

Expressions: Roots and Powers

Squares

The **square** of a number is that number multiplied by itself. The square of a number n is the number $n \cdot n = n^2$.

The exponent represents the operation—multiplication—and indicates the number of factors, which in this case is 2. Consider this example:

$$4 \cdot 4 = 16$$

$$4 \cdot 4 = 4^2 \quad x \cdot x = x^2$$

4 is the factor two times. x is the factor two times.

2 is the exponent. 2 is the exponent.

Squares can be represented in three different ways: exponential form, expanded form, and evaluated form.

Exponential form $4^2 \quad x^2$

Expanded form $4 \cdot 4 \quad x \cdot x$

Evaluated form 16

You can read an equation involving squares in several ways. For example, the expression $4^2 = 16$ can be read:

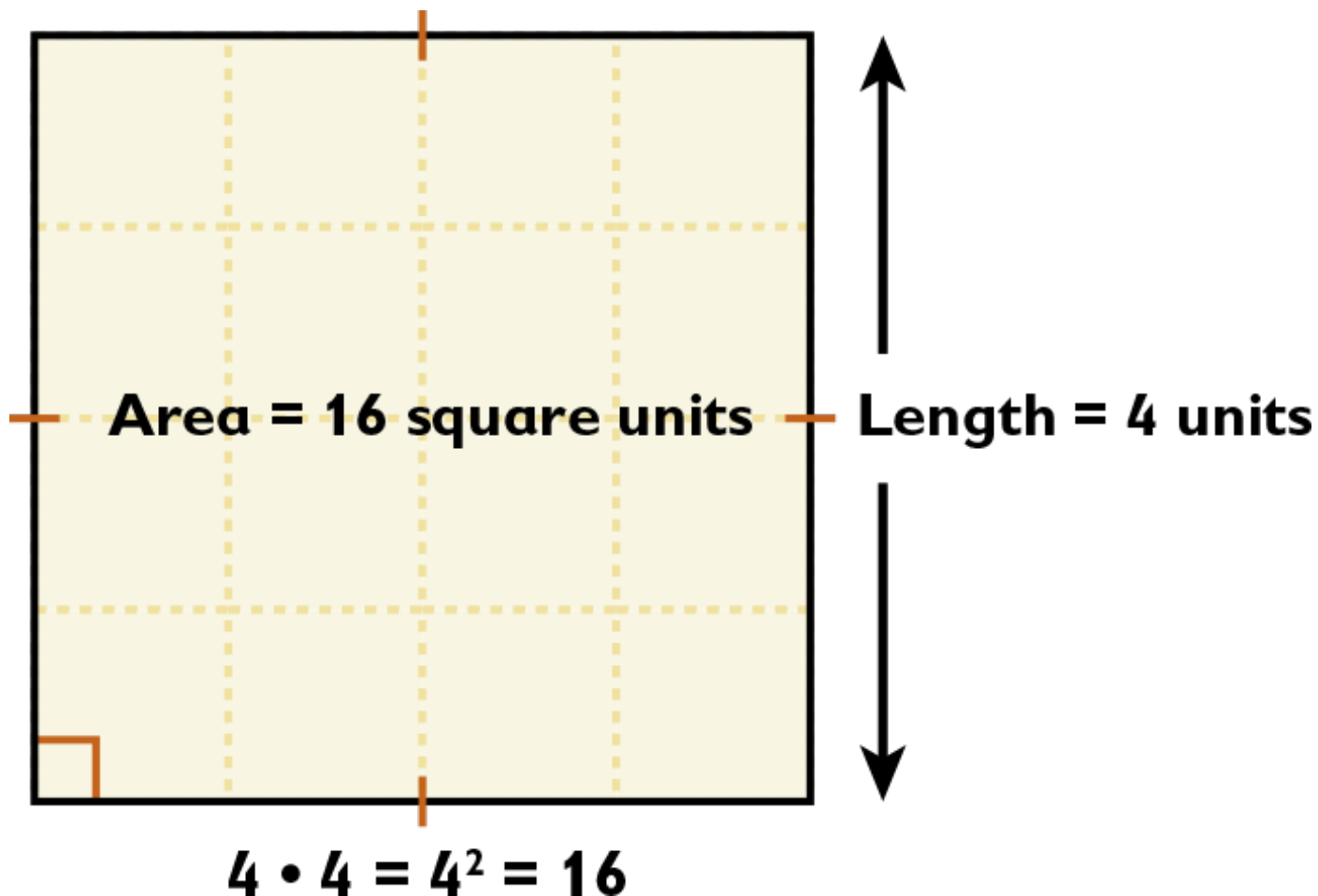
“The square of 4 is 16.”

