.RP.2.b

8. a. The scales from the smallest model plane to largest are:

   1 : 108, 1 : 72, 1 : 48, 1 : 32

   Real Spitfire planes are all the same size. So, for each scale, the larger the second value is, the greater the difference between each unit length of the actual plane compared to each unit length that the model represents.

   The relative proportion of the model planes to the actual plane decreases as the second value in the scale increases.

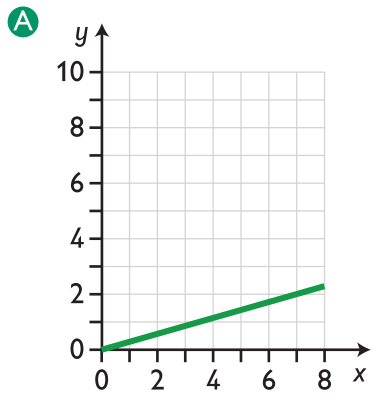
   b. Each scale is a constant of proportionality. Scale and constant of proportionality refer to the same thing when describing the relationship between an actual Spitfire and a scale model.

# LESSON 4: GRAPHS

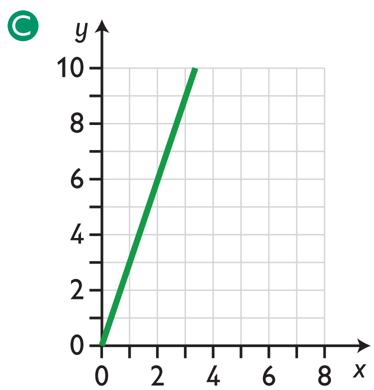
ANSWERS

## ANSWERS

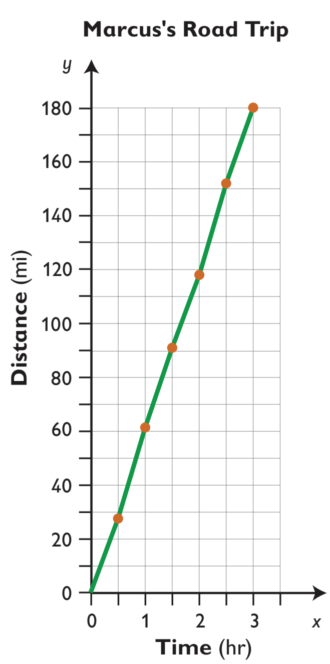
7.RP.2.a **1.**




7.RP.2.b **2.**




7.RP.2.a **3.**



# LESSON 4: GRAPHS

## ANSWERS

7.RP.2.a **4.** (B) is not

7.RP.2.b **5.** Marcus traveled the fastest during the 2.0–2.5 time interval. During this half hour, he traveled 34 mi for an average speed of 68 mi/hr.

| **Time** (hr) | **Distance** (mi) | **Distance This Interval** (mi) | **Speed** (mi : hr) |
|---|---|---|---|
| 0.5 | 28 | 28 | 28 : 0.5 = 56 : 1 |
| 1.0 | 61 | 33 | 33 : 0.5 = 66 : 1 |
| 1.5 | 91 | 30 | 30 : 0.5 = 60 : 1 |
| 2.0 | 118 | 27 | 27 : 0.5 = 54 : 1 |
| 2.5 | 152 | 34 | 34 : 0.5 = 68 : 1 |
| 3.0 | 180 | 28 | 28 : 0.5 = 56 : 1 |

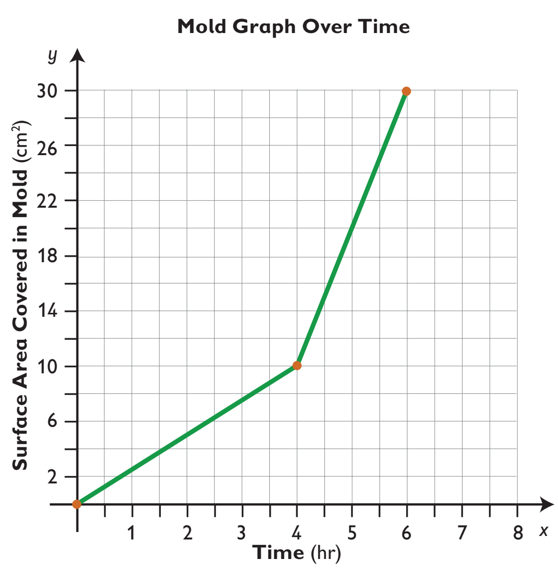
7.RP.2.b **6.**

a. The two graphs have only the points (0, 0) and (4, 150) in common.

b. The rest of the points are not exactly the same.

7.RP.2.a **7.**

a.




b. The growth of the mold is not proportional over time because the rate of growth increases after 4 hours

# LESSON 4: GRAPHS

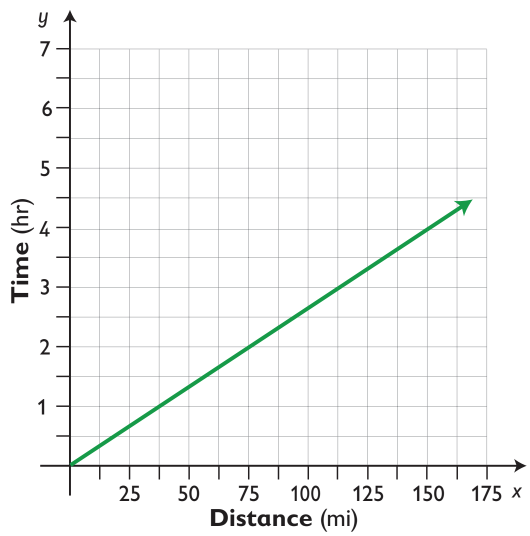
ANSWERS

7.RP.2.b **8.** The graph is a straight line starting at the origin and passing through the points (1, 37.5), (2, 75), (3, 112.5), and (4, 150).

