Lesson 1

Problem 1

Which one of these shapes is not like the others? Explain what makes it different by representing each width and height pair with a ratio.

Solution

C is different from A and B. For both A and B, the width:height ratio is 5:4. However, for C, the width is 10 units and the height is 6 units, so the width:height ratio is 5:3.

Problem 2

In one version of a trail mix, there are 3 cups of peanuts mixed with 2 cups of raisins. In another version of trail mix, there are 4.5 cups of peanuts mixed with 3 cups of raisins. Are the ratios equivalent for the two mixes? Explain your reasoning.

Solution

Yes, since 3 times 1.5 is 4 and 2 times 1.5 is 3.

Problem 3

(from Unit 1, Lesson 12)

For each object, choose an appropriate scale for a drawing that fits on a regular sheet of paper. Not all of the scales on the list will be used.

<table>
<thead>
<tr>
<th>Objects</th>
<th>Scales</th>
</tr>
</thead>
<tbody>
<tr>
<td>A. A person</td>
<td>1. 1 in : 1 ft</td>
</tr>
<tr>
<td>B. A football field (120 yards by $53\frac{1}{3}$ yards)</td>
<td>2. 1 cm : 1 m</td>
</tr>
<tr>
<td>C. The state of Washington</td>
<td>3. 1: 1000</td>
</tr>
<tr>
<td>(about 240 miles by 360 miles)</td>
<td>4. 1 ft: 1 mile</td>
</tr>
</tbody>
</table>
D. The floor plan of a house                     5. 1: 100,000
E. A rectangular farm (6 miles by   6. 1 mm: 1 km
   2 mile)                                      7. 1: 10,000,000

Solution
Answers vary. Sample responses:

1. 1 in :1 ft
2. 1: 1000
3. 1: 10,000,000
4. 1 cm: 1 m
5. 1: 100,000

Problem 4
(from Unit 1, Lesson 11)
Which scale is equivalent to 1 cm to 1 km?

A. 1 to 1000
B. 10,000 to 1
C. 1 to 100,000
D. 100,000 to 1
E. 1 to 1,000,000

Solution
C

Problem 5
(from Grade 7, Unit 2, Lesson 5)
1. Find 3 different ratios that are equivalent to \( 7 : 3 \).

2. Explain why these ratios are equivalent.

Solution
1. Answers vary. Sample response: \( 14 : 6, 21 : 9, 28 : 12 \)

2. Answers vary. Sample response: 7 and 3 are each multiplied by 2, 3, and 4, respectively.

Lesson 2

Problem 1
When Han makes chocolate milk, he mixes 2 cups of milk with 3 tablespoons of chocolate syrup. Here is a table that shows how to make batches of different sizes.

<table>
<thead>
<tr>
<th>cups of milk</th>
<th>tablespoons of chocolate syrup</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>3</td>
</tr>
<tr>
<td>8</td>
<td>12</td>
</tr>
<tr>
<td>1</td>
<td>3/2</td>
</tr>
<tr>
<td>10</td>
<td>15</td>
</tr>
</tbody>
</table>

\( \times 4 \)
Use the information in the table to complete the statements. Some terms are used more than once.

1. The table shows a proportional relationship between ___________ and ___________.
2. The scale factor shown is ___________.
3. The constant of proportionality for this relationship is ___________.
4. The units for the constant of proportionality are ___________ per ___________.

Bank of Terms: tablespoons of chocolate syrup, 4, cups of milk, cup of milk, \( \frac{3}{2} \)

**Solution**

1. cups of milk, tablespoons of chocolate syrup
2. 4
3. \( \frac{3}{7} \)
4. tablespoons of chocolate syrup, cup of milk

**Problem 2**

A certain shade of pink is created by adding 3 cups of red paint to 7 cups of white paint.

1. How many cups of red paint should be added to 1 cup of white paint?

<table>
<thead>
<tr>
<th>cups of white paint</th>
<th>cups of red paint</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td></td>
</tr>
<tr>
<td>7</td>
<td>3</td>
</tr>
</tbody>
</table>

2. What is the constant of proportionality?

**Solution**

1. \( \frac{3}{7} \) cups of red paint
2. \( \frac{3}{7} \)

**Problem 3**

(from Unit 1, Lesson 12)

A map of a rectangular park has a length of 4 inches and a width of 6 inches. It uses a scale of 1 inch for every 30 miles.

1. What is the actual area of the park? Show how you know.

2. The map needs to be reproduced at a different scale so that it has an area of 6 square inches and can fit in a brochure. At what scale should the map be reproduced so that it fits on the brochure? Show your reasoning.

**Solution**

1. 21,600 square miles. Sample reasoning: The area on the map is 24 square inches. 1 square inch represents 900 square miles, since \( 30 \cdot 30 = 900 \). The actual area is \( 24 \cdot 900 \), which equals 21,600 square miles.
2. 1 inch to 60 miles. Sample explanations:
   - If 21,600 square miles need to be represented by 6 square inches, each square inch needs to represent 3,600 square miles: \(21,600 \div 6 = 3,600\). This means each 1-inch side of the square needs to be 60 miles.
   - The area of this new map is \(\frac{1}{4}\) of the first map, since 6 is \(\frac{1}{4}\) of 24. This means each 1 inch square has to represent 4 times as much area than the first. \(900 \cdot 4 = 3,600\). If each 1 inch square represents 3,600 square miles, every 1 inch represents 60 miles.

