Expressions: Integer Exponents

Review of Exponential Form

Exponential form is a simple and useful way to express a number that is multiplied by itself many times. For example:

6^{10} means (6 • 6 • 6 • 6 • 6 • 6 • 6 • 6 • 6 • 6).

6 is the repeated factor. It is also known as the *base*.

10 tells you that there are 10 repeated factors. 10 is the *exponent*.

Parentheses are essential when you express a negative number in exponential form. Consider the number -4^6. Without parentheses, you read this number as follows:

$-(4 \cdot 4 \cdot 4 \cdot 4 \cdot 4 \cdot 4) = -4{,}096$

If you want to represent "−4 raised to the power of 6," you must use parentheses:

$(-4)^6 = (-4)(-4)(-4)(-4)(-4)(-4) = +4{,}096$

Basic Definitions

For any positive integer n and $b \neq 0$, consider these definitions:

- Definition of b^n: $b^n = b \cdot b \cdot b \cdot \ldots \cdot b$ (The factor b occurs n times.)
- Definition of b^{-n}: b^{-n} = 1bn

The first definition shows the basic meaning of exponential notation as repeated multiplication.

The second definition introduces a negative exponent to stand for a reciprocal: b^{-1}= 1b.

Operations With Exponents

To multiply different powers of the same base, *add* the exponents:

$b^n \bullet b^m = b^{n+m}$ where n and m are any positive integers

To divide different powers of the same base, *subtract* the exponents:

bnbm = b^{n-m} where n and m are any positive integers and $b \neq 0$

To take a power of a power, *multiply* the exponents:

$(b^n)^m = b^{n \bullet m}$ where n and m are any positive integers

Negative Exponents

Exponents can be negative. The notation b^{-n} is defined as 1bn and the notation b^n is defined as 1b–n, where $b \neq 0$. This relationship is called the *negative exponent property*.

The following example demonstrates why this property is true.

$2^3 \div 2^{-7}$ = 2327=2•2•22•2•2•2•2•2•2

=12•2•2•2 (by cancellation of factors)

= 12−4 or 2^{-4}

$2^3 \bullet 2^{-7} = 2^{3+(-7)} = 2^{-4}$ = 12−4

In addition:

12−4=1÷124=1•241= 2^4, so it is also true that 2^4 = 12−4.

You can simplify and evaluate expressions involving negative exponents using the same rules that you use for positive exponents, fractional exponents, and the zero exponent. Consider these examples:

$9^{-3} \div 9^5 = 9^{(-3 + 5)} = 9^2$

$(4^{-6})^2 = 4^{(6 \cdot -2)} = 4^{-12}$

$(2^{-6} \cdot x^3)^{-5} = 2^{(-6) \cdot (-5)} \cdot x^{3 \cdot (-5)} = 2^{30} \cdot x^{-15}$ = 230x15

(9−54)−2=(9(−5)·(−2)41·(−2))=9104−2=910·42

A negative base raised to a positive power *x* is equal to the negative number multiplied by itself *x* times. A base raised to a negative exponentis equal to 1 divided by that base raised to a positive exponent. For example, compare $(-4)^5$ and 4^{-5}:

$(-4)^5 = (-4) \cdot (-4) \cdot (-4) \cdot (-4) \cdot (-4) = -1{,}024$

4−5=145=11,024

The Zero Exponent

Any real, nonzero base raised to the power of zero has the value 1. Thus, in general, $b^0 = 1$, where $b \neq 0$.

For example, consider the expression $4^6 \div 4^6$. Using subtraction of exponents, the simplification is 4646=40.

4•4•4•4•4•44•4•4•4•4•4=1

Representing the numerator and the denominator in expanded form and canceling factors gives the answer 1. It follows that $4^0 = 1$.

Scientific Notation

Scientific notation is a way of compactly representing very large and very small numbers. A number written in scientific notation is written as a number between 1 and 10 multiplied by a power of 10.

$a \times 10^n$ where $1 \leq a < 10$ and n is an integer

Here are some examples:

0.024=2.4100=2.4102=2.4•1102=2.4•10−2

$12{,}560{,}800 = 1.2560800 \bullet 10{,}000{,}000 = 1.25608 \bullet 10^7$