Each $y$-value can be found by multiplying the corresponding $x$-value by 37.5 ($y = 37.5x$), so the constant of proportionality is 37.5.

## Challenge Problem

7.RP.2.b **9.** a.



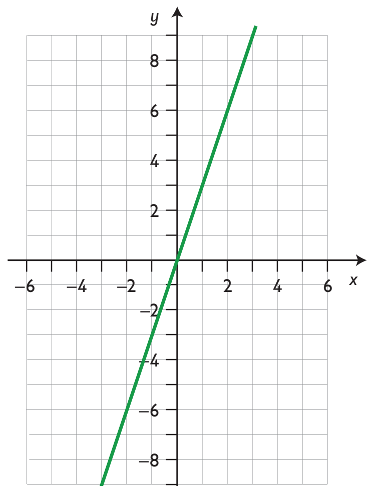

b. The constant of proportionality for this graph is $\frac{1}{37.5}$. This value is the inverse of the constant of proportionality for the original graph.

# LESSON 5: A STRAIGHT LINE GRAPH

ANSWERS

## ANSWERS

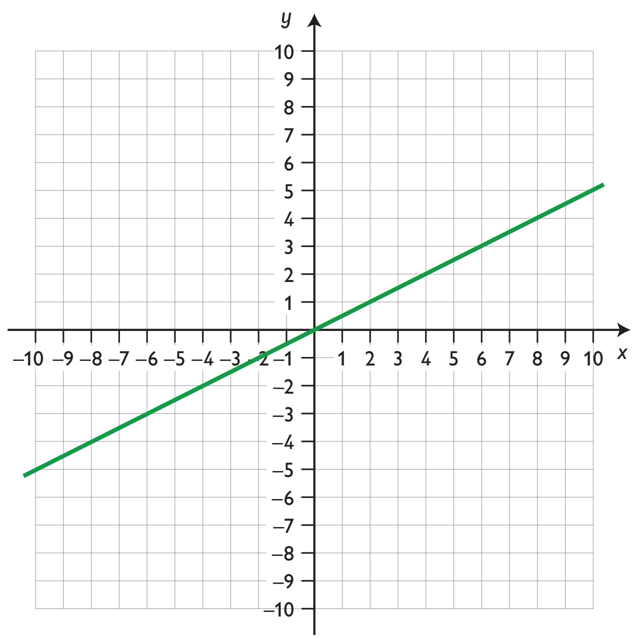
7.RP.2.a **1.** C



7.RP.2 **2.**

- A A proportional relationship can be expressed by the formula $x = ky$.
- B A proportional relationship always includes the point (0, 0).
- E The graph of a proportional relationship will always be a straight line that passes through the origin.

7.RP.2.a **3.** a.



b. This graph does represent a proportional relationship because the graph is a straight line that passes through the origin.

# LESSON 5: A STRAIGHT LINE GRAPH

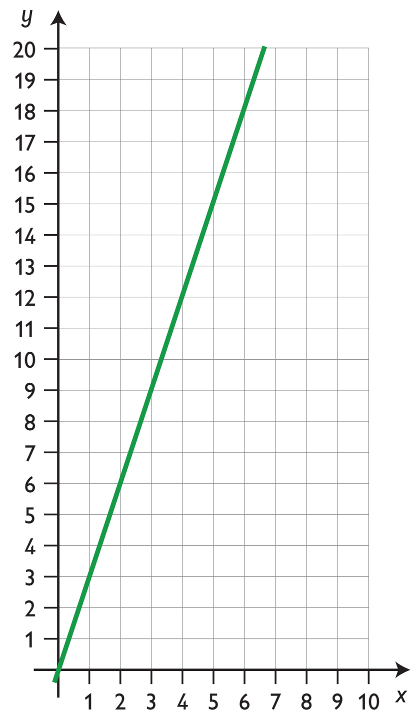
ANSWERS

7.RP.1
7.RP.2

**4.** a.

| $x$ | $\frac{4}{3}$ | $\frac{7}{3}$ | 4 | $\frac{20}{3}$ |
|---|---|---|---|---|
| $y$ | 4 | 7 | 12 | 20 |

b.




7.RP.2.c **5.** A proportional relationship can be represented by the function $y = kx$, where $y$ is the dependent variable, x is the independent variable, and k is the constant of proportionality. Therefore, when $x = 1$, $y = k(1)$, or $y = k$.

7.RP.2.a **6.** A is

7.RP.2.c **7.** If $y$ gives the number of coins and $x$ gives the height of the stack, then the formula is $y = 8x$.

7.RP.2.c **8.** In the formula for Stack B, there is a smaller constant of proportionality than for Stack A.

For Stack A, the constant of proportionality is given by: $\frac{8}{1} = 8$

For Stack B, the constant of proportionality is given by: $k = \frac{8}{1.5} = \frac{16}{3} = 5\frac{1}{3}$

# LESSON 5: A STRAIGHT LINE GRAPH

ANSWERS

7.RP.2.a **9.** Graphs B and D represent proportional relationships.

All four graphs are straight lines, but only these two graphs can be extended to pass through the origin.

## Challenge Problem

7.RP.2
7.RP.2.d **10.** Suppose the graph of a proportional relationship does not pass through the origin. The ratio of the two variables, $\frac{y}{x}$, must be equal to some nonzero constant, $k$. However, because the graph of this relationship does not pass through the origin, it must intersect the $x$-axis at some point $(x, 0)$ with $x \neq 0$. Therefore, $\frac{y}{x} = 0$, which is a contradiction.

# LESSON 6: RATES AND PROPORTIONS

ANSWERS

## ANSWERS

7.RP.2 **1.** Karen could buy 5 cans of peaches for $10.

7.RP.2 **2.** Jack could read 60 pages in 75 min.

7.RP.2 **3.** Maya will have earned 32 days of vacation after working exactly 2 years.

