MATH GRADE 7 UNIT 3

CONSTRUCTIONS AND ANGLES

ANSWERS
FOR EXERCISES

LESSON 2: ANGLES

ANSWERS

ANSWERS

7.G.5 1. (A) ∠1 and ∠2
(D) ∠3 and ∠4

7.G.5 2. (A) ∠1 and ∠4
(C) ∠2 and ∠3

7.G.5 3. (D) ∠3 and ∠4
(E) ∠6 and ∠1

7.G.5 4. The measure of ∠3 is 127°

7.G.5 5. (C) 145°

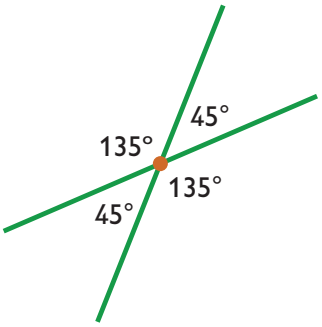
7.G.5 6. (D) ∠1 and ∠4

7.G.5 7. (A) ∠4 and ∠2

7.G.5 8. The angle opposite the 45° angle measures 45° as well because it is a vertical angle. Vertical angles are congruent.

The angle adjacent to the 45° angle and the 45° angle are supplementary angles because each has an uncommon side that lies on the same line. The angle adjacent to the 45° angle measures 135°. 45° + 135° = 180°

So, the measures of the other three angles are 45°, 135°, and 135°.



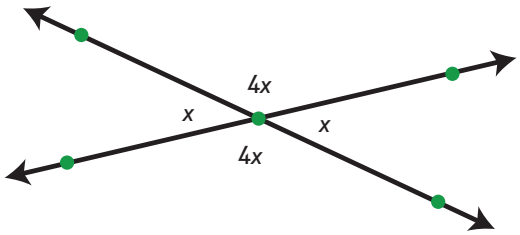
The measures of all four angles add up to 360°. 45° + 135° + 45° + 135° = 360°

LESSON 2: ANGLES

ANSWERS

Challenge Problem

7.G.5 9.



The two angles that are described must be supplementary. (If they were vertical, they would be congruent.) If x is the first angle, the second angle is $4x$.

$x + 4x = 180°$

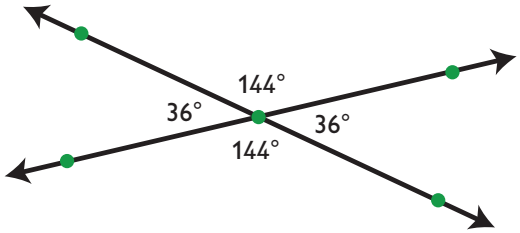
$5x = 180°$

$x = 36°$

Second angle:

$4x = 4(36°) = 144°$

So, one angle measures 36° and the second angle measures 144°. The other two angles have the same measures because they are vertical to the first two angles. The measures of the four angles are 36°, 144°, 36°, and 144°.



LESSON 3: PROPERTIES OF PARALLELOGRAMS ANSWERS

ANSWERS

7.G.2 1. C Rectangle

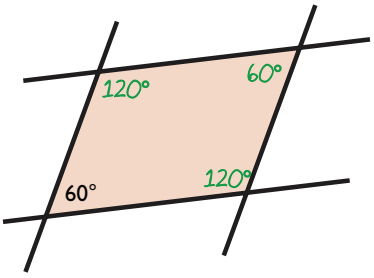
7.G.5 2. B $\angle 3$
D $\angle 5$
E $\angle 7$

7.G.2 3. C $\overline{AC}$ and $\overline{BD}$
D $\overline{BC}$ and $\overline{AD}$

7.G.5 4. Since the figure is a parallelogram, opposite angles are congruent; thus, there will be a 60° angle opposite the given angle.

Adjacent angles are supplementary, so the angles to either side of the given angle will measure 180° – 60° = 120°.