**Problem 4**
(from Unit 1, Lesson 6)
Noah drew a scaled copy of Polygon P and labeled it Polygon Q.

If the area of Polygon P is 5 square units, what scale factor did Noah apply to Polygon P to create Polygon Q? Explain or show how you know.

**Solution**
The area of polygon Q is 45 square units, so the area has scaled by a factor of 9, since 5 \(\cdot\) 9 = 45. Since the area of a scaled copy varies from the original area by the square of the scale factor, the scale factor is 3.

**Problem 5**
(from Grade 7, Unit 2, Lesson 5)
Select all the ratios that are equivalent to each other.

- A. 4 : 7
- B. 8 : 15
- C. 16 : 28
- D. 2 : 3
- E. 20 : 35

**Solution**
A, C, and E

**Lesson 3**

**Problem 1**
Noah is running a portion of a marathon at a constant speed of 6 miles per hour.

Complete the table to predict how long it would take him to run different distances at that speed, and how far he would run in different time intervals.
Problem 2

One kilometer is 1000 meters.

1. Complete the tables. What is the interpretation of the constant of proportionality in each case?

<table>
<thead>
<tr>
<th>meters</th>
<th>kilometers</th>
</tr>
</thead>
<tbody>
<tr>
<td>1,000</td>
<td>1</td>
</tr>
<tr>
<td>250</td>
<td></td>
</tr>
<tr>
<td>12</td>
<td></td>
</tr>
<tr>
<td>1</td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>kilometers</th>
<th>meters</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1,000</td>
</tr>
<tr>
<td>5</td>
<td></td>
</tr>
<tr>
<td>20</td>
<td></td>
</tr>
<tr>
<td>0.3</td>
<td></td>
</tr>
</tbody>
</table>

The constant of proportionality tells us that:

2. What is the relationship between the two constants of proportionality?

Solution

1. a.  

<table>
<thead>
<tr>
<th>meters</th>
<th>kilometers</th>
</tr>
</thead>
<tbody>
<tr>
<td>1,000</td>
<td>1</td>
</tr>
<tr>
<td>250</td>
<td>0.25</td>
</tr>
<tr>
<td>12</td>
<td>0.012</td>
</tr>
<tr>
<td>1</td>
<td>0.001</td>
</tr>
</tbody>
</table>

0.001 kilometers per meter

b.
<table>
<thead>
<tr>
<th>kilometers</th>
<th>meters</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1,000</td>
</tr>
<tr>
<td>5</td>
<td>5,000</td>
</tr>
<tr>
<td>20</td>
<td>20,000</td>
</tr>
<tr>
<td>0.3</td>
<td>300</td>
</tr>
</tbody>
</table>

1000 meters per kilometer

2. 0.001 and 1000 are reciprocals of each other. This is easier to see if 0.001 is written as \( \frac{1}{1000} \).

**Problem 3**

Jada and Lin are comparing inches and feet. Jada says that the constant of proportionality is 12, Lin says it is \( \frac{1}{12} \). Do you agree with either one of them? Why or why not?

**Solution**

Jada is saying that there are 12 inches for every 1 foot. Lin is saying that there are \( \frac{1}{12} \) foot for every 1 inch.

**Problem 4**

(from Unit 1, Lesson 12)

The area of the Mojave desert is 25,000 square miles. A scale drawing of the Mojave desert has an area of 10 square inches. What is the scale of the map?

**Solution**

1 inch to 50 miles

**Problem 5**

(from Unit 1, Lesson 11)

Which of these scales is equivalent to the scale 1 cm to 5 km? Select all that apply.

A. 3 cm to 15 km  
B. 1 mm to 150 km  
C. 5 cm to 1 km  
D. 5 mm to 2.5 km  
E. 1 mm to 500 m

**Solution**

A, D, E

**Problem 6**

(from Unit 2, Lesson 1)

Which one of these pictures is not like the others? Explain what makes it different using ratios.
Solution

M is different from L and N. The width:height ratios for the outsides of the pictures are all equivalent to 5:4. However, the width:height ratios of the insides of L and N both have a 3:4 ratio of width:height, while the inside of M has a width of 4 units and a height of 8 units, making its ratio 1:2.

Alternatively, the ratio of height to thickness at the widest part for L and N are both 4:1. But M has a height of 8 units and a thickness of 3 units, making that ratio 8:3.

Lesson 4

Problem 1

A certain ceiling is made up of tiles. Every square meter of ceiling requires 10.75 tiles. Fill in the table with the missing values.

<table>
<thead>
<tr>
<th>square meters of ceiling</th>
<th>number of tiles</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td></td>
</tr>
<tr>
<td>10</td>
<td>107.5</td>
</tr>
<tr>
<td>a</td>
<td>100</td>
</tr>
</tbody>
</table>

Solution

<table>
<thead>
<tr>
<th>square meters of ceiling</th>
<th>number of tiles</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>10.75</td>
</tr>
<tr>
<td>10</td>
<td>107.5</td>
</tr>
<tr>
<td>9.3</td>
<td>100</td>
</tr>
<tr>
<td>a</td>
<td>10.75 \cdot a</td>
</tr>
</tbody>
</table>

Problem 2

On a flight from New York to London, an airplane travels at a constant speed. An equation relating the distance traveled in miles, d, to the number of hours flying, t, is \( t = \frac{1}{100}d \). How long will it take the airplane to travel 800 miles?