7.RP.1
7.RP.2 **4.** Lucy needs $\frac{2}{3}$ gal more paint to cover a total of 1,100 ft$^2$.

$1,100 - 900 = 200$

$\frac{3}{900} = \frac{x}{200}$

$900x = 600$

$x = \frac{2}{3}$

7.RP.1
7.RP.2 **5.** If Marcus has 40 shirts, then he has 25 pairs of jeans.

unit rate = 5 ÷ 8
= 0.625

number of jeans = 0.625 • 40
= 25

7.RP.1
7.RP.2 **6.** The two towns are 42.5 mi apart.

constant of proportionality $= \frac{5}{1} = 5$

The formula for the distance between the two towns in miles, *d*, in terms of the number of inches on the map, *n*, is:

$d = 5n$
$d = 5 \cdot 8.5$
$d = 42.5$

7.RP.1
7.RP.2 **7.** B $\frac{7 \text{ frogs}}{25 \text{ ft}^2} = \frac{x}{43,560 \text{ ft}^2}$

7.RP.1
7.RP.2.b **8.**

a. $\frac{4}{15}$

b. 3.75 : 1

c. 0.267 : 1

# LESSON 6: RATES AND PROPORTIONS

ANSWERS

## Challenge Problem

7.RP.1
7.RP.2

**9.**

a. Maya's robot can evaluate 5,280 equations in 1 hr.

Method 1—Setting up a proportion and solving for the missing value

$$\frac{572 \text{ equations}}{6.5 \text{ min}} = \frac{x}{60 \text{ min}}$$

$$60 \bullet \frac{572 \text{ equations}}{6.5 \text{ min}} = \frac{x}{\cancel{60} \text{ min}} \bullet \cancel{60}$$

$$\frac{34{,}320 \text{ equations}}{6.5 \text{ min}} = x$$

$$5{,}280 = x$$

Method 2—Finding the unit rate and multiplying

$$\begin{aligned} \text{unit rate} &= 572 \text{ equations} \div 6.5 \\ &= 88 \text{ equations/min} \end{aligned}$$

$$\begin{aligned} \text{total equations} &= \text{unit rate} \bullet \text{time} \\ &= 88 \bullet 60 \\ &= 5{,}280 \end{aligned}$$

Method 3—Writing and solving a formula using the constant of proportionality

$$\text{constant of proportionality} = \frac{572}{6.5} = 88$$

The formula for number of equations, $n$, in terms of time, $t$, is:

$n = 88t$

For $t = 60$, $n = 88(60)$ or 5,280 equations

b. Share your observations with a classmate.

# LESSON 7: UNIT RATES

ANSWERS

## ANSWERS

7.RP.1 **1.** 1.2 laps per minute

7.RP.1 **2.** The rate of pumping gas is $\frac{3.4 \text{ gal}}{1.5 \text{ min}} \approx 2.266$ gal/min.

7.RP.1 **3.** The price of carrot raisin salad is $6.99 per pound.

7.RP.1 **4.** Marcus drank $\frac{1}{20}$ gal/min.

7.RP.2.c **5.**
- **A** $y = 20x$
- **C** $x = \frac{1}{20}y$

7.RP.1 **6.** Maya's walking speed is $2\frac{1}{4}$ mph.

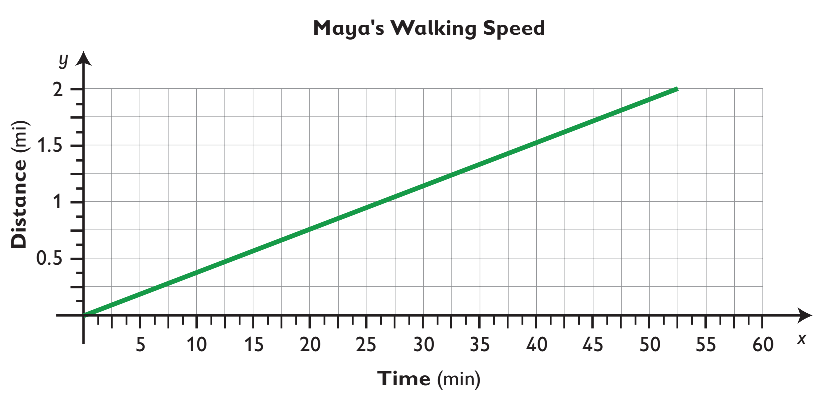
7.RP.1, 7.RP.2 **7.** Maya could walk $5\frac{5}{8}$ miles in $2\frac{1}{2}$ hours.

$$y = 2\tfrac{1}{4} \bullet 2\tfrac{1}{2}$$
$$= \frac{9}{4} \bullet \frac{5}{2}$$
$$= 5\tfrac{5}{8}$$

7.RP.2.c **8.** $y = \frac{9}{4}x$ , where $y$ is the distance in miles and $x$ is the time in hours

$y = 0.0375x$, where $y$ is the distance in miles and $x$ is the time in minutes

7.RP.2 **9.**



# LESSON 7: UNIT RATES

ANSWERS

7.RP.2.b **10.** The constant of proportionality is Maya's walking speed in miles per hour. $2\frac{1}{4}$ mph.

7.RP.1 **11.**

| **Day** | **Volume of Gas** (gal) | **Price** ($) |
|---|---|---|
| Monday | 58.00 | 127.60 |
| Tuesday | 7.00 | 15.40 |
| Wednesday | 11.40 | 25.08 |
| Thursday | 19.00 | 41.80 |
| Friday | 15.00 | 33.00 |

7.RP.2.c **12.** $y = 2.2x$, where $x$ gives the gallons of gas and $y$ the price

## Challenge Problem

7.RP.2.c **13.**

a. $c = 0.75w$, where $c$ is cost and $w$ is weight

b. $c = 0.75(10)$
$= 7.5$

10 lb of tomatoes cost $7.50.

c.
$$\frac{1.5}{2} = \frac{x}{10}$$
$$\cancel{10} \cdot \frac{1.5}{\cancel{2}} = \frac{x}{\cancel{10}} \cdot \cancel{10}$$
$$5 \cdot 1.5 = x$$
$$7.5 = x$$

d. Discuss which method you prefer with a classmate.