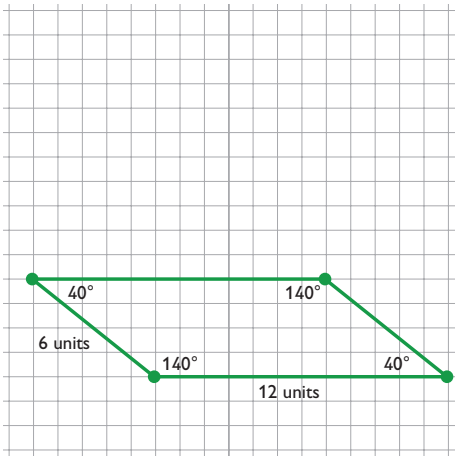
The other three angles measure 120°, 60°, and 120°.



7.G.5 5. Since the figure is a parallelogram, opposite angles are congruent and adjacent angles are supplementary. There must be a 90° angle opposite the known angle and 90° angles next to the known angle, because their measures must add up to 180°.

So, the other three angles must all measure 90°.

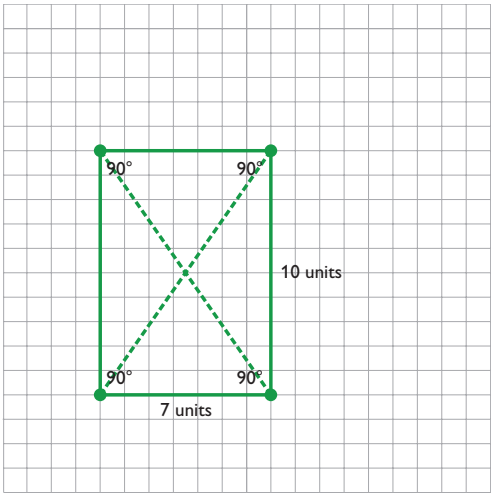
7.G.2 6.



LESSON 3: PROPERTIES OF PARALLELOGRAMS

ANSWERS

7.G.2 **7.**

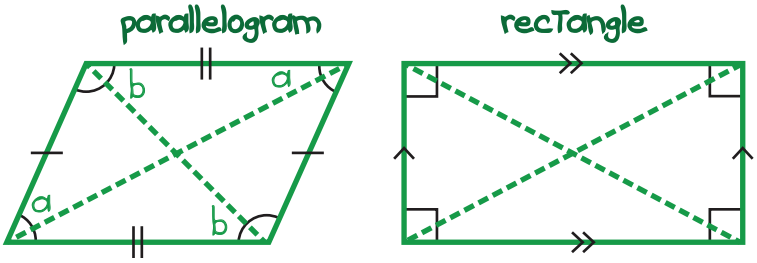


7.G.5 **8.** **D** 300°

Challenge Problem

7.G.2 **9.** A parallelogram and a rectangle both have four sides, with two pairs of opposite sides that are congruent. The two pairs of opposite sides are also parallel. The four interior angle measures add up to 360°.

A parallelogram differs from a rectangle in that it has two pairs of congruent angles, while a rectangle has four 90° angles. In a parallelogram, the diagonals bisect one another whereas the diagonals in a rectangle bisect one another and are congruent.



LESSON 4: PROPERTIES OF QUADRILATERALS

ANSWERS

ANSWERS

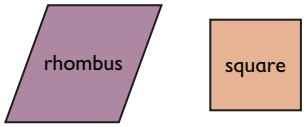
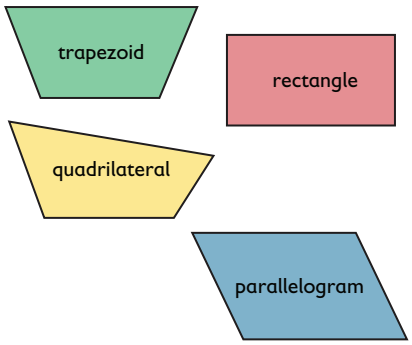
7.G.2 1. Ⓐ Square

7.G.5 2. The sum of the angle measures of all the angles in this quadrilateral is 360°.

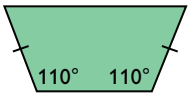
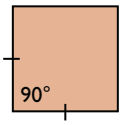
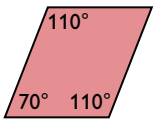
7.G.5 3. Ⓓ 140°

7.G.5 4. Ⓓ 145°

7.G.2 5.