Solution

1.6 hours since \( \frac{1}{100} \cdot 800 = 1.6 \)

Problem 3

Each table represents a proportional relationship. For each, find the constant of proportionality, and write an equation that represents the relationship.
Solution

1. Constant of proportionality: 4. Equation: \( P = 4s \)

2. Constant of proportionality: 3.14 Equation: \( C = 3.14d \)

Problem 4
(from Unit 1, Lesson 11)
A map of Colorado says that the scale is 1 inch to 20 miles or 1 to 1,267,200. Are these two ways of reporting the scale the same? Explain your reasoning.

Solution
Yes. Sample response: There are 12 inches in a foot and 5280 feet in 1 mile, so that's 63,360 inches in a mile and 1,267,200 inches in 20 miles.

Problem 5
(from Unit 1, Lesson 3)
Here is a polygon on a grid.

1. Draw a scaled copy of the polygon using a scale factor \( \frac{3}{2} \). Label the copy A.

2. Draw a scaled copy of the polygon with a scale factor \( \frac{1}{2} \). Label it B.

3. Is Polygon A a scaled copy of Polygon B? If so, what is the scale factor that takes B to A?

Solution

1.

2.
3. Yes, A is a scaled copy of B with a scale factor of 6.

Lesson 5

Problem 1

The table represents the relationship between a length measured in meters and the same length measured in kilometers.

1. Complete the table.

2. Write an equation for converting the number of meters to kilometers. Use $x$ for number of meters and $y$ for number of kilometers.

Solution

1. Complete the table.

<table>
<thead>
<tr>
<th>meters</th>
<th>kilometers</th>
</tr>
</thead>
<tbody>
<tr>
<td>1,000</td>
<td>1</td>
</tr>
<tr>
<td>3,500</td>
<td>3.5</td>
</tr>
<tr>
<td>500</td>
<td>0.5</td>
</tr>
<tr>
<td>75</td>
<td>0.075</td>
</tr>
<tr>
<td>1</td>
<td>0.001</td>
</tr>
<tr>
<td>$x$</td>
<td>$0.001x$</td>
</tr>
</tbody>
</table>

2. $y = 0.001x$ or equivalent

Problem 2

Concrete building blocks weigh 28 pounds each. Using $b$ for the number of concrete blocks and $w$ for the weight, write two equations that relate the two variables. One equation should begin with $w =$ and the other should begin with $b =$.

Solution

$w = 28b$ and $b = \frac{1}{28}w$

Problem 3

A store sells rope by the meter. The equation $p = 0.8L$ represents the price $p$ (in dollars) of a piece of nylon rope that is $L$ meters long.
1. How much does the nylon rope cost per meter?

2. How long is a piece of nylon rope that costs $1.00?

**Solution**

1. $0.80 or dollar or dollar.

2. 1.25 meters or \( \frac{5}{4} \) meters or \( 1\frac{1}{4} \) meters.

**Problem 4**
(from Unit 2, Lesson 4)
The table represents a proportional relationship. Find the constant of proportionality and write an equation to represent the relationship.

<table>
<thead>
<tr>
<th>( a )</th>
<th>( y )</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>( \frac{5}{4} )</td>
</tr>
<tr>
<td>3</td>
<td>1</td>
</tr>
<tr>
<td>10</td>
<td>( 5 )</td>
</tr>
<tr>
<td>12</td>
<td>4</td>
</tr>
</tbody>
</table>

Constant of proportionality: __________

Equation: \( y = \) __________

**Solution**

Constant of proportionality: \( \frac{1}{3} \)  Equation: \( y = \frac{1}{3}x \)

**Problem 5**
(from Unit 1, Lesson 8)
On a map of Chicago, 1 cm represents 100 m. Select all statements that express the same scale.

A. 5 cm on the map represents 50 m in Chicago.

B. 1 mm on the map represents 10 m in Chicago.

C. 1 km in Chicago is represented by 10 cm on the map.

D. 100 cm in Chicago is represented by 1 m on the map.

**Solution**

B, C

**Lesson 6**

**Problem 1**
A car is traveling down a highway at a constant speed, described by the equation \( d = 65t \), where \( d \) represents the distance, in miles, that the car travels at this speed in \( t \) hours.

1. What does the 65 tell us in this situation?

2. How many miles does the car travel in 1.5 hours?

3. How long does it take the car to travel 26 miles at this speed?

**Solution**

1. The car travels 65 miles in 1 hour. Or, the car is traveling 65 miles per
hour. Or, 65 miles per hour is the constant of proportionality.

2. The car travels 97.5 miles in 1.5 hours.

3. It takes the car \( \frac{3}{5} \) of an hour, or 0.4 hours, or 24 minutes to travel 26 miles.

**Problem 2**

Elena has some bottles of water that each holds 17 fluid ounces.

1. Write an equation that relates the number of bottles of water \( b \) to the total volume of water \( w \) in fluid ounces.

