

## LESSON 1: PAPER FOLDING

## EXERCISES

## EXERCISES

1. Write three things you already know about angles.

Share your work with a classmate.

Does your classmate understand what you wrote?

2. Write your wonderings about angles.

Share your wonderings with a classmate.

3. Think about your work in the math units you have completed this year so far. Describe something you are especially proud of and that you would like to repeat in this unit.

For example, consider how you participated in discussions, ways you worked with partners or helped a classmate, a new math concept you learned, questions you shared that helped you better understand a topic, a challenge or exercise that you enjoyed completing, and so on.

4. In this lesson, you folded paper to create a parallelogram and considered the resulting figures.

What aspect(s) of this activity were challenging?

How did it affect your figures if you did not carefully line up the edges of your paper before making a fold? If you did not encounter this issue during the lesson, try the activity again and test what happens if you do not carefully align the edges before folding. Write down your observations of how not being precise affects the figures you create.

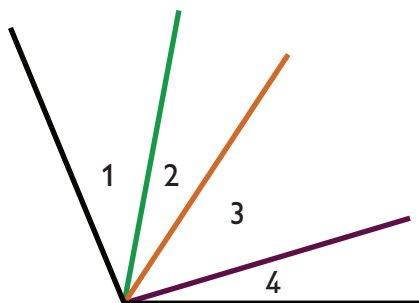


## LESSON 2: ANGLES

## EXERCISES

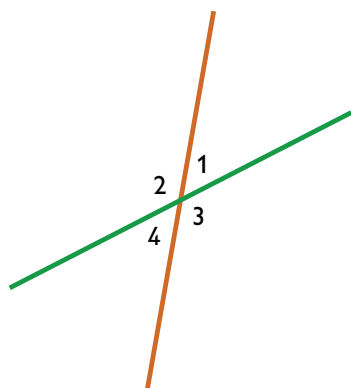
## EXERCISES

1. Which angles are adjacent? There may be more than one pair of adjacent angles.



- ☐ A  $\angle 1$  and  $\angle 2$     ☐ B  $\angle 2$  and  $\angle 4$     ☐ C  $\angle 1$  and  $\angle 4$     ☐ D  $\angle 3$  and  $\angle 4$

2. Which angles are vertical angles? There may be more than one pair of vertical angles.

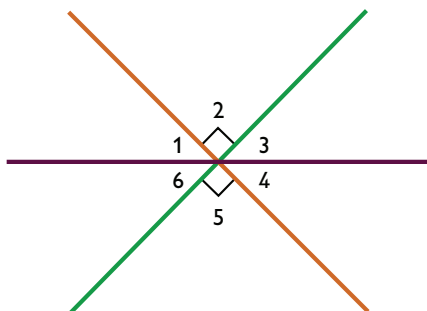


- ☐ A  $\angle 1$  and  $\angle 4$   
☐ B  $\angle 1$  and  $\angle 3$   
☐ C  $\angle 2$  and  $\angle 3$   
☐ D  $\angle 2$  and  $\angle 4$   
☐ E  $\angle 3$  and  $\angle 4$

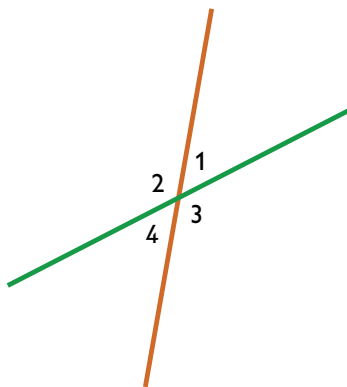
## LESSON 2: ANGLES

## EXERCISES

3. Which angles are complementary? There may be more than one pair of complementary angles.



- (A)  $\angle 1$  and  $\angle 2$
  - (B)  $\angle 2$  and  $\angle 5$
  - (C)  $\angle 3$  and  $\angle 5$
  - (D)  $\angle 3$  and  $\angle 4$
  - (E)  $\angle 6$  and  $\angle 1$
4. If the measure of  $\angle 1$  is  $53^\circ$ , what is the measure of  $\angle 3$ ?

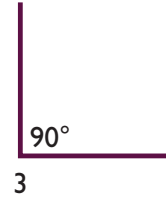
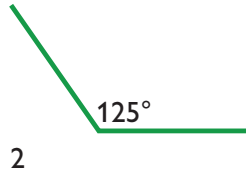
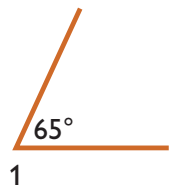


5. Two angles are adjacent, and the noncommon sides lie on the same line. If one angle measures  $35^\circ$ , what is the measure of the other angle?
- (A)  $35^\circ$
  - (B)  $55^\circ$
  - (C)  $145^\circ$
  - (D) Cannot be determined