“4 squared is 16.”

“4 raised to the second power is 16.”

“4 to the second power is 16.”

The square of a number can be represented geometrically as an area: the area of a square with side length s is the number s^2 . Consider this figure:



The side length of the square is 4 units. The area of the square is $4 \cdot 4 = 4^2 = 16$, or 16 square units.

Square Roots

The **square root** of a number n is a number that has a square equal to n . So, the number m is a square root of n only if $m \cdot m = n$.

For example, consider the expression $4 \cdot 4$:

$$4 \cdot 4 = 16$$

It is also true that $-4 \cdot -4 = (-4)^2 = 16$.

$$\sqrt{16} = \pm 4$$

4 is called the **positive square root** of 16. -4 is called the **negative square root** of 16.

Every positive, nonzero real number has two square roots, one positive and one negative. If m is a square root of n , then $-m$ is also a square root of n . By convention, when you use the square root symbol, $\sqrt{}$, you are referring to the positive square root.

The square root of a negative number is not a real number, because squaring a positive root gives a positive result and squaring a negative root also gives a positive result.

Rational and Irrational Square Roots

Some square roots are **rational**. For example:

$$\sqrt{81} = 9 \quad \sqrt{0.25} = 0.5$$

Other square roots are **irrational**. Irrational square roots cannot be evaluated exactly but can be expressed as decimal approximations to a given number of decimal places. An example of an irrational square root is $\sqrt{3}$.

$$\sqrt{3} \approx 1.73205$$

The square of $\sqrt{3}$ equals 3, but the square of 1.73205 does not.

Sometimes an answer expressed in the form $\sqrt{3}$ is acceptable.

Cubes

The **cube** of a number is that number multiplied by its square. The cube of a

number n is the number $n \cdot n \cdot n = n^3$.

The exponent represents the operation—multiplication—and indicates the number of factors, which in this case is 3. Consider this example:

$$4 \cdot 4 \cdot 4 = 64$$

$$4 \cdot 4 \cdot 4 = 4^3 \quad x \cdot x \cdot x = x^3$$

$$4^3 = 64$$

4 is the factor three times. x is the factor three times.

3 is the exponent. 3 is the exponent.

Cubes can be represented in three different ways: exponential form, expanded form, and evaluated form.

Exponential form $4^3 \quad x^3$

Expanded form $4 \cdot 4 \cdot 4 \quad x \cdot x \cdot x$

Evaluated form 64

You can read an equation involving cubes in several ways. For example, the expression

$4^3 = 64$ can be read:

“The cube of 4 is 64.”