2. How much water is in 51 bottles?

3. How many bottles does it take to hold 51 fluid ounces of water?

**Solution**

1. \( w = 17b \) or \( b = \frac{1}{17}w \)

2. 867 fluid ounces, because \( 17 \cdot 51 = 867 \)

3. 3 bottles, because \( 51 \div 17 = 3 \)

**Problem 3**

(from Unit 2, Lesson 5)

There are about 1.61 kilometers in 1 mile. Let \( x \) represent a distance measured in kilometers and \( y \) represent the same distance measured in miles. Write two equations that relate a distance measured in kilometers and the same distance measured in miles.

**Solution**

\( x = 1.61y \) and \( y = \frac{1}{1.61}x \) or \( y = 0.62x \)

**Problem 4**

(from Unit 2, Lesson 2)

In Canadian coins, 16 quarters is equal in value to 2 toonies.

<table>
<thead>
<tr>
<th>number of quarters</th>
<th>number of toonies</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td></td>
</tr>
<tr>
<td>16</td>
<td>2</td>
</tr>
<tr>
<td>20</td>
<td></td>
</tr>
<tr>
<td>24</td>
<td></td>
</tr>
</tbody>
</table>

1. Fill in the table.

2. What does the value next to 1 mean in this situation?

**Solution**

1. 

<table>
<thead>
<tr>
<th>number of quarters</th>
<th>number of toonies</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>( \frac{1}{8} )</td>
</tr>
<tr>
<td>16</td>
<td>2</td>
</tr>
<tr>
<td>20</td>
<td>2.5</td>
</tr>
<tr>
<td>24</td>
<td>3</td>
</tr>
</tbody>
</table>
2. \( \frac{1}{8} \) means that one-eighth of a toonie is worth the same as 1 quarter.

**Problem 5**

(from Unit 2, Lesson 2)

Each table represents a proportional relationship. For each table:

1. Fill in the missing parts of the table.
2. Draw a circle around the constant of proportionality.

<table>
<thead>
<tr>
<th>( x )</th>
<th>( y )</th>
<th>( a )</th>
<th>( b )</th>
<th>( m )</th>
<th>( n )</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>10</td>
<td>12</td>
<td>3</td>
<td>5</td>
<td>3</td>
</tr>
<tr>
<td>3</td>
<td>15</td>
<td>20</td>
<td>5</td>
<td>10</td>
<td>6</td>
</tr>
<tr>
<td>7</td>
<td>35</td>
<td>40</td>
<td>10</td>
<td>30</td>
<td>18</td>
</tr>
<tr>
<td>1</td>
<td>( \frac{5}{2} )</td>
<td>1</td>
<td>( \frac{1}{3} )</td>
<td>( \frac{1}{2} )</td>
<td>( \frac{1}{3} )</td>
</tr>
</tbody>
</table>

**Solution**

<table>
<thead>
<tr>
<th>( x )</th>
<th>( y )</th>
<th>( a )</th>
<th>( b )</th>
<th>( m )</th>
<th>( n )</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>10</td>
<td>12</td>
<td>3</td>
<td>5</td>
<td>3</td>
</tr>
<tr>
<td>3</td>
<td>15</td>
<td>20</td>
<td>5</td>
<td>10</td>
<td>6</td>
</tr>
<tr>
<td>7</td>
<td>35</td>
<td>40</td>
<td>10</td>
<td>30</td>
<td>18</td>
</tr>
<tr>
<td>1</td>
<td>( \frac{5}{2} )</td>
<td>1</td>
<td>( \frac{1}{3} )</td>
<td>( \frac{1}{2} )</td>
<td>( \frac{1}{3} )</td>
</tr>
</tbody>
</table>

**Problem 6**

(from Unit 1, Lesson 4)

Describe some things you could notice in two polygons that would help you decide that they were not scaled copies.

**Solution**

If they were not the same shape (for example, if one was a triangle and one was a square), they could not be scaled copies. I could find an angle measure in one that was not an angle measure of the other. I could find that a different scale factor would have to be used on one part of the pair than on another.

**Lesson 7**

**Problem 1**

Decide whether each table could represent a proportional relationship. If the relationship could be proportional, what would the constant of proportionality be?

1. How loud a sound is depending on how far away you are

<table>
<thead>
<tr>
<th>distance to listener (ft)</th>
<th>sound level (dB)</th>
</tr>
</thead>
<tbody>
<tr>
<td>5</td>
<td>85</td>
</tr>
<tr>
<td>10</td>
<td>79</td>
</tr>
<tr>
<td>20</td>
<td>73</td>
</tr>
<tr>
<td>40</td>
<td>67</td>
</tr>
</tbody>
</table>

**Solution**

<table>
<thead>
<tr>
<th>volume (fluid ounces)</th>
<th>cost ($)</th>
</tr>
</thead>
<tbody>
<tr>
<td>16</td>
<td>$1.49</td>
</tr>
<tr>
<td>20</td>
<td>$1.59</td>
</tr>
<tr>
<td>30</td>
<td>$1.89</td>
</tr>
</tbody>
</table>
1. Not proportional since the ratio of distance to listener to sound level is not always the same.

2. Not proportional since the ratio of volume to cost is not always the same.

**Problem 2**

A taxi service charges $1.00 for the first $\frac{1}{10}$ mile then $0.10 for each additional $\frac{1}{10}$ mile after that.