## LESSON 2: ANGLES

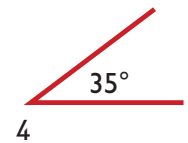
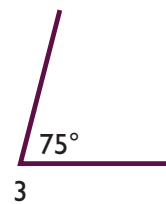
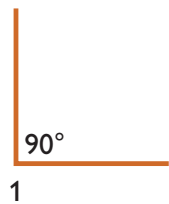
## EXERCISES

6. Which angles are complementary?



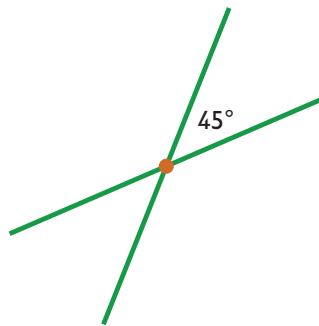
- Ⓐ  $\angle 1$  and  $\angle 2$     Ⓑ  $\angle 2$  and  $\angle 3$     Ⓒ  $\angle 3$  and  $\angle 4$     Ⓓ  $\angle 1$  and  $\angle 4$

7. Which angles are complementary?



- Ⓐ  $\angle 4$  and  $\angle 2$     Ⓑ  $\angle 3$  and  $\angle 1$     Ⓒ  $\angle 2$  and  $\angle 3$     Ⓓ  $\angle 1$  and  $\angle 4$

8. Two lines intersect and one of the four angles formed measures  $45^\circ$ .



What are the measures of the other three angles? Explain how you know.

### Challenge Problem

9. Two lines intersect, and one of the resulting angles is four times as large as another of the angles.

What are the measures of the four angles? Explain how you know.

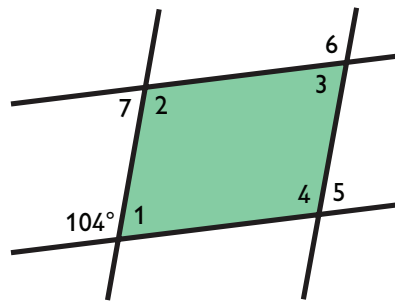


## LESSON 3: PROPERTIES OF PARALLELOGRAMS

## EXERCISES

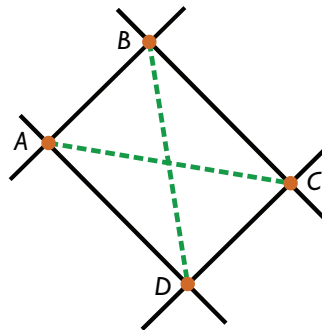
## EXERCISES

- In a certain parallelogram, the diagonals are congruent but they are not perpendicular to one another. What is the most specific name for this figure?
  - ☐ A Quadrilateral
  - ☐ B Parallelogram
  - ☐ C Rectangle
  - ☐ D Square
- Consider this parallelogram.



Which angles measure  $76^\circ$ ? There may be more than one correct angle.

- ☐ A  $\angle 2$
  - ☐ B  $\angle 3$
  - ☐ C  $\angle 4$
  - ☐ D  $\angle 5$
  - ☐ E  $\angle 7$
- Consider this rectangle.



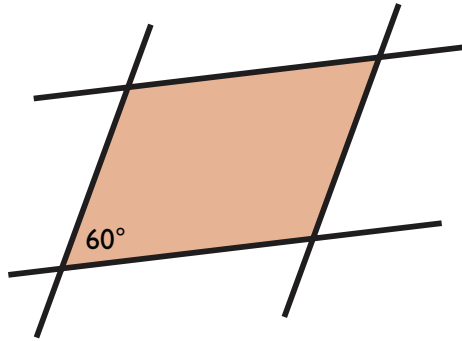
Which line segments are congruent? There may be more than one pair of congruent line segments.

- ☐ A  $\overline{AC}$  and  $\overline{DC}$
- ☐ B  $\overline{AB}$  and  $\overline{AD}$
- ☐ C  $\overline{AC}$  and  $\overline{BD}$
- ☐ D  $\overline{BC}$  and  $\overline{AD}$
- ☐ E  $\overline{BC}$  and  $\overline{AB}$

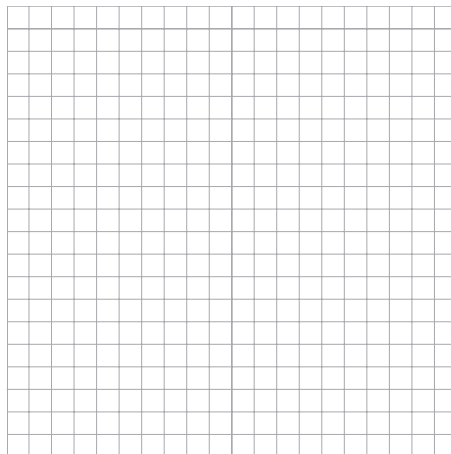
## LESSON 3: PROPERTIES OF PARALLELOGRAMS

## EXERCISES

4. In a certain parallelogram, one angle measures  $60^\circ$ . What are the measures of the other three interior angles? Explain how you know.