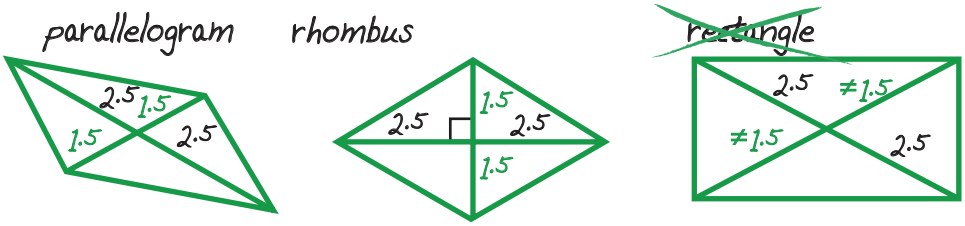
Diagonals Are Perpendicular	Diagonals Are Not Perpendicular
	

7.G.5 6.

Quadrilateral	Missing Angle Measures
	140°
	270°
	90°
	70°

LESSON 4: PROPERTIES OF QUADRILATERALS ANSWERS

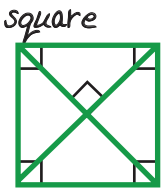
7.G.2 **7.** The figure is a parallelogram because the diagonals bisect each other. More specifically, it may also be a rhombus if the diagonals happen to be perpendicular. The figure cannot be a rectangle because a rectangle has congruent diagonals and 3 in. ≠ 5 in.



7.G.2
7.G.5 **8.** Since all four angles are congruent, they must each measure 90°.

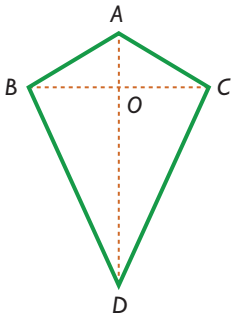
90° + 90° + 90° + 90° = 360°

So, the figure is a rectangle. However, since the diagonals are perpendicular to each other, it must also be a rhombus and, more specifically, a square.

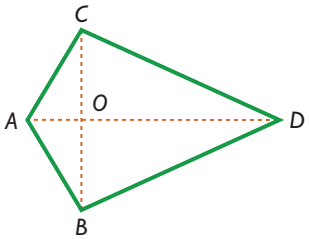


Challenge Problem

7.G.2 **9.** **a.** True. The diagonals of a kite are perpendicular to each other. When you try many cases, like the figure below, they are also perpendicular.



b. Sometimes true. If a figure is a kite, one of the diagonals bisects a pair of opposite angles. The diagonal *AD* bisects the angles but the diagonal *BC* does not.



c. True. A rhombus is a kite. Since by definition a kite has two pairs of equal sides adjacent to each other, and a rhombus is a parallelogram with four equal sides, the rhombus has two pairs of equal sides adjacent to each other. In fact, it has four pairs of equal sides adjacent to each other.

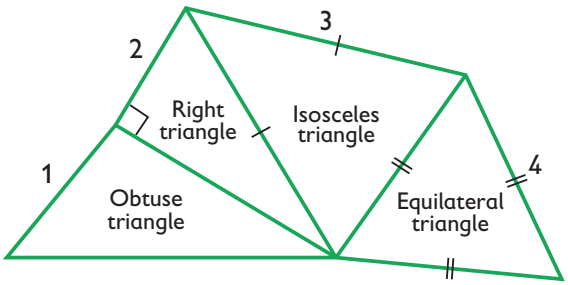
LESSON 5: BUILDING TRIANGLES

ANSWERS

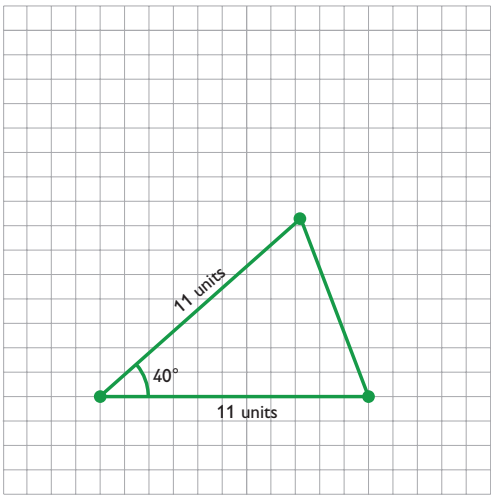
ANSWERS

7.G.2 1. **B** The measure of the third angle is 30°

7.G.2 2.