Fill in the table with the missing information then determine if this relationship between distance traveled and price of the trip is a proportional relationship.

<table>
<thead>
<tr>
<th>distance traveled (mi)</th>
<th>price (dollars)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\frac{1}{10}$</td>
<td>1.80</td>
</tr>
<tr>
<td>2</td>
<td>2.90</td>
</tr>
<tr>
<td>$3 \frac{1}{10}$</td>
<td>4.00</td>
</tr>
<tr>
<td>10</td>
<td>10.90</td>
</tr>
</tbody>
</table>

This is not a proportional relationship since the ratio of price to distance traveled is not always the same.

**Problem 3**

A rabbit and turtle are in a race. Is the relationship between distance traveled and time proportional for either one? If so, write an equation that represents the relationship.

**Turtle’s run:**

<table>
<thead>
<tr>
<th>distance (meters)</th>
<th>time (minutes)</th>
</tr>
</thead>
<tbody>
<tr>
<td>108</td>
<td>2</td>
</tr>
<tr>
<td>405</td>
<td>7.5</td>
</tr>
<tr>
<td>540</td>
<td>10</td>
</tr>
<tr>
<td>1,768.5</td>
<td>32.75</td>
</tr>
</tbody>
</table>

**Rabbit’s run:**

<table>
<thead>
<tr>
<th>distance (meters)</th>
<th>time (minutes)</th>
</tr>
</thead>
<tbody>
<tr>
<td>800</td>
<td>1</td>
</tr>
<tr>
<td>900</td>
<td>5</td>
</tr>
<tr>
<td>1,107.5</td>
<td>20</td>
</tr>
<tr>
<td>1,524</td>
<td>32.5</td>
</tr>
</tbody>
</table>

**Solution**

The distance might be proportional to the time for the turtle. The equation would be $d = 54 + t$, where $d$ represents the distance traveled in meters and $t$ is the time in minutes.

**Problem 4**

(from Unit 2, Lesson 2)

For each table, answer: What is the constant of proportionality?

1.  
2.  
3.  
4.  
Solution

1. 7 (or $\frac{1}{7}$)

2. 120 (or $\frac{1}{1375}$)

3. $\frac{1}{2}$ (or 25)

4. $2\frac{1}{2}$ (or $\frac{3}{4}$)

Problem 5
(from Unit 1, Lesson 4)
Kiran and Mai are standing at one corner of a rectangular field of grass looking at the diagonally opposite corner. Kiran says that if the field were twice as long and twice as wide, then it would be twice the distance to the far corner. Mai says that it would be more than twice as far, since the diagonal is even longer than the side lengths. Do you agree with either of them?

Solution

Kiran is correct. If we scale the length and width of a rectangle by a factor of 2, then the diagonal will also scale by a factor of 2.

Lesson 8

Problem 1

The relationship between a distance in yards ($y$) and the same distance in miles ($m$) is described by the equation $y = 1760m$.

1. Find measurements in yards and miles for distances by filling in the table.

<table>
<thead>
<tr>
<th>distance measured in miles</th>
<th>distance measured in yards</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td></td>
</tr>
<tr>
<td>5</td>
<td></td>
</tr>
<tr>
<td>3,520</td>
<td></td>
</tr>
<tr>
<td>17,600</td>
<td></td>
</tr>
</tbody>
</table>

2. Is there a proportional relationship between a measurement in yards and a measurement in miles for the same distance? Explain why or why not.

Solution

1. 
2. There is a proportional relationship. The constant of proportionality is 1760 yards per mile.

**Problem 2**

Decide whether or not each equation represents a proportional relationship.

1. The remaining length \(L\) of 120-inch rope after \(x\) inches have been cut off: \(120 - x = L\)

2. The total cost \(t\) after 8% sales tax is added to an item's price \(p\): \(1.08p = t\)

3. The number of marbles each sister gets \(x\) when \(m\) marbles are shared equally among four sisters: \(x = \frac{m}{4}\)

4. The volume \(V\) of a rectangular prism whose height is 12 cm and base is a square with side lengths \(s\) cm: \(V = 12s^2\)

**Solution**

1. no

2. yes

3. yes

4. no

**Problem 3**

1. Use the equation \(y = \frac{3}{2}x\) to fill in the table.

<table>
<thead>
<tr>
<th>(x)</th>
<th>(y)</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>3</td>
</tr>
<tr>
<td>3</td>
<td>4.5</td>
</tr>
<tr>
<td>6</td>
<td>9</td>
</tr>
</tbody>
</table>

Is \(y\) proportional to \(x\) and \(y\)? Explain why or why not.

2. Use the equation \(y = 3.2x + 5\) to fill in the table.

<table>
<thead>
<tr>
<th>(x)</th>
<th>(y)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>8.2</td>
</tr>
<tr>
<td>2</td>
<td>11.4</td>
</tr>
<tr>
<td>4</td>
<td>17</td>
</tr>
</tbody>
</table>

Is \(y\) proportional to \(x\) and \(y\)? Explain why or why not.

**Solution**

1. 
Yes, there is a proportional relationship between $x$ and $y$ since $\frac{5}{2} = \frac{15}{6}$ in each row.

2.