5. In a certain parallelogram, one angle measures  $90^\circ$ . What are the measures of the other three angles? Explain how you know.
6. Draw a quadrilateral by following these steps.
- Draw a horizontal line segment that is 12 units long.
  - On the left side of the line segment, use a protractor to construct an obtuse angle measuring  $140^\circ$  that opens to the right. The vertex of the angle should be at the left end of the horizontal line segment. Make the line segment that extends from the angle 6 units long.
  - On the right side of the horizontal line segment, construct an angle that is supplementary to the first angle and opens to the left. Create the angle by drawing a line segment that intersects the right end of the horizontal line. Make the line segment 6 units long.
  - Draw a line segment to connect the ends of the two line segments that are 6 units long.

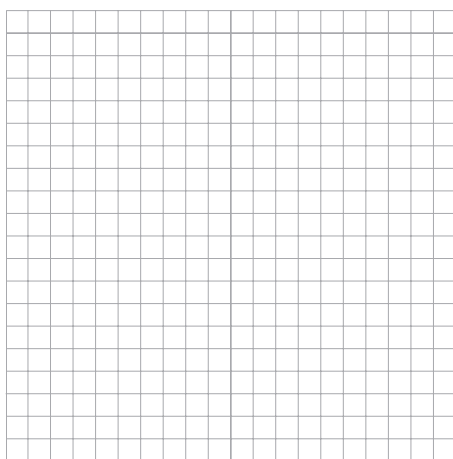




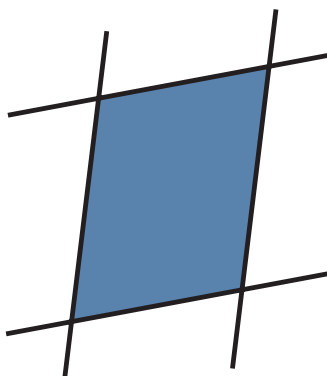
## LESSON 3: PROPERTIES OF PARALLELOGRAMS

## EXERCISES

7. Draw a quadrilateral by following these steps.
- Draw a vertical line segment that is 10 units long.
  - On the upper end of the line segment, construct a right angle that opens to the right. Draw a line segment that intersects the top end of the vertical line segment; make the line segment 7 units long.
  - On the lower end of the vertical line segment, construct an angle that is supplementary to the first angle and opens to the right. Draw a line segment that intersects the lower end of the vertical line segment; make the line segment 7 units long.
  - Draw a line segment to connect the ends of the two line segments that are 7 units long.
  - Draw the diagonal lines that connect the four vertices of your quadrilateral.



8. This parallelogram has angle measures of  $120^\circ$  and  $60^\circ$ . At any of the four vertices of this parallelogram, there are three more angles outside the figure. Which measure could be the sum of these three angle measures?



- |                 |                 |
|-----------------|-----------------|
| (A) $90^\circ$  | (B) $120^\circ$ |
| (C) $270^\circ$ | (D) $300^\circ$ |

## LESSON 3: PROPERTIES OF PARALLELOGRAMS

## EXERCISES

### Challenge

---

9. What properties of a parallelogram and a rectangle are the same?  
What properties of a parallelogram and a rectangle can be different?  
Draw images to support your explanation.

## LESSON 4: PROPERTIES OF QUADRILATERALS

## EXERCISES

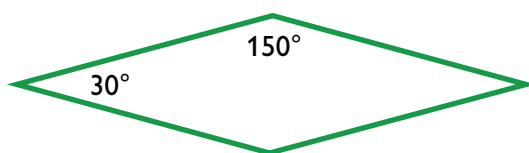
## EXERCISES

1. A certain quadrilateral has congruent and perpendicular diagonals.

What is the most specific name for this figure?

- ☐ A Square      ☐ B Rectangle      ☐ C Rhombus      ☐ D Parallelogram

2. Consider this quadrilateral.



What is the sum of the angle measures of all the angles in this quadrilateral?

\_\_\_\_\_°

3. A certain quadrilateral has two right angles and a  $40^\circ$  angle.

What is the measure of the fourth angle?