7.G.2 3.



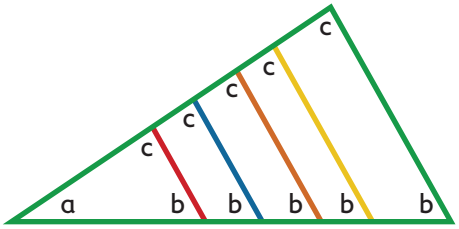
7.G.2 4. The sum of the two other angles is 90°

7.G.2 5. In order to form a triangle, the angle measures must add up to 180°.

$\angle a + \angle b + \angle c = 180°$

If the sum of the angle measures is 180°, an infinite number of triangles can be constructed. All of the triangles will be similar to each other.

This relationship can be shown using parallel sides for the third side of a triangle.

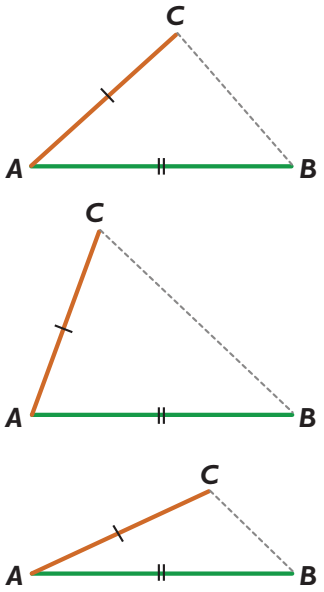


LESSON 5: BUILDING TRIANGLES

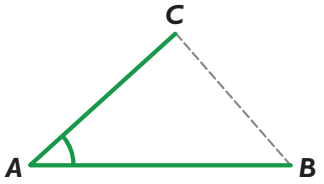
ANSWERS

7.G.2 **6.** If two side lengths of a triangle are known, an infinite number of triangles can be formed because the angle between the known sides can be any measure, allowing the third side to be any length.

Here are three examples.



7.G.2 **7.** If the lengths of two sides and the measure of the included angle are given, there is only one side length possible for the third side. That means there is only one possible triangle that can be formed.



7.G.2 **8.** $\angle 2$ is supplementary with the 117° angle because they form a straight line.

$180° - 117° = 63°$

The measures of three angles in the triangle must add up to 180°. To find the measure of $\angle 1$, add the two known angle measures and subtract from 180°.

$48° + 63° = 111°$

$180° - 111° = 69°$

So, the measure of $\angle 1$ is 69°. The measure of $\angle 2$ is 63°.

LESSON 5: BUILDING TRIANGLES

ANSWERS

Challenge Problem

7.G.2 **9.** Since ΔDEF is equilateral, the three angles are congruent and measure 60° each.

$180° \div 3 = 60°$, or $3 \cdot 60° = 180°$

$\angle EFG$ measures 120° because it forms a straight line with $\angle EFD$.

$180° - 60° = 120°$

ΔEFG is an isosceles triangle, so it has two congruent angles.

Subtract 120° from 180° to find the sum of the congruent angle measures.

$180° - 120° = 60°$

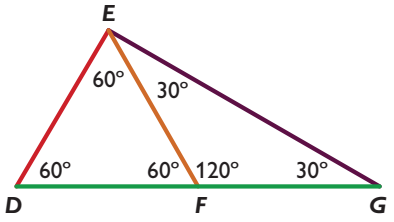
Divide the sum by 2 to find the measure of each angle.

$60° \div 2 = 30°$

$\angle DEG$ measures 90° because it is the sum of the two adjacent angles, $\angle DEF$ and $\angle FEG$.

$60° + 30° = 90°$

Since ΔDEG has a 90° angle, it is a right triangle. All of the side lengths are different, so it is also a scalene triangle.