No, there is no proportional relationship between $x$ and $y$. In the first row $\frac{8.2}{1} = 8.2$ but in the second row $\frac{5.7}{2} = 2.85$.

**Problem 4**

(from Unit 2, Lesson 6)

To transmit information on the internet, large files are broken into packets of smaller sizes. Each packet has 1,500 bytes of information. An equation relating packets to bytes of information is given by $b = 1500p$ where $p$ represents the number of packets and $b$ represents the number of bytes of information.

1. How many packets would be needed to transmit 30,000 bytes of information?

2. How much information could be transmitted in 30,000 packets?

3. Each byte contains 8 bits of information. Write an equation to represent the relationship between the number of packets and the number of bits.

**Solution**

1. 20 packets

2. 45,000,000 bytes

3. $x = 12,000p$

**Lesson 9**

**Problem 1**

For each situation, explain whether you think the relationship is proportional or not. Explain your reasoning.

1. The weight of a stack of standard 8.5x11 copier paper vs. number of sheets of paper.

2. The weight of a stack of different-sized books vs. the number of books in the stack.
Solution
1. There is a proportional relationship between weight and number of sheets of paper. Each piece of paper has the same weight. To find the weight of a stack, multiply the number of sheets of paper by the weight of a single sheet of paper. (In case it comes up: We're assuming for this question that each piece of paper is the same weight. Manufacturing being what it is, though, we acknowledge that's not true.)

2. The relationship between the number of books and the weight of the stack is not proportional. Each book has a different weight, the weight of the stack can't be determined by multiplying the number of books by the weight of one book.

Problem 2
Every package of a certain toy also includes 2 batteries.

1. Are the number of toys and number of batteries in a proportional relationship? If so, what are the two constants of proportionality? If not, explain your reasoning.

2. Use \( t \) for the number of toys and \( b \) for the number of batteries to write two equations relating the two variables.

\[
b = \quad t =
\]

Solution
1. Yes. 2 and \( \frac{1}{2} \) are the constants of proportionality

2. \( b = 2t \) and \( t = \frac{1}{2}b \)

Problem 3
Lin and her brother were born on the same date in different years. Lin was 5 years old when her brother was 2.

1. Find their ages in different years by filling in the table.

<table>
<thead>
<tr>
<th>Lin's age</th>
<th>Her brother's age</th>
</tr>
</thead>
<tbody>
<tr>
<td>5</td>
<td>2</td>
</tr>
<tr>
<td>6</td>
<td></td>
</tr>
<tr>
<td>15</td>
<td></td>
</tr>
<tr>
<td></td>
<td>25</td>
</tr>
</tbody>
</table>

2. Is there a proportional relationship between Lin's age and her brother's age? Explain your reasoning.

Solution
1.
<table>
<thead>
<tr>
<th>Lin's age</th>
<th>Her brother's age</th>
</tr>
</thead>
<tbody>
<tr>
<td>5</td>
<td>2</td>
</tr>
<tr>
<td>6</td>
<td>3</td>
</tr>
<tr>
<td>15</td>
<td>12</td>
</tr>
<tr>
<td>28</td>
<td>25</td>
</tr>
</tbody>
</table>

2. There is no proportional relationship. Every year, they each age by one year, so the ratio of their ages changes every year.

Problem 4
(from Unit 2, Lesson 8)
A student argues that \( y = \frac{x}{9} \) does not represent a proportional relationship between \( x \) and \( y \) because we need to multiply one variable by the same constant to get the other one and not divide it by a constant. Do you agree or disagree with this student?

Solution
Disagree. Dividing by 9 is the same as multiplying by \( \frac{1}{9} \). We can look at the equation \( y = \frac{1}{9}x \) as \( y = \frac{x}{9} \). Also, \( \frac{y}{x} = \frac{1}{9} \) is constant for all corresponding values of \( x \) and \( y \).

Problem 5
(from Unit 1, Lesson 3)
Quadrilateral A has side lengths 3, 4, 5, and 6. Quadrilateral B is a scaled copy of Quadrilateral A with a scale factor of 2. Select all of the following that are side lengths of Quadrilateral B.

A. 5
B. 6
C. 7
D. 8
E. 9

Solution
B and D because the side lengths of Quadrilateral B would be 6, 8, 10, and 12.

Lesson 10
Problem 1
Which graphs could represent a proportional relationship? Explain how you decided.
Solution
A, C. Both are graphs of proportional relationships. Both graphs are part of a straight line that passes through the origin.

Problem 2
A lemonade recipe calls for $\frac{1}{4}$ cup of lemon juice for every cup of water.

1. Use the table to answer these questions.
   a. What does $x$ represent?
   b. What does $y$ represent?
   c. Is there a proportional relationship between $x$ and $y$?

<table>
<thead>
<tr>
<th>$x$</th>
<th>$y$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>$\frac{1}{4}$</td>
</tr>
<tr>
<td>2</td>
<td>$\frac{1}{2}$</td>
</tr>
<tr>
<td>3</td>
<td>$\frac{3}{4}$</td>
</tr>
<tr>
<td>4</td>
<td>1</td>
</tr>
<tr>
<td>5</td>
<td>$1\frac{1}{2}$</td>
</tr>
<tr>
<td>6</td>
<td>$1\frac{1}{4}$</td>
</tr>
</tbody>
</table>

2. Plot the pairs in the table in a coordinate plane.

Solution
1. a. $x$ represents the cups of water
   b. $y$ represents the cups of lemon juice
   c. Yes

2.