- ☐ A  $40^\circ$       ☐ B  $60^\circ$       ☐ C  $120^\circ$       ☐ D  $140^\circ$

4. A certain quadrilateral has these angle measures:  $45^\circ$ ,  $60^\circ$ , and  $110^\circ$ .

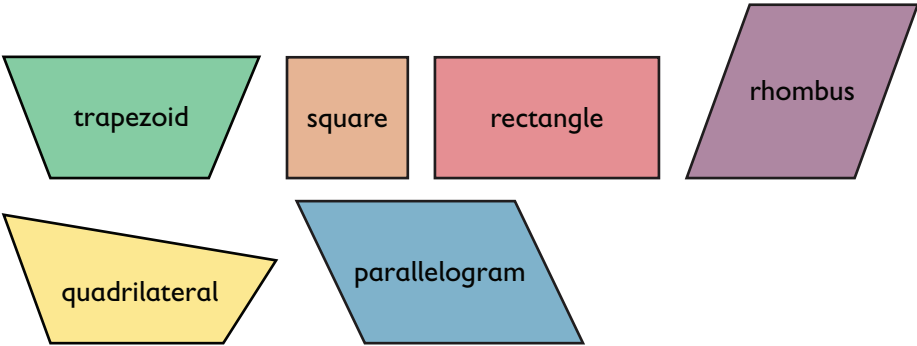
What is the measure of the fourth angle?

- ☐ A  $35^\circ$       ☐ B  $45^\circ$       ☐ C  $110^\circ$       ☐ D  $145^\circ$

LESSON 4: PROPERTIES OF QUADRILATERALS

EXERCISES

5. Match each quadrilateral to the appropriate column.

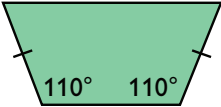
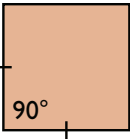

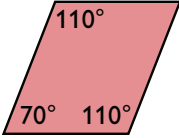


Diagonals Are Perpendicular	Diagonals Are Not Perpendicular

6. For each quadrilateral, enter the missing angle measure or the sum of the missing angle measures.

LESSON 4: PROPERTIES OF QUADRILATERALS

EXERCISES

Quadrilateral	Missing Angle Measures
	<div><div></div>°</div>
	<div><div></div>°</div>
	<div><div></div>°</div>
	<div><div></div>°</div>

7. The diagonals of a certain quadrilateral are 3 in. and 5 in. Each diagonal divides the other diagonal into two congruent segments. What is this figure? Explain how you know.
8. In a certain quadrilateral, all four angles are congruent. If the diagonals are perpendicular to each other, what is the figure? Explain how you know.

Challenge Problem

9. A kite is a special type of quadrilateral.  
Which of these statements are true? Justify each answer.
- a. If a figure is a kite, then the diagonals are perpendicular.
- b. If a figure is a kite, then the diagonals bisects a pair of opposite angles.
- c. A rhombus is a kite.



## LESSON 5: BUILDING TRIANGLES

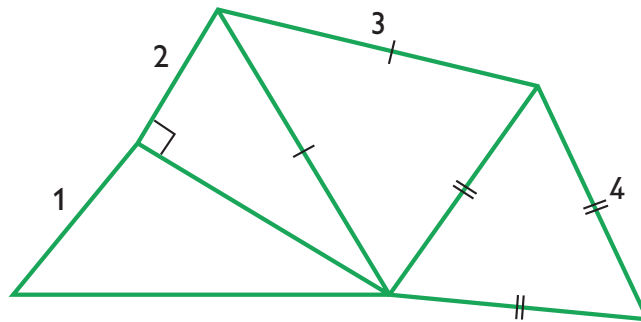
## EXERCISES

## EXERCISES

1. The angle measures of two angles in a certain triangle are  $50^\circ$  and  $100^\circ$ .  
What is the measure of the third angle?

**A**  $10^\circ$       **B**  $30^\circ$       **C**  $100^\circ$       **D**  $210^\circ$

2. In this diagram, there are four triangles.



Match each triangle type to the corresponding triangle on the figure.

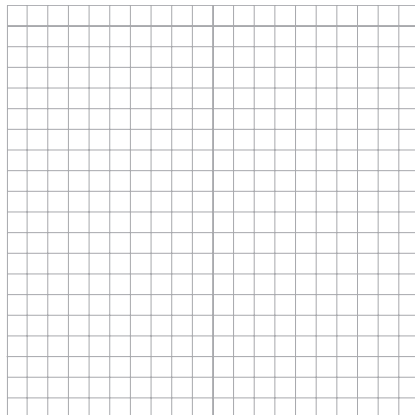
Isosceles  
triangle

Right  
triangle

Equilateral  
triangle

Obtuse  
triangle

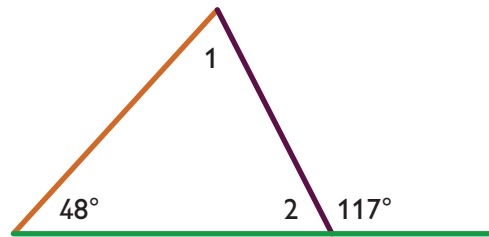
3. Draw a triangle by following these steps.
- Draw a horizontal line segment that is 11 units long.
  - On the left side of the line segment, use a protractor to construct an acute angle measuring  $40^\circ$  that opens to the right. The vertex of the angle should be at the left end of the horizontal line segment. Make the line segment that extends from the angle 11 units long.
  - Draw a line segment to connect the ends of the two other line segments.