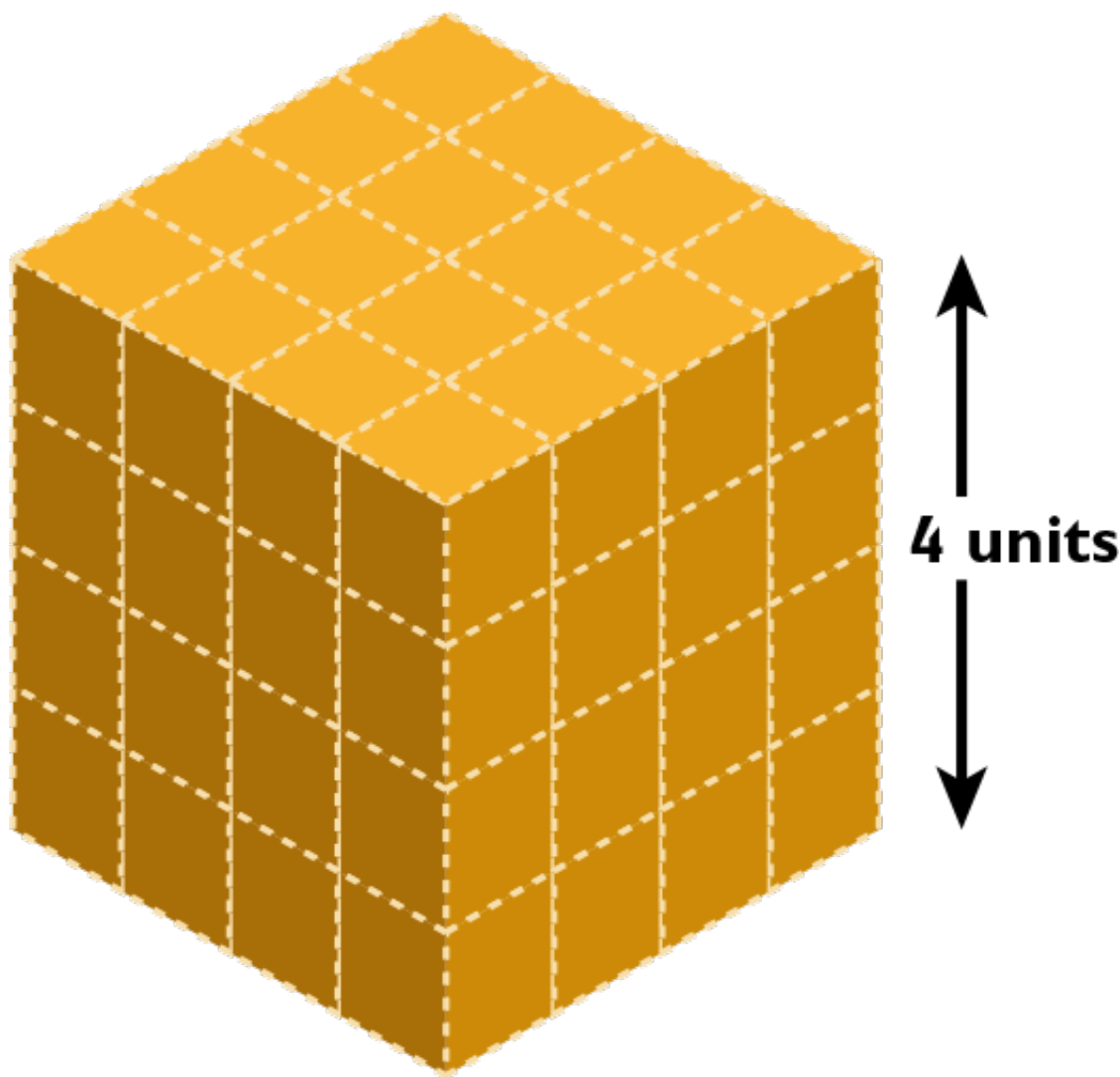
“4 cubed is 64.”

“4 raised to the third power is 64.”

“4 to the third power is 64.”

You can represent the cube of a number geometrically as a volume: the

volume of a cube with side length s is the number s^3 . Consider this figure:



The side length of the cube is 4 units. The volume of the cube is $4 \cdot 4 \cdot 4 = 4^3 = 64$, or 64 cube units.

Cube Roots

The **cube root** of a number n is a number whose cube equals n . So, the number m is a cube root of n only if $m \cdot m \cdot m = n$.

For example, the cube root of 64 is 4:

$$\sqrt[3]{64} = 4$$

Because 4 is a rational number, $\sqrt[3]{64}$ is called a **rational cube root**.

The cube root symbol is $\sqrt[3]{}$. When you write a cube root, such as $\sqrt[3]{64}$, make sure that the exponent is written small enough and within the “hook” of the symbol so that it is not interpreted incorrectly—for example, as $3\sqrt[3]{64}$.

Relationship of Powers and Roots

The operations of “raising to a power” and “taking a root” are inverse operations in the sense that each undoes the other.

Taking the n th root undoes raising to the power n .

$$\sqrt[n]{b^n} = b \text{ (in general, } \sqrt[n]{b^n} = b \text{)}$$

Raising to the power n undoes taking an n th root.

$$(b^n)^{1/n} = b \text{ (in general, } (b^n)^{1/n} = b \text{)}$$