LESSON 6: PROPERTIES OF TRIANGLES

ANSWERS

ANSWERS

7.G.5 1.
- B Right triangle
- D Isosceles triangle

7.G.5 2.
- C 40° and 100°
- F 70° and 70°

7.G.5 3.
- A 58° and 58°
- B 32° and 84°
- E 40° and 76°

7.G.2 4.
- C Two of the sides must be the same length.

7.G.2 5.
- B All three angles are the same measure.

7.G.2 6.
- D The sum of the lengths of the two shorter sides must be greater than the length of the third side.

7.G.2 7.
- A 5 cm
- B 13 cm
- C 27 cm

7.G.5 8.

No matter how high the bridge tilts up, $\angle EDF$ will measure 90°. Without more information, you cannot determine the exact measure of $\angle DEF$.

You know that the angle measures of a triangle must add up to 180°. So, the sum of the measures of $\angle DEF$ and $\angle DFE$ equals 90° because 180 – 90 = 90°.

Since the sum of the measures of $\angle DEF$ and $\angle DFE$ is 90°, the measure of $\angle DEF$ is greater than 0° and less than 90°.

7.G.2 9.

Maya misses the point that the angle measures need to be taken into account. Without measuring the angle measures, her construction is unlikely to result in a congruent triangle.

LESSON 6: PROPERTIES OF TRIANGLES

ANSWERS

Challenge Problem

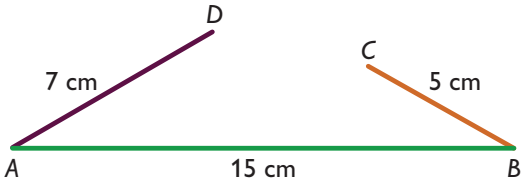
7.G.2

10. No, you cannot form a triangle with these sticks.

The two shorter lengths, 5 cm and 7 cm, add up to 12 cm. But the third stick is longer than that sum. 15 cm > 12 cm

To form a triangle, the lengths of the two shorter sides must add up to more than the length of the third side.

Here is a diagram showing why these sticks cannot form a triangle.



No matter how you arrange the three sticks, they cannot form a triangle.
If I imagine swinging points *C* and *D* to any position, they will never reach one another to complete the triangle.

LESSON 7: ANGLE SUMS

ANSWERS

ANSWERS

7.G.5 1. B 900°

7.G.5 2. C Octagon

7.G.5 3. B 130° and 140°

7.G.5 4. No, the angle sum is not 720°.

Although the figure is divided into four triangles, the total angle measure of the quadrilateral has not changed and remains 360°.

The four angles that meet at the center of the parallelogram are not part of the angles of the parallelogram. The sum of the measures of those angles is 360° because they make a complete circle.

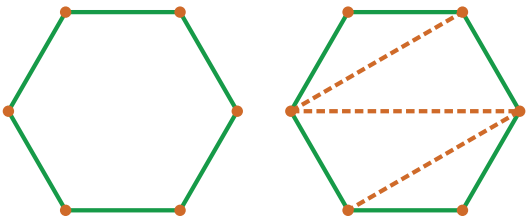
So, if the four triangle sums are added to get 720°, you must also subtract 360°.

720° – 360° = 360°

7.G.5 5. The angle sum is not correct because 2,400° is not evenly divisible by 180°.

However, 2,340° is a multiple of 180° (13 • 180°), and the student could have easily made an accumulated error of 60° while measuring the 15 angles with a protractor. The error would be about 4° per angle.

7.G.2
7.G.5 6. Any hexagon can be divided into four triangles, as shown in this diagram.



The measures of all of the angles in each of the four triangles are included in the total measure of the interior angles of the hexagon, so you can just add their measures.

180° + 180° + 180° + 180° = 720°

7.G.5 7. $\angle ABC = 154.29°$

7.G.5 8. $\angle 1 = 120°$

7.G.5 9. C 1,440°

LESSON 7: ANGLE SUMS

ANSWERS

Challenge Problem

7.G.5 10. a. Here are two examples.