## LESSON 5: BUILDING TRIANGLES

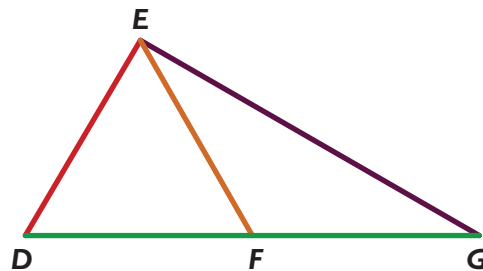
## EXERCISES

4. In a right triangle, there is a  $90^\circ$  angle. What is the sum of the two other angles?  
\_\_\_\_\_°
5. If all three angle measures are given, under what conditions can a triangle be constructed? How many triangles can be made? Explain your thinking.
6. If the lengths of two sides of a triangle are given, how many different (noncongruent) triangles can be formed? Explain your thinking.
7. If the lengths of two sides of a triangle are given *and* the measure of the angle between the sides is given, how many triangles can be formed? Explain your thinking.
8. What are the measures of  $\angle 1$  and  $\angle 2$ ? Explain how you know.



## Challenge Problem

9.  $\triangle DEF$  is an equilateral triangle, and  $\triangle EFG$  is an isosceles triangle.



What are the three angle measures of  $\triangle DEG$  and what type of triangle is it? Explain how you know.



## LESSON 6: PROPERTIES OF TRIANGLES

## EXERCISES

## EXERCISES

1. The angle measures of two angles in a certain triangle are both  $45^\circ$ . What type of triangle is it? There may be more than one correct triangle.
  - A Acute triangle
  - B Right triangle
  - C Scalene triangle
  - D Isosceles triangle
  - E Equilateral triangle
  
2. One of the angle measures in a certain isosceles triangle is  $40^\circ$ . Which of these pairs of angle measures could be the other two angle measures? There may be more than one correct pair of angle measures.
  - A  $60^\circ$  and  $60^\circ$
  - B  $30^\circ$  and  $110^\circ$
  - C  $40^\circ$  and  $100^\circ$
  - D  $75^\circ$  and  $75^\circ$
  - E  $40^\circ$  and  $40^\circ$
  - F  $70^\circ$  and  $70^\circ$
  
3. One of the angle measures in a certain acute triangle is  $64^\circ$ . Which of these pairs of angle measures could be the other two angle measures? There may be more than one correct pair of angle measures.
  - A  $58^\circ$  and  $58^\circ$
  - B  $32^\circ$  and  $84^\circ$
  - C  $26^\circ$  and  $90^\circ$
  - D  $73^\circ$  and  $73^\circ$
  - E  $40^\circ$  and  $76^\circ$
  - F  $16^\circ$  and  $100^\circ$
  
4. A certain triangle has only two congruent angles. Which statement must be true about the sides of the triangle?
  - A All three sides must be the same length.
  - B All three sides must be different lengths.
  - C Two of the sides must be the same length.
  - D Two of the sides must be shorter than the third side.

## LESSON 6: PROPERTIES OF TRIANGLES

## EXERCISES

5. A certain triangle has three congruent sides. Which statement must be true about the angles of the triangle?
- ☐ A All three angles are different measures.
  - ☐ B All three angles are the same measure.
  - ☐ C Two angles are the same measure, and the third angle is larger.
  - ☐ D Two angles are the same measure, and the third angle is smaller.
6. Which statement must be true of three side lengths in order for them to make a triangle?
- ☐ A All three side lengths must be different lengths.
  - ☐ B All three side lengths must add up to 180.
  - ☐ C The sum of the lengths of the two shorter sides must be less than the length of the third side.
  - ☐ D The sum of the lengths of the two shorter sides must be greater than the length of the third side.
7. The side lengths of a certain triangle are 13 cm and 15 cm. Which of these side lengths could be used for the third side? There may be more than one correct side length.
- ☐ A 5 cm      ☐ B 13 cm      ☐ C 27 cm      ☐ D 30 cm      ☐ E 38 cm

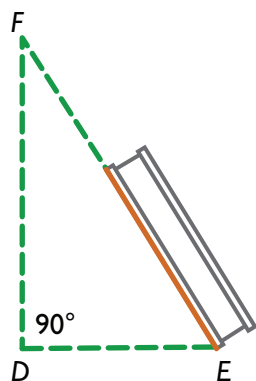
## LESSON 6: PROPERTIES OF TRIANGLES

## EXERCISES

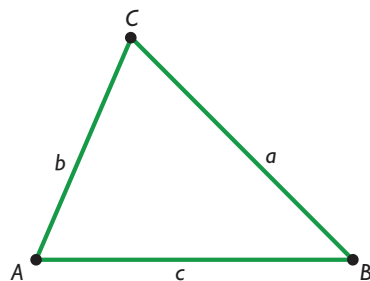
8. Here is a picture of a single-leaf bascule bridge—a type of drawbridge—in open position.