# LESSON 8: NON-PROPORTIONAL RELATIONSHIPS

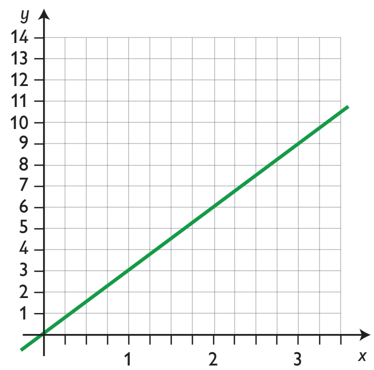
ANSWERS

## ANSWERS

7.RP.2.a **1.** C

| Time | 1 | 3 | 6 | 8 | 12 |
|---|---|---|---|---|---|
| Earnings | \$14.75 | \$44.25 | \$88.50 | \$118.00 | \$177.00 |

E




7.RP.2.a **2.** A is

7.RP.2.a **3.** B non-proportional

7.RP.2.a **4.** It is a non-proportional relationship. Time 0 has a height of 14 in. So, the graph of this relationship does not pass through the origin.

7.RP.2.a **5.** It represents a non-proportional relationship. A graph with $k = 0$ does not represent a proportional relationship.

7.RP.2.a **6.** No, it is a non-proportional relationship. The customer still spends \$30 even if he or she does not visit the park. So, the graph of this relationship does not pass through the origin.

7.RP.2.a **7.** $y = \frac{1}{x}$ Non-proportional relationship

$\frac{1}{4}p = l$ Proportional relationship

$C = 2\pi r$ Proportional relationship

$p - 3 = 2x$ Non-proportional relationship

$y = x - 1$ Non-proportional relationship

# LESSON 8: NON-PROPORTIONAL RELATIONSHIPS ANSWERS

7.RP.2.a **8.** The relationship between cost and games played would become proportional if there were no admission fee. If Jack had paid only the per-game charge, he would have paid $0.50 for each of his 25 games and spent a total of $12.50.

## Challenge Problem

7.RP.2.a **9.**

a. Graph B shows the relationship between the amount of gas used by the van and the miles Maya and her mom traveled.

b. Graph B shows a proportional relationship because it goes through the origin and is a straight line (1 gal of gas is used every 26 mi). Graph A, however, is a non-proportional relationship, because even though it is a straight line it does not pass through the origin.

c. In Graph A, the volume of gas is decreasing as the distance increases. The van starts out with 25 gal in the tank and the amount of remaining gas decreases over the distance of the trip; the van is using the gas at a consistent rate. So, the graph shows a decreasing linear relationship where every 26 mi 1 fewer gallon remains in the tank.

   In Graph B, the gas is increasing as the distance increases. At the beginning of the trip, they haven't used any gas; as they drive farther and farther, their gas use increases at a constant rate. So, the graph shows an increasing linear relationship where every 26 mi 1 more gallon has been used.

# LESSON 9: USING PROPORTIONS

ANSWERS

## ANSWERS

7.RP.2.d **1.** (D) The point (2, 7) does not lie on the graph. The point (4, 16) does lie on the graph.

7.RP.2.d **2.** The point (2.5, 10) represents a volume of 10 $cm^3$ for a height of 2.5 cm.
The point (3, 12) represents a volume of 12 $cm^3$ for a height of 3 cm.

7.RP.2 **3.** The volume of the sand increases at a constant rate because the cross-sectional area of the cylinder does not change.

7.RP.2.b **4.** The constant of proportionality, or unit rate, is 4.

7.RP.2 **5.** $h = 25$ cm

7.RP.2 **6.** The volume of the sand is not proportional to the height for any container in which the cross-sectional area changes with *h*. One example is a cone, such as in an hourglass.

7.RP.2.a **7.** Maya is correct.

7.RP.2.b **8.** $k = \frac{1}{6}$

7.RP.2.c **9.** $p = \frac{1}{6}t$

### Challenge Problem

7.RP.2 **10.**

a. Any relationship in which one variable does not change at a constant rate with the other is not proportional.

b. A proportional relationship is a relationship in which two variables have a ratio that stays the same all the time.

# LESSON 10: ANALYZING GRAPHS

ANSWERS

## ANSWERS

7.RP.2.b **1.**

- **C** The constant of proportionality for line *b* is greater than the constant of proportionality for line *c*.
- **D** The constant of proportionality for line *e* is less than the constant of proportionality for line *d*.

7.RP.2.b **2.** Graph A

7.RP.2.b **3.**

a. Graph A: $k = 1.6$

b. Graph B: $k = 0.9$

7.RP.2.a **4.** Here is an example table.

| $x$ | $y$ |
|---|---|
| 1 | 0.9 |
| 2 | 1.8 |
| 5 | 4.5 |
| 10 | 9 |
| 15 | 13.5 |

7.RP.2.c **5.** $y = 0.9x$

7.RP.2.b **6.** Line *B* represents the plywood.

7.RP.2.b
7.RP.2.c **7.** The constant of proportionality is:

$$k = \frac{4}{5}$$

$$h = \frac{4}{5}n$$

The formula is $h = \frac{4}{5}n$, where *h* is the height and *n* is the number of pieces.

# LESSON 10: ANALYZING GRAPHS

ANSWERS

7.RP.2.b
7.RP.2.c

**8.** The constant of proportionality for two-by fours is:

$k = \frac{8}{5}$

$h = \frac{8}{5}n$

The formula is $h = \frac{8}{5}n$, where $h$ is the height and $n$ is the number of pieces.