Example One

For each polygon, one triangle section can be analyzed since they are all the same. The triangles are the same because the diagonal segments are all congruent and the third sides, the polygon sides, are all congruent.

This also means each triangle is an isosceles triangle and that the two angles at the polygon side are congruent. The angle measures in the polygons are known, and any two angles at the polygon side add to equal this measure. The third angle at the center is found by subtracting the polygon angle measure from 180°.

Square: 180° – 90° = 90°

Hexagon: 180° – 120° = 60°

Octagon: 180° – 135° = 45°

Example Two

For each polygon, the sum of the measures of the angles at the center is 360° because they make a complete circle. For each polygon, each of the triangles that divide it is the same because the diagonal segments are all congruent and the third sides, the polygon sides, are all congruent.

This also means that the angles in the center of each polygon divide the 360° into equal size angles. The angles at the center are found by dividing 360° by the number of triangles.

Square: 360° ÷ 4° = 90°

Hexagon: 360° ÷ 6° = 60°

Octagon: 360° ÷ 8° = 45°

b. The angles at the center are getting smaller as the number of sides increases. The angle measures will get closer and closer to zero as the polygon angle gets closer and closer to 180°.

The polygon would be getting closer to a circle and the segments going to the center are getting closer to being radii.

LESSON 8: FINDING ANGLES

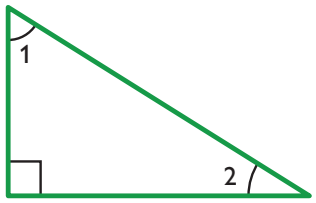
ANSWERS

ANSWERS

7.G.5 1. D 130°

7.G.2 2. C Rhombus

7.G.5 3. B



7.G.5 4. $x = 40°$

7.G.5 5. $x = 20°$

7.G.5 6. $\angle 1 = 22.5°$

7.G.5 7. The sum of the interior angles of an octagon is 1,080°.

180° • 6 = 1,080°

Since there are 8 angles, the measure of each angle is 1,080° ÷ 8 = 135°.

The sides of the figure inside the octagon form the base of four isosceles triangles with the octagon. The measure of each acute angle (such as ∠1) of these isosceles triangles is 22.5°.

22.5° + 90° + 22.5° = 135°

Using the same process all around the octagon, you can determine that each angle of the inside figure measures 90°. Each side is the same length because the sides are part of congruent triangles.

So, the figure inside the octagon is a square.

7.G.5 8. $\angle ACB = 64°$

7.G.5 9. $\angle ACB = 106°$

LESSON 8: FINDING ANGLES

ANSWERS

Challenge Problem

7.G.5

10. Since ΔABC is isosceles, it has two congruent angles, so 1 also measures 75°.

The last angle in ΔABC must measure 30° because 75° + 75° + 30° = 180°.
This 30° angle and the measure of $\angle 3$ total 45°, so $\angle 3$ measures 15°.

45° – 30° = 15°

$\angle 2$ also measures 15° since ΔBCD is isosceles.

This leaves 150° for the measure of $\angle 4$. 15° + 15° + 150° = 180°.