When the leaf of the drawbridge opens, the open end of the bridges makes  $EDF$  measuring  $90^\circ$  (see figure). What is the measure of  $DEF$ , which is the angle the leaf of the bridge makes with the horizontal plane? Explain your reasoning.



9. Maya wants to construct this triangle.



She starts by using a ruler to measure the lengths of  $BC$ ,  $AC$ , and  $AB$ .

Maya says, “I just need to draw lines the same length as the three I just measured, and I’m done.”

Explain why she may be wrong.

### Challenge Problem:

10. You are given three sticks measuring 5 cm, 7 cm, and 15 cm.

Can you arrange the sticks on the ground to form a triangle? Explain why or why not. Draw a sketch to support your explanation.



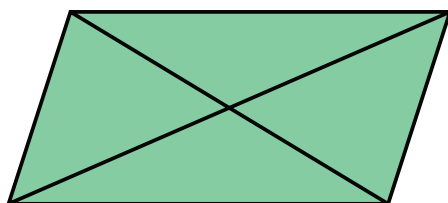
## LESSON 7: ANGLE SUMS

## EXERCISES

## EXERCISES

1. What is the sum of the angle measures of a regular heptagon (7-sided polygon)?  
**A**  $128.6^\circ$       **B**  $900^\circ$       **C**  $1,080^\circ$       **D**  $1,260^\circ$
2. A certain regular polygon has angle measures of  $135^\circ$ . What type of polygon is it?  
**A** Pentagon      **B** Hexagon      **C** Octagon      **D** Decagon
3. A certain pentagon has three right angles. What could the other two angles measure?  
**A**  $90^\circ$  and  $90^\circ$   
**B**  $130^\circ$  and  $140^\circ$   
**C**  $20^\circ$  and  $150^\circ$   
**D** There is not enough information.
4. If a parallelogram is divided by its two diagonals, is the angle sum of the parallelogram  $720^\circ$ ?

Explain your reasoning.

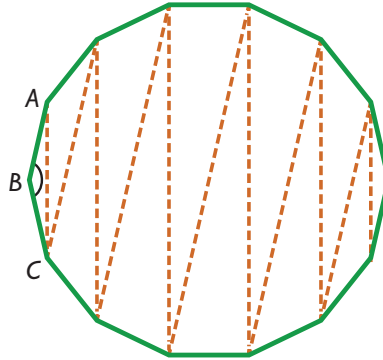


5. A classmate measures all of the angles in a polygon and gets a total of  $2,400^\circ$ . However, the angle sum is not correct. Explain why the sum is not correct and what the correct angle sum might be.
6. Explain why the sum of the interior angles of a hexagon is  $720^\circ$ . Show your work and explain your reasoning.

## LESSON 7: ANGLE SUMS

## EXERCISES

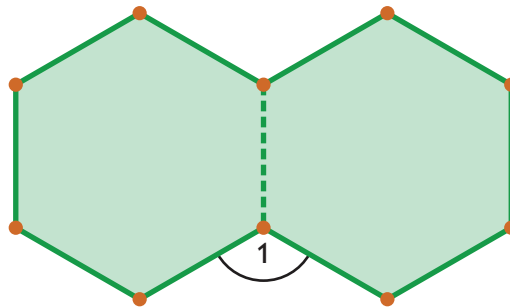
7. This is a regular tetradecagon (14-sided polygon).



What is the measure of  $\angle ABC$ ? Round your answer to the nearest hundredth.

$\angle ABC = \underline{\hspace{1cm}}^\circ$

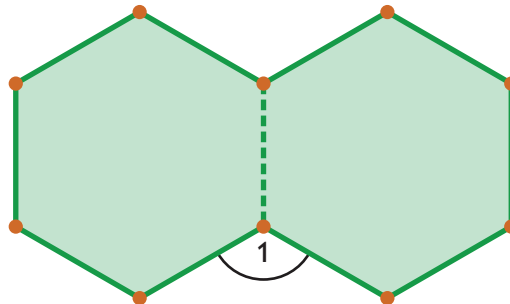
8. This polygon is composed of two regular hexagons.



What is the measure of  $\angle 1$ ?

$\angle 1 = \underline{\hspace{1cm}}^\circ$

9. This polygon is composed of two regular hexagons.



What is the sum of the interior angle measures in the whole polygon?