7.RP.2.b
7.RP.2.d

**9.** C The drywall graph would not be as steep as line *B*; it would be between line *B* and the *x*-axis.

## Challenge Problem

7.RP.2.b
7.RP.2.d

**10.** a. The graphs represent the same data, but the axes are flipped. The first graph shows that 26 mi can be travelled with 1 gal, and the second shows that 1 gal is needed to travel 26 mi.

I know they represent the same data in different ways because the points marked are inverses of one another.

Points on first graph: (1, 26), (2, 52), (3, 78)

Points on second graph: (26, 1), (52, 2), (78, 3)

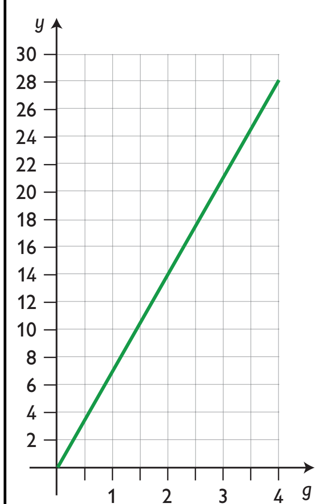
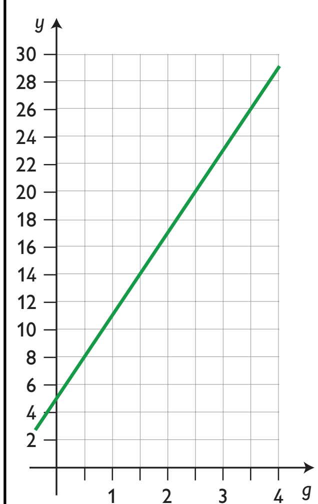
b. The constant of proportionality for the first graph is $\frac{26}{1}$; for the second graph, it is $\frac{1}{26}$. What I notice about the two constants of proportionality is that they are inverses of one another.

# LESSON 11: PUTTING IT TOGETHER 1

ANSWERS

## ANSWERS

7.RP.2 **2.**

<table>
<tr><th></th><th>Proportional Relationship</th><th>Non-Proportional Relationship</th></tr>
<tr><td>Word Problem</td><td>Maya started a business called Maya’s Lawn Mowing Service. She charges $7 each time she mows someone’s lawn.</td><td>Jack started a lawn mowing business, too. Each family that signs up for his service pays an initial fee of $5. Then, each time he mows a family’s lawn he charges $6.</td></tr>
<tr><td>Table</td><td>A constant of proportionality, or a constant ratio, exists.
<table>
<tr><th>Lawn Mowings</th><th>Income</th><th>Ratio</th></tr>
<tr><td>1</td><td>$7</td><td>7 : 1 = 7</td></tr>
<tr><td>2</td><td>$14</td><td>14 : 2 = 7</td></tr>
<tr><td>3</td><td>$21</td><td>21 : 3 = 7</td></tr>
<tr><td>4</td><td>$28</td><td>28 : 4 = 7</td></tr>
</table></td><td>A constant of proportionality, or constant ratio, does not exist.
<table>
<tr><th>Lawn Mowings</th><th>Income</th><th>Ratio</th></tr>
<tr><td>1</td><td>$11</td><td>11 : 1 = 11</td></tr>
<tr><td>2</td><td>$17</td><td>17 : 2 = 8.5</td></tr>
<tr><td>3</td><td>$23</td><td>23 : 3 ≈ 7.67</td></tr>
<tr><td>4</td><td>$29</td><td>25 : 4 = 7.25</td></tr>
</table></td></tr>
<tr><td>Graph</td><td>Graph is linear.<br><br>Graph goes through origin (0, 0).<br>

</td><td>Graph is either linear or nonlinear.<br><br>Graph does not go through origin (0, 0).<br>

</td></tr>
<tr><td>Equation</td><td>Equation follows formula:<br>$y = kx$<br>$i = 7m$<br>where $i$ is income, 7 is the price per lawn mowing, and $m$ is number of lawn mowings</td><td>Equation does not follow formula:<br>$y = kx$<br>$i = 6m + 5$<br>where $i$ is income, 6 is the price per lawn mowing, $m$ is number of lawn mowings, and 5 is the initial fee</td></tr>
</table>

# LESSON 15: CONNECTION TO PERCENT

ANSWERS

## ANSWERS

7.RP.3 **1.** \$37.45

7.RP.3 **2.** \$52.43

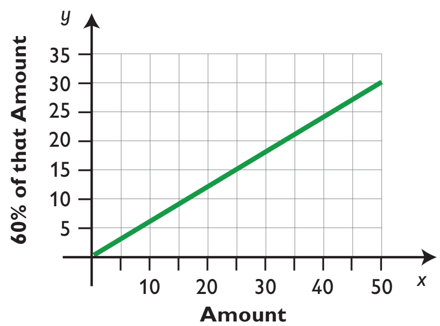
7.RP.2.c 7.RP.3 **3.** $\$3 + \$3(0.07) = \$3.21$

7.RP.2.c 7.RP.3 **4.** $\$15 + \$15(0.07) = \$16.05$

7.RP.3 **5.** C 30

7.RP.2.c 7.RP.3 **6.** $y = 0.6x$, where $x$ is any amount, and $y$ is 60% of any amount