<table>
<thead>
<tr>
<th>$x$</th>
<th>$y$</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>3</td>
</tr>
<tr>
<td>4</td>
<td>6</td>
</tr>
<tr>
<td>5</td>
<td>7</td>
</tr>
<tr>
<td>8</td>
<td>10</td>
</tr>
</tbody>
</table>

Problem 3
(from Unit 2, Lesson 7)
Decide whether each table could represent a proportional relationship. If the relationship could be proportional, what would be the constant of proportionality?

1. The sizes you can print a photo
2. The distance from which a lighthouse is visible.

<table>
<thead>
<tr>
<th>height of a lighthouse (feet)</th>
<th>distance it can be seen (miles)</th>
</tr>
</thead>
<tbody>
<tr>
<td>20</td>
<td>6</td>
</tr>
<tr>
<td>45</td>
<td>9</td>
</tr>
<tr>
<td>70</td>
<td>11</td>
</tr>
<tr>
<td>95</td>
<td>13</td>
</tr>
<tr>
<td>150</td>
<td>16</td>
</tr>
</tbody>
</table>

**Solution**

1. Not proportional since the ratio of width to height is not always the same.

2. Not proportional since the ratio of height to distance is not always the same.

**Problem 4**

(from Unit 2, Lesson 9)

Select all of the pieces of information that would tell you $x$ and $y$ have a proportional relationship. Let $y$ represent the distance between a rock and a turtle's current position in meters and $x$ represent the number of minutes the turtle has been moving.

- A. $y = 3x$
- B. After 4 minutes, the turtle has walked 12 feet away from the rock.
- C. The turtle walks for a bit, then stops for a minute before walking again.
- D. The turtle walks away from the rock at a constant rate.

**Solution**

A, D

**Lesson 11**

**Problem 1**

There is a proportional relationship between the number of months a person has had a streaming movie subscription and the total amount of money they have paid for the subscription. The cost for 6 months is $47.94. The point (6, 47.94) is shown on the graph below.

1. What is the constant of proportionality in this relationship?
2. What does the constant of proportionality tell us about the situation?

3. Add at least three more points to the graph and label them with their coordinates.

4. Write an equation that represents the relationship between $C$, the total cost of the subscription, and $m$, the number of months.

Solution

1. $7.99$

2. The movie streaming service costs $7.99 for one month of service.

3. 

4. $C = 7.99m$

Problem 2

The graph shows the amounts of almonds, in grams, for different amounts of oats, in cups, in a granola mix. Label the point $(1, k)$ on the graph, find the value of $k$, and explain its meaning.

Solution
The point (1, 25) is on the graph. It means that for each cup of oats there are 25 grams of almonds in the granola mix.

**Problem 3**

(from Unit 2, Lesson 9)

To make a friendship bracelet, some long strings are lined up then taking one string and tying it in a knot with each of the other strings to create a row of knots. A new string is chosen and knotted with the all the other strings to create a second row. This process is repeated until there are enough rows to make a bracelet to fit around your friend’s wrist.

Are the number of knots proportional to the number of rows? Explain your reasoning.

**Solution**

Yes, since each row will have the same number of knots in it, the number of knots will always be a multiple of the number of rows.

**Problem 4**

(from Unit 2, Lesson 9)

What information do you need to know to write an equation relating two quantities that have a proportional relationship?

**Solution**

A constant of proportionality and variables for the quantities.

**Lesson 12**

**Problem 1**

Match each equation to its graph.

- A. \(y = 2x\)
- B. \(y = \frac{4}{5}x\)
- C. \(y = \frac{1}{4}x\)
- D. \(y = \frac{2}{5}x\)
- E. \(y = \frac{4}{5}x\)
- F. \(y = \frac{3}{5}x\)
Problem 2

The graphs below show some data from a coffee shop menu. One of the graphs shows cost (in dollars) vs. drink volume (in ounces), and one of the graphs shows calories vs. drink volume (in ounces).

1. Which graph is which? Give them the correct titles.

2. Which quantities appear to be in a proportional relationship? Explain how you know.

3. For the proportional relationship, find the constant of proportionality. What does that number mean?

Solution

1. The first graph is cost vs volume, and the second graph is calories vs volume. You can tell because the y-values are appropriate for cost in dollars on the first graph, and the y-values are appropriate for calories on the second.

2. It appears there is a proportional relationship between calories and volume. The points appear to lie on a line that would pass through the origin. Also, it makes sense that every one ounce would contain the same number of calories. Regarding the cost relationship, the points do not appear to lie on a precise line, and the line definitely would not pass through the origin. This makes sense because there is more to the cost of a cup of coffee than the amount of coffee.

3. The constant of proportionality is 15 calories per ounce, which can be found using $\frac{150}{10}$ or $\frac{360}{24}$. It means the coffee drink contains 15 calories in 1 ounce.

Problem 3

Lin and Andre biked home from school at a steady pace. Lin biked 1.5 km and it took her 5 minutes. Andre biked 2 km and it took him 8 minutes.