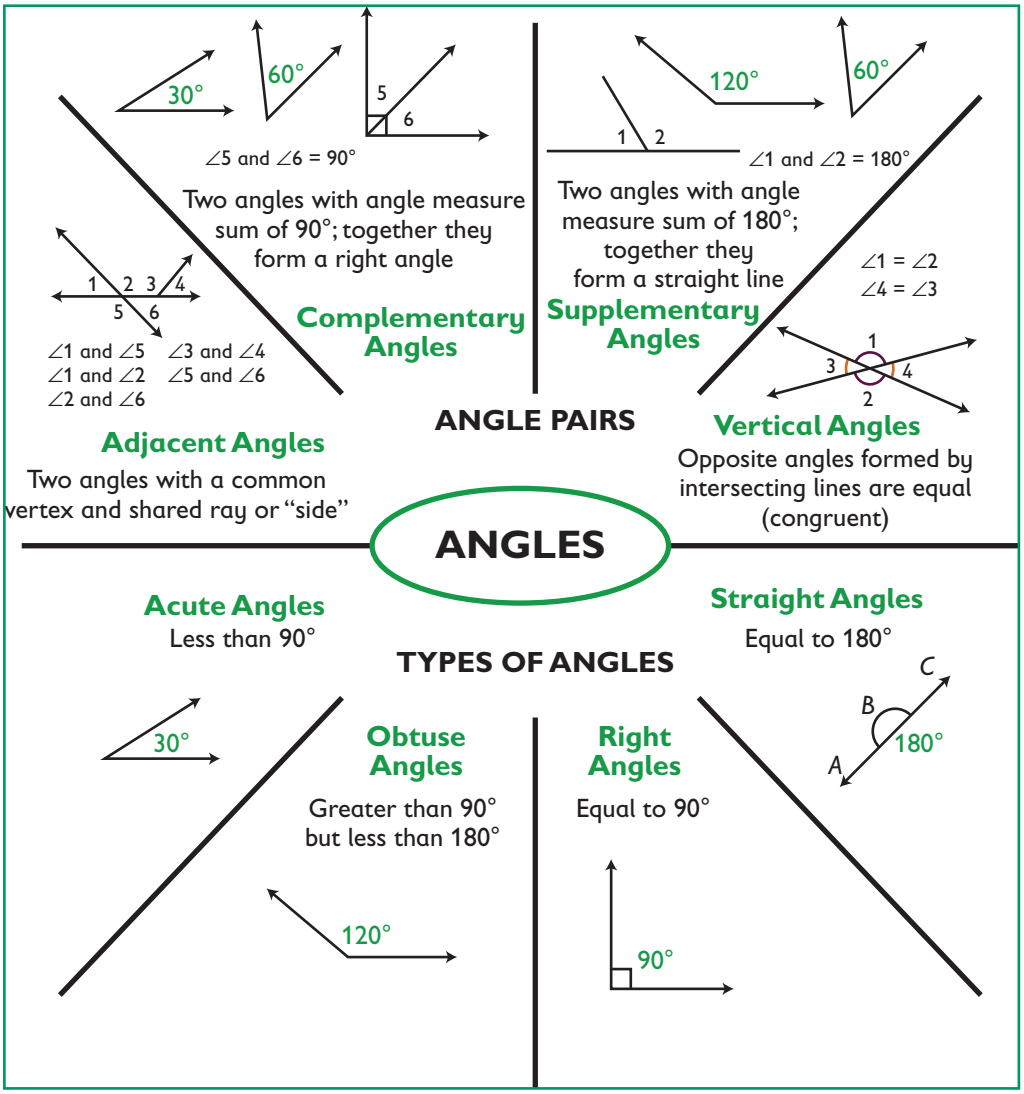
LESSON 9: PUTTING IT TOGETHER

ANSWERS

ANSWERS

7.G.2
7.G.5

3. Here is one example.

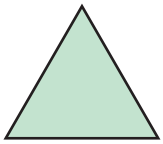
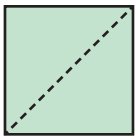
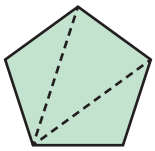
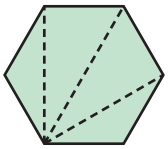
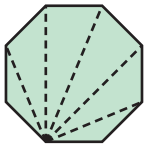


LESSON 9: PUTTING IT TOGETHER

ANSWERS

7.G.2
7.G.5

4.

Shape	Number of Sides	Number of Triangles	Angle Sum– Interior Angles	Each Interior Angle Measure (if regular polygon)	Angle Sum– Exterior Angles	Each Exterior Angle Measure (if regular polygon)
Triangle 	3	1	180°	60°	360°	120°
Quadrilateral 	4	2	360°	90°	360°	90°
Pentagon 	5	3	540°	108°	360°	72°
Hexagon 	6	4	720°	120°	360°	60°
Octagon 	8	6	1,080°	135°	360°	45°
Any Polygon	n	$n - 2$	$(n - 2)180°$	$(n - 2)\frac{180°}{n}$ or 180° – exterior angle measure	360°	$\frac{360°}{n}$ or 180° – interior angle measure