7.RP.2.c 7.RP.3 **7.**




7.RP.3 **8.** C \$3.22

7.RP.2.c 7.RP.3 **9.** $y = 1.2x$, where $x$ is any dollar amount, and $y$ is 120% of any amount

or

$y = x(1 + 0.2)$, where $x$ is any dollar amount and $y$ is the amount plus 20% of the amount

or

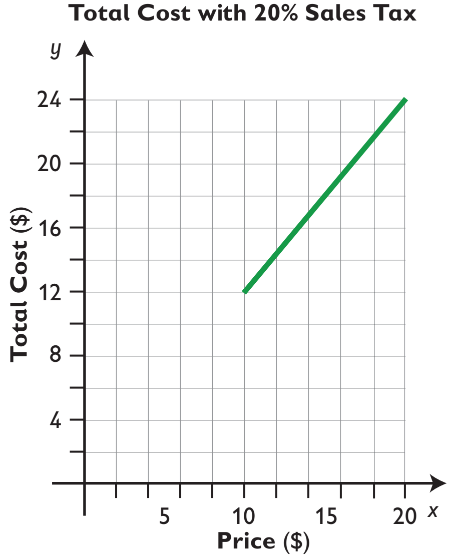
$y = x + 0.2x$, where $x$ is any dollar amount and $y$ is the amount plus 20% of the amount

# LESSON 15: CONNECTION TO PERCENT

ANSWERS

7.RP.3 **10.**




## Challenge Problem

7.RP.3 **11.** When writing a total cost formula, such as for sales tax, I must find the sum of the original cost and the total sales tax. Since the sales tax is the product of some decimal and the original cost, I can use the distributive property to factor out the original cost and write the total cost as the product of the original cost and 1 plus the sales tax decimal.

For the \$3 box of pencils:

$\$3 + \$3(0.07) = \$3(1 + 0.07) = \$3(1.07) = \$3.21$

For the \$15 calculator:

$\$15 + \$15(0.07) = \$15(1 + 0.07) = \$15(1.07) = \$16.05$

# LESSON 16: MORE ABOUT SALES TAX

ANSWERS

## ANSWERS

7.RP.3 **1.**

(A) $t = (0.0875 \cdot 15) + 15$

(B) $t = 15(0.0875 + 1)$

(E) $t = 1.0875 \cdot 15$

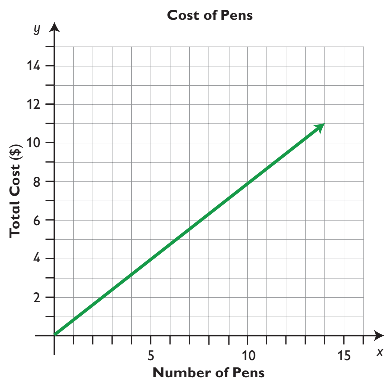
7.RP.3 **2.**

(D)

| Starting Amount | Percent Change | Final Amount |
|---|---|---|
| $19.00 | 7% increase | $20.33 |

7.RP.3 **3.**

(B)




7.RP.3 **4.**

| Starting Amount | Percent Change | Final Amount |
|---|---|---|
| $80 | 6.5% increase | $85.20 |

7.RP.2.c
7.RP.3 **5.**

If $y$ gives the price after sales tax and $x$ is the price before sales tax, then the sales tax, $t$, is given by this equation:

$$\frac{y}{x} - 1 = t$$

I subtract 1 from $\frac{y}{x}$ because this term includes 100% of the price before sales tax ($x$) plus $t$% of the price before sales tax ($t \cdot x$).

7.RP.3 **6.**

24 students should pass the test on their first try. Because you are finding 75% of a given value, there is a percent change of (75 – 100)% = –25%, or a 25% decrease.

| Starting Amount | Percent Change | Final Amount |
|---|---|---|
| 32 students | 25% decrease | 24 students |

# LESSON 16: MORE ABOUT SALES TAX

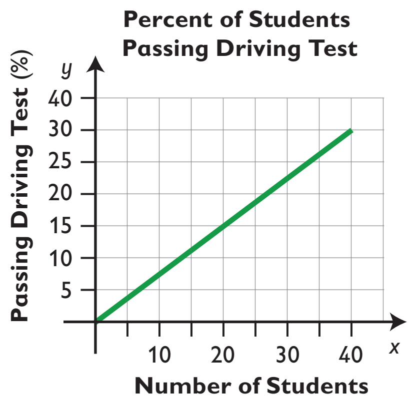
ANSWERS

7.RP.2.c
7.RP.3

**7.** If $x$ gives the total number of students in the class, then the number of students who pass on their first try, $y$, is given by:
$y = 0.75x$.

7.RP.2.c
7.RP.3

**8.**



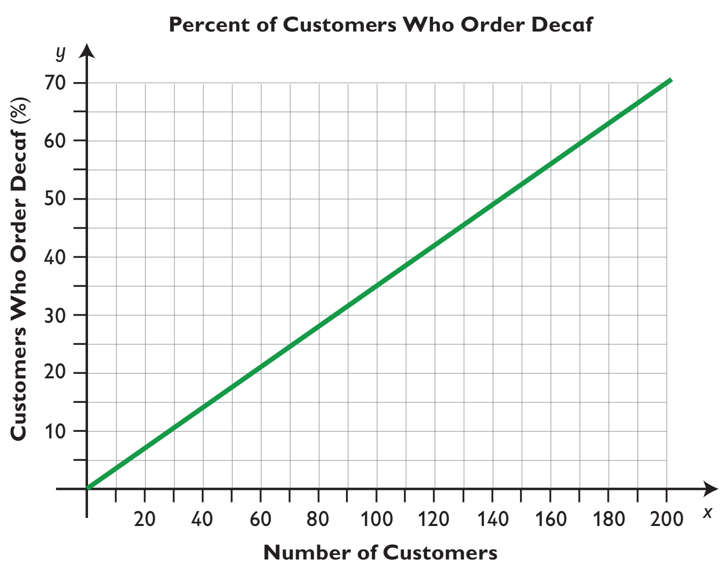

7.RP.2.c
7.RP.3

**9.** Because 65% of customers order regular coffee, (100 – 65)% = 35% of the customers order decaf.

If $x$ gives the total number of customers Lucy's step-brother has on any given morning, then the number of customers who order decaffeinated coffee, $y$, is given by this equation:
$y = 0.35x$.

