# Irrational Numbers

Decimals that do not terminate or repeat are called *irrational numbers*. *Irrational* means "not rational."

You have encountered two kinds of irrational numbers in your math studies thus far: the square root of a number that is not a perfect square and the number π. Another example of an irrational number is 0.123456789101112.... The pattern in these digits could go on forever, but it is not a repeating pattern, so the number is not a rational number.

## The Number π

The fact that the number π is not rational was first proved about 100 hundred years ago. The main way of finding a value for π is to use a calculator. To 10 digits, the value is: π ≈ 3.141592654. The decimal representation of π provides approximations to π as rational numbers by simply using only a few decimal places. For example, π ≈ 3.14 = 314100.

It is also possible to approximate π independently of the decimal system (i.e., in the form ab, where *a* and *b* are integers and *b* is *not* a power of 10). Here are the four approximations using the smallest integers *a* and *b*:

31    227    333106    355113

These numbers get very close to π. For example, the fourth one is approximately 3.141592920, accurate to 7 digits. However, *no* rational approximation to π is exact, no matter how large the integers that are used.

## Square Roots

Square roots that are not perfect squares, such as the number 7‾√, are irrational numbers.

As with the number $\pi$, you can make a rational approximation to a square root. One method of doing this is known as the “divide-and-average” method. For example, consider $\sqrt{7}$:

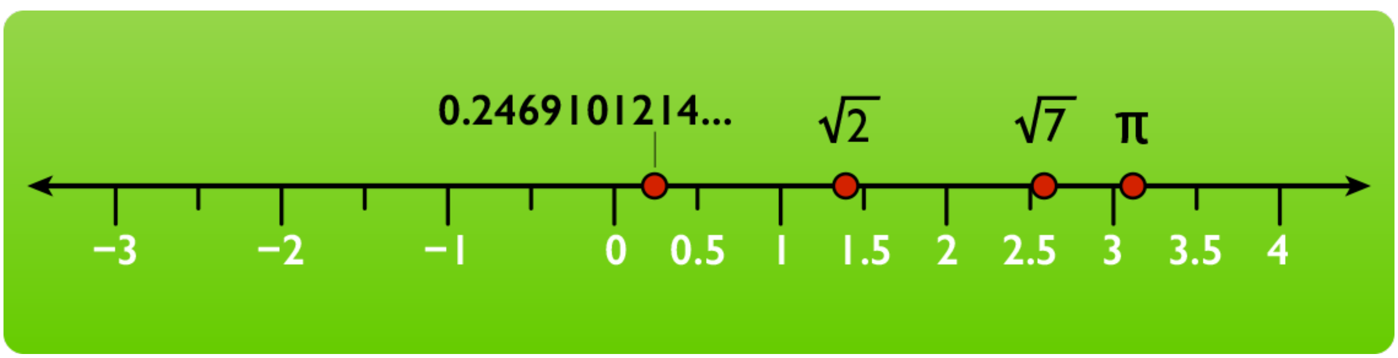
- Guess $\sqrt{7} \approx 2.5$. Then *divide* 7 by this guess: $\frac{7}{2.5} = 2.8$.
- *Average* 2.5 and 2.8, which is 2.65. This is the next approximation.
- Guess $\sqrt{7} = 2.65$. Then *divide* 7 by this guess: $\frac{7}{2.65} \approx 2.642$.
- *Average* 2.65 and 2.641, which is 2.6455. This is the next approximation. (Note: 2.6455 squared is about 6.9987, very close to 7, after only two steps.)

### Comparing Irrational Numbers on a Number Line

Because you can find approximations of irrational numbers, you can compare them. Consider the following irrational numbers:

$\pi \quad 0.2469101214\ldots \quad \sqrt{2} \quad \sqrt{7}$

The following number line shows the approximate locations of each of these numbers:




From the number line, you can see that:

$0.246910\ldots < \sqrt{2} < \sqrt{7} < \pi$