1. Draw a graph with two lines that represent the bike rides of Lin and Andre.
2. For each line, highlight the point with coordinates \((1, k)\) and find \(k\).

3. Who was biking faster?

**Solution**

Lin and Andre biked home from school at a steady pace. Lin biked 1.5 km and it took her 5 minutes. Andre biked 2 km and it took him 7 minutes.

1. [Graph]

2. For Lin's graph, \(k = 0.3\). For Andre's graph, \(k = \frac{2}{7}\).

3. Lin is going slightly faster at 0.3 km per minutes. Andre is going \(\frac{2}{7}\) or 0.25 km per hour.

**Lesson 13**

**Problem 1**

At the supermarket you can fill your own honey bear container. A customer buys 12 oz of honey for $5.40.

1. How much does honey cost per ounce?

2. How much honey can you buy per dollar?

3. Write two different equations that represent this situation. Use \(h\) for ounces of honey and \(c\) for cost in dollars.

4. Choose one of your equations, and sketch its graph. Be sure to label the axes.

**Solution**

1. $0.45 per ounce

2. About 2.2 ounces

3. \(c = 0.45h\; h = 2.2c\)

4. Students should have one of two linear graphs going through the origin. Graph 1: \(c = 0.45h\); horizontal axis label: \(h\), honey (ounces); vertical axis label \(c\), cost ($); Graph 2: \(h = 2.2c\); horizontal axis label: \(c\), cost ($); vertical axis label: \(h\), honey (ounces)
The point \((3, \frac{1}{2})\) lies on the graph representing a proportional relationship. Which of the following points also lie on the same graph?

A. \((1, 0.4)\)
B. \((1.5, \frac{6}{10})\)
C. \((\frac{2}{3}, 3)\)
D. \((4, \frac{12}{5})\)
E. \((15, 6)\)

**Solution**

A, B, E

**Problem 3**

A trail mix recipe asks for 4 cups of raisins for every 6 cups of peanuts. There is proportional relationship between the amount of raisins, \(r\) (cups), and the amount of peanuts, \(p\) (cups), in this recipe.

1. Write the equation for the relationship that has constant of proportionality greater than 1. Graph the relationship.
2. Write the equation for the relationship that has constant of proportionality less than 1. Graph the relationship.

**Solution**

1. \(p = \frac{2}{3}r\). Students should have a graph of \(p = \frac{2}{3}r\), label horizontal axis \(r\) (or "raisins (cups)") and vertical axis \(p\) (or "peanuts (cups)"). Since this is a proportional relationship, the graph should be linear and go through the origin.

2. \(r = \frac{3}{2}p\). Students should have a graph of \(r = \frac{3}{2}p\), label horizontal axis \(p\) and vertical axis \(r\). Since this is a proportional relationship, the graph should be linear and go through the origin. The slope of this graph should be less steep than the previous graph.

**Problem 4**

(from Unit 2, Lesson 11)

Here is a graph that represents a proportional relationship.

1. Come up with a situation that could be represented by this graph.
2. Label the axes with the quantities in your situation.
3. Give the graph a title.
4. Choose a point on the graph. What do the coordinates represent in your situation?

**Solution**

Answers vary. Sample response:

1. For every 2 gallons of gray paint created, 1 gallon of black paint is
used.


3. Title: Amount of Black Paint Needed to Create Gray Paint

4. The point (60, 30) means, in order to make 60 gallons of gray paint, 30 gallons of black paint is needed.

Lesson 14
Problem 1

The equation \( c = 2.95g \) shows how much it costs to buy gas at a gas station on a certain day. In the equation, \( c \) represents the cost in dollars, and \( g \) represents how many gallons of gas were purchased.

1. Write down at least four (gallons of gas, cost) pairs that fit this relationship.

2. Create a graph of the relationship.

3. What does 2.95 represent in this situation?

4. Jada's mom remarks, “You can get about a third of a gallon of gas for a dollar.” Is she correct? How did she come up with that?

Solution

1. Answers vary. Sample response:

<table>
<thead>
<tr>
<th>gallons of gas (g)</th>
<th>cost in dollars (c)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>2.95</td>
</tr>
<tr>
<td>2</td>
<td>5.90</td>
</tr>
<tr>
<td>10</td>
<td>29.50</td>
</tr>
<tr>
<td>20</td>
<td>59.00</td>
</tr>
</tbody>
</table>

2. Answers vary. Sample response:

3. One gallon of gas costs $2.95. Or, gas costs $2.95 per gallon. Or, 2.95 dollars per gallon is the constant of proportionality.

4. Since 2.95 is close to 3, Jada's mom reasoned that if it cost about 3 dollars per gallon, the reciprocal rate must be \( \frac{1}{3} \) gallon per dollar.

Problem 2
There is a proportional relationship between a volume measured in cups and the same volume measured in tablespoons. 3 cups is equivalent to 48 tablespoons, as shown in the graph.

1. Plot and label at least two more points that represent the relationship.

2. Use a straightedge to draw a line that represents this proportional relationship.

3. For which value y is (1, y) on the line you just drew?

4. What is the constant of proportionality for this relationship?

5. Write an equation representing this relationship. Use $c$ for cups and $t$ for tablespoons.

Solution

1. See below

2. See below

3. 16

4. 16 tablespoons per cup

5. $t = 16c$
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