7.RP.2.c
7.RP.3

**10.**



# LESSON 16: MORE ABOUT SALES TAX

ANSWERS

## Challenge Problem

7.RP.2.c
7.RP.3

**11.**

a. In the formula $y = 0.35x$, $y$ gives the number of people who order decaffeinated coffee. So, I would solve for $x$ using the $y$-value of 54:

$$54 = 0.35x \rightarrow \frac{54}{0.35} = x \rightarrow x \approx 154 \text{ customers}$$

b. To use the graph, I would draw a horizontal line intersecting the $y$-axis at $y = 54$. Where my horizontal line meets the graph of the formula, I would draw a vertical line down to the $x$-axis. The $x$-value for this location gives the total number of customers.

# LESSON 17: PERCENT INCREASE

ANSWERS

## ANSWERS

7.RP.3 **1.** 71.5

7.RP.3 **2.** The original number is 25.

7.RP.3 **3.** There is a 50% increase in the average length of a business phone call.

7.RP.3 **4.** There is a 36% increase in the price of a pair of socks.

7.RP.3 **5.** This year's target amount is $2,967.

$$\begin{aligned} t &= 2{,}580(1 + 0.15) \\ &= 2{,}580 + 387 \\ &= 2{,}967 \end{aligned}$$

7.RP.3 **6.** Karen paid about a 15.8% tip.

7.RP.3 **7.** D 0.20

7.RP.3 **8.** B 1.20

7.RP.3 **9.** The minimum wage in California is $8.00 per hour.

### Challenge Problem

7.RP.3 **10.** £2.8125 of the amount I owe will pay for the tip.

Here is one example.

I use the tip percentage and the total to find the total tip (£11.25).

$$86.25 = 1.15x$$

$$\frac{1}{1.15} \cdot 86.25 = \cancel{1.15}x \cdot \frac{1}{\cancel{1.15}}$$

$$75 = x$$

$$86.25 - 75 = 11.25$$

Then, I divide the total tip by 4 to find the tip per person of £2.8125.

# LESSON 18: PERCENT DECREASE

ANSWERS

## ANSWERS

| | | |
|---|---|---|
| 7.RP.3 | 1. | (D) The decrease is about 5%. |
| 7.RP.3 | 2. | 56.25 |
| 7.RP.3 | 3. | The original number is 50. |
| 7.RP.3 | 4. | (B) 20% |
| 7.RP.3 | 5. | (A) the same as |
| 7.RP.3 | 6. | (A) less than 50% |
| 7.RP.3 | 7. | The percent decrease is 70%. |
| 7.RP.3 | 8. | There is about a 26.47% decrease in the weight of the sand. |
| 7.RP.3 | 9. | The truck collects 13.9 tons of recyclable materials on its second run. |
| 7.RP.3 | 10. | The sale price of the dress is $64.50. |
| 7.RP.3 | 11. | The edited version is 2 min and 48 sec. |
| 7.RP.3 | 12. | The original price of the skates was $620. |
| 7.RP.3 | 13. | The class average is 80.16%, or 64.13 points out of 80. |

# LESSON 18: PERCENT DECREASE

ANSWERS

## Challenge Problem

7.RP.2.c
7.RP.3

14.

a. The coat is on sale for \$96 after both discounts.

b. For an item costing \$$x$, its price is given by the expression $0.48x$.

$x(1 - 0.2)(1 - 0.4)$
$x(0.8)(0.6)$
$0.48x$

c. The final sale price is 52% off the original price.

$0.48 = 1 - d$, where $d$ is the discount (%) off the original price
$d + 0.48 = 1 - d + d$
$d + 0.48 = 1$
$d + 0.48 - 0.48 = 1 - 0.48$
$d = 1 - 0.48$
$d = 0.52$
$d = 52\%$

# LESSON 19: MISTAKES WITH PERCENTS

ANSWERS

## ANSWERS

7.RP.3 **1.**

**A** Lucy says, "If you eat 1 serving, you are eating 260 calories. Of those calories, 120 calories are from fat. So, the percent of calories from fat is 120 ÷ 260, or about 46%."

**B** Marcus says, "No matter how many servings you have, the ratio of calories from fat to total calories stays the same: 120 : 260 or 6 : 13."

**D** Sophie says, "If you reduce the serving size by 50%, you reduce the calories from fat by 50%."

7.RP.3 **2.**

**B** Jack says, "If you eat $\frac{3}{4}$ of a serving, instead of getting 20% of your daily value of total fat you would get 15% of your daily value, because 15% is 75% of 20%.

7.RP.3 **3.**

**B** Karen says, "If you eat $\frac{3}{4}$ of a serving, instead of getting 28% of your daily value of sodium you would get 75% of 28%."

7.RP.3 **4.**

The sale price of the shoes is $25.

7.RP.3 **5.**

Jack is incorrect.

Sophie pays $\frac{1}{4}$ of the original price, because the shoes are discounted $\frac{3}{4}$ of the original price. She paid $25 for the shoes. If she had paid $\frac{3}{4}$ of the original price, she would have paid $75.

7.RP.3 **6.**

Sophie is correct.

A sale of 75% means each item is 25% of its original price.

So, the ratio of the sale price to the original price is 25 : 100, or 1 : 4.

7.RP.3 **7.**

It is possible that some of the students watched both sports. In fact, all students who watched hockey could have also watched basketball. So, the school reporter cannot simply add together the two percents.

It would have been more accurate for the student to report that between 70% and 80% of the students did not watch either sport.

7.RP.3 **8.**

Karen added the percents she got on each test: 20% + 60% = 80%. If she got lower scores on each test, it doesn't make sense that her overall score would be higher.

I can test her reasoning by using a specific number for how many questions there were on each test. If there were 100 questions on each test, then Karen got 20 out of 100 questions on one test plus 60 out of 100 on the second test. That means Karen scored 80 out of 200 questions total, or 40%.