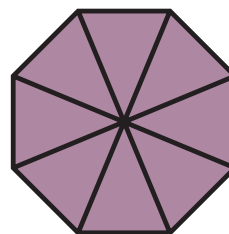
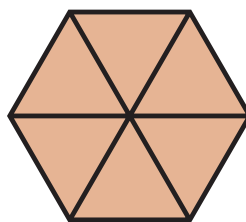
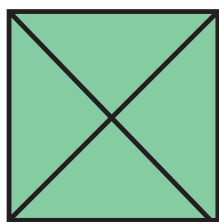
- Ⓐ  $1,080^\circ$ 
 Ⓑ  $1,260^\circ$ 
 Ⓒ  $1,440^\circ$ 
 Ⓓ  $1,620^\circ$

## LESSON 7: ANGLE SUMS

## EXERCISES

## Challenge Problem

10. For each regular polygon shown, the diagonals meet in the center and divide one another into two equal segments.



- What is the measure of each angle at the center of the polygon?
- As the number of sides increases, what is the angle measure of each angle at the center of the polygon getting closer to?



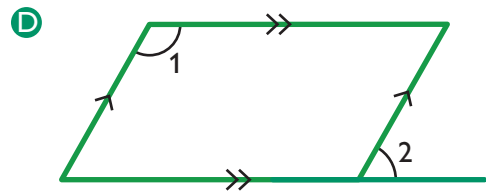
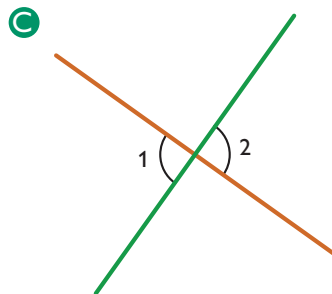
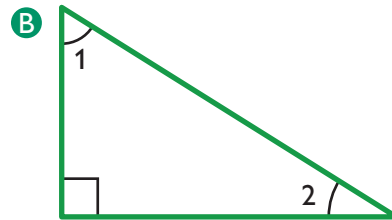
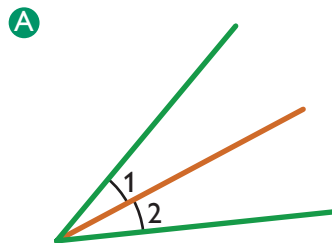


## LESSON 8: FINDING ANGLES

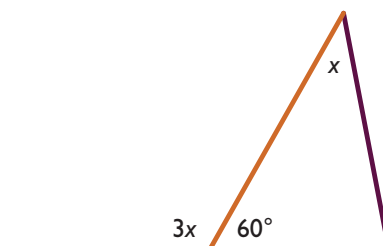
## EXERCISES

## EXERCISES

- The known angle measures in a certain pentagon are  $95^\circ$ ,  $105^\circ$ ,  $90^\circ$ , and  $120^\circ$ . What is the measure of the remaining angle?  
 (A)  $90^\circ$       (B)  $110^\circ$       (C)  $120^\circ$       (D)  $130^\circ$
- The diagonals of a certain quadrilateral are perpendicular to each other and divide each other into equal segments. What is the most specific name for the figure?  
 (A) Parallelogram  
 (B) Rectangle  
 (C) Rhombus  
 (D) Trapezoid
- In which figure are  $\angle 1$  and  $\angle 2$  complementary?



- Solve for  $x$ .

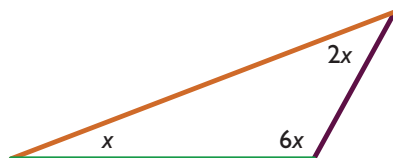


$$x = \underline{\hspace{2cm}}^\circ$$

## LESSON 8: FINDING ANGLES

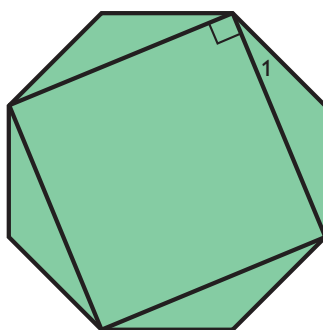
## EXERCISES

5. Solve for  $x$ .



$$x = \underline{\hspace{1cm}}^\circ$$

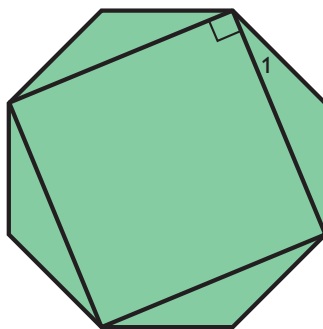
6. Consider this regular octagon.



Find the measure of  $\angle 1$ .

$$\angle 1 = \underline{\hspace{1cm}}^\circ$$

7. Consider this regular octagon.

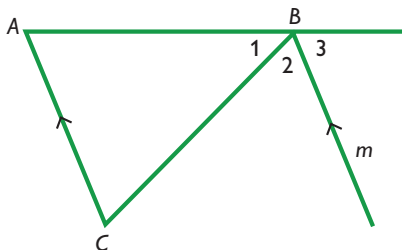


What figure is inside the octagon? Explain how you know.

## LESSON 8: FINDING ANGLES

## EXERCISES

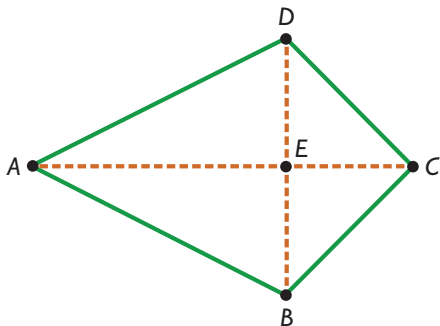
8. In this figure, line  $m$  is parallel to  $\overline{AC}$ ,  $\angle 1 = \angle 3$ , and  $\angle 2 = 64^\circ$ .



Calculate the measure of  $\angle ACB$ .

$$\angle ACB = \underline{\hspace{2cm}}^\circ$$

9. This quadrilateral is a kite, meaning  $\overline{AD} \cong \overline{AB}$  and  $\overline{BC} \cong \overline{DC}$



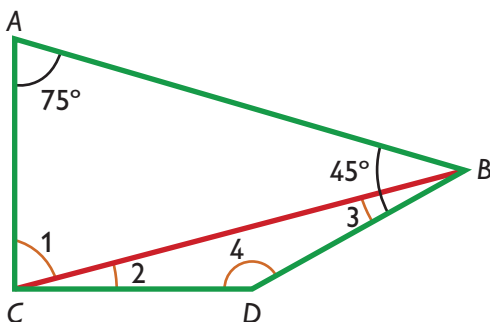
$$\angle BAE = 28^\circ \text{ and } \angle EDC = 44^\circ.$$

What is the measure of  $\angle ABC$ ?

$$\angle ABC = \underline{\hspace{2cm}}^\circ$$

### Challenge Problem

10. Figure ABCD is an irregular quadrilateral and  $\triangle ABC$  and  $\triangle BCD$  are isosceles triangles.  $\angle CAB$  measures  $75^\circ$  and  $\angle ABD$  measures  $45^\circ$ .



What are the measures of  $\angle 1$ ,  $\angle 2$ ,  $\angle 3$ , and  $\angle 4$ ?

Explain how you determined each angle measure.



## LESSON 9: PUTTING IT TOGETHER

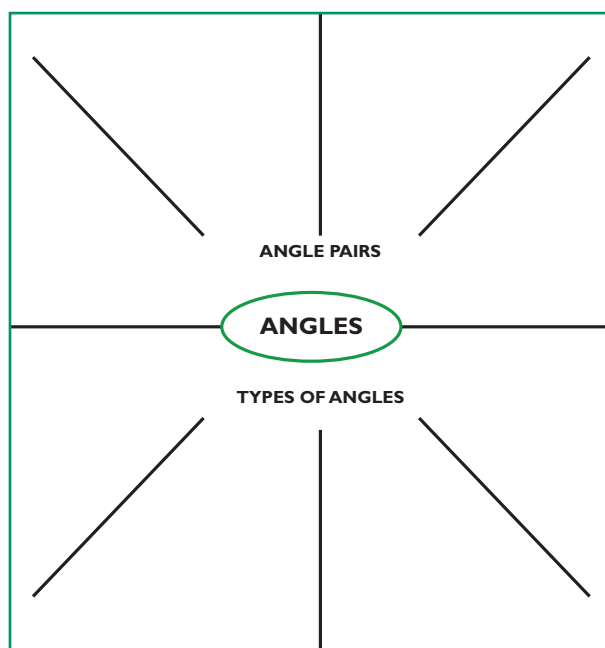
## EXERCISES

## EXERCISES

1. Read your work on the Self Check task. What would you do differently if you were starting the Self Check task now?
2. Review the notes you took during the lessons about diagonals in parallelograms, rhombuses, and rectangles. Add any additional ideas you have about the topic to your notes.
3. Read your notes and think about your work in this unit involving angles (for example, acute angles) and angle pairs (for example, complementary angles).

In your Notebook, create a graphic organizer to show facts about angles and angle pairs. Organize your graphic organizer in a way that will allow you to use it as a reference throughout the rest of the school year.

Use a chart similar to the one shown, or create your own. Make sure to include diagrams and formulas. (If you prefer, add the types of angles and angle pairs to your math vocabulary list.)



## LESSON 9: PUTTING IT TOGETHER

## EXERCISES

4. Consider your work in this unit involving the angle measures and angle sums of triangles, quadrilaterals, and regular polygons.

In your Notebook, create a graphic organizer to show facts about polygons and angles. Organize your visual graphic organizer in a way that will allow you to use it as a reference throughout the rest of the school year.

Use a chart similar to the one shown, or create your own.

Shape	Number of Sides	Number of Triangles	Angle Sum—Interior Angles	Each Interior Angle Measure (if regular polygon)	Angle Sum—Exterior Angles	Each Exterior Angle Measure (if regular polygon)
Triangle						
Quadrilateral						
Pentagon						
Hexagon						
Octagon						
Any Polygon						

5. Complete any exercises from this unit that you have not finished.