# LESSON 19: MISTAKES WITH PERCENTS

ANSWERS

7.RP.3 **9.** On Blowout Sale Friday, the price of every item was 30% of its original price. When the store owners increased prices the next day, they multiplied 50% times a much smaller number. So, it doesn't make sense that the price would only be 80% of the original.

$b = (1 - 0.7)p$, where $b$ is the blowout sale price and $p$ is the original price ($20)
$b = (0.3)20$
$b = 6$
The sale price of a $20 item is $6.

$n = (1 + 0.5)b$, where $n$ is the new price on Saturday and $b$ is the blowout sale price ($6)
$n = (1.5)6$
$n = 9$
The new price of a $6 sale item (that was originally $20) is $9.

Comparing the new price to the original price, I find that the new price is 45% of the original price.

$$\frac{9}{20} = 0.45 = 45\%$$

So, the new prices on Saturday are 55% lower (1 – 0.45) than they were before Blowout Sale Friday.

## Challenge Problem

7.RP.3 **10.** Lucy has not made any mistakes with her calculations, and she hasn't done anything wrong. She has made several mistakes in her reasoning, though.

One mistake is that she thinks the graphs show that the two savings plans represent proportional relationships. The amount gained increases each year. The graphs appear linear, at least for the first few years, but they aren't actually proportional.

If the plans represented proportional relationships, you would find the total gain using the formula $y = kx$.

Plan 1: $y = kx$, where $y$ is the total gain, $k$ is the constant of proportionality, and $x$ is time in years. In this case, $k$ would be $(0.03 \cdot s)$, where $s$ is the original amount.
Year 1: $y = (0.03 \cdot 300) \cdot 1 = \$9$
Year 2: $y = (0.03 \cdot 300) \cdot 2 = \$18$

Plan 2: $y = kx$, where $y$ is the total gain, $k$ is the constant of proportionality, and $x$ is time in years. In this case, $k$ would be $(0.02 \cdot s)$, where $s$ is the original amount.
Year 1: $y = (0.02 \cdot 300) \cdot 1 = \$6$
Year 2: $y = (0.02 \cdot 300) \cdot 2 = \$12$
Year 3: $y = (0.02 \cdot 300) \cdot 3 = \$18$

*(continues)*

# LESSON 19: MISTAKES WITH PERCENTS

ANSWERS

7.RP.3 **10.** *(continued)*

However, in this situation, you don't multiply the percent increase by the original amount each year. You multiply it by that year's starting amount. For example, in the first plan in year 1, you calculate a 3% increase of $300. In year 2, you calculate 3% of $309.

So, even though the plans use constant percent increases, each year those percents are multiplied by increasing amounts. This is an example of exponential growth.

Another mistake Lucy made is to add each year's percent increase to get a total percent increase of 6%.

Plan 1, 3% + 3% = 6%

Plan 2, 2% + 2% + 2% = 6%

In Plan 1, the total percent increase is actually:
$318.27 ÷ $300.00 = 1.0609, which is a 6.09% increase

In Plan 2, the total percent increase is actually:
$318.36 ÷ $300.00 = 1.0612, which is a 6.12% increase

# LESSON 20: PUTTING IT TOGETHER— PART 2

ANSWERS

## ANSWERS

| | | |
|---|---|---|
| 7.RP.3 | 1. | C 1.64 |
| 7.RP.3 | 2. | A $\frac{6}{9}$<br>C $\frac{2}{3}$ |
| 7.RP.3 | 3. | B 1.2<br>C 20% increase<br>E $\frac{6}{5}$ |
| 7.RP.3 | 4. | 11 books |
| 7.RP.3 | 5. | A $\frac{1}{0.125}$ |
| 7.RP.2.a<br>7.RP.2.c | 6. | a. 4 mi<br>b. $t = 9.5d$ or $d = \frac{1}{9.5}t$ |
| 7.RP.3 | 7. | 6% increase |
| 7.RP.2d | 8. | The point (10, 95) indicates her desired time of 95 min after 10 km. |
| 7.RP.3 | 9. | 30% decrease |
| 7.RP.3 | 10. | 42.9% increase |
| 7.RP.3 | 11. | The percent increase and the percent decrease are not the same because you are taking percents of different values. For the percent decrease, you are taking a percent of 25. For the percent increase, you are taking a percent of 17.5. |

# LESSON 20: PUTTING IT TOGETHER—PART 2 ANSWERS

## Challenge Problem

7.RP.3 **12.** Here is one student's work.

I made a table of sample values to test the equation.

| ***n*** (number) | ***p*** (percent) | ***r*** (product) |
|---|---|---|
| 10 | 1 | 0.1 |
| 10 | 10 | 1 |
| 10 | 50 | 5 |
| 10 | 99 | 9.9 |
| 10 | 100 | 10 |
| 10 | 101 | 10.1 |
| 10 | 110 | 11 |
| 10 | 150 | 15 |
| 10 | 199 | 19.9 |
| 10 | 200 | 20 |

The product of a number, $n$, times a percent, $p$, is greater than the number, $n$, when $p$ is greater than 100.

The product of a number, $n$, times a percent, $p$, is less than the number, $n$, when $p$ is less than 100.

Another way to say this is that 100% of any number is the number itself. Anything over 100% of a number is greater than the number. Anything below 100% of a number is less than the number.

# LESSON 21: PUTTING IT TOGETHER—PART 3 ANSWERS

## ANSWERS

7.RP.2
7.RP.3

**2.** Here is one student's work.

My grandparents live in Guatemala. There the money they use is quetzales and centavos. When I visited last year, I had to convert back and forth between U.S. dollars and Guatemalan quetzales all the time. The exchange rate changed frequently, but it was around $1 to Q7.65. I used the exchange rate to figure out how much something would cost in dollars. Also, when I wanted to exchange dollars to get cash in quetzales, I would compare different rates at different banks to find the best place to exchange my money.