Rational Numbers: Decimals

A *rational number* is defined as any number that can be expressed in the form ab, where *a* and *b* are integers and $b \neq 0$. Here are some examples of rational numbers:

12

3 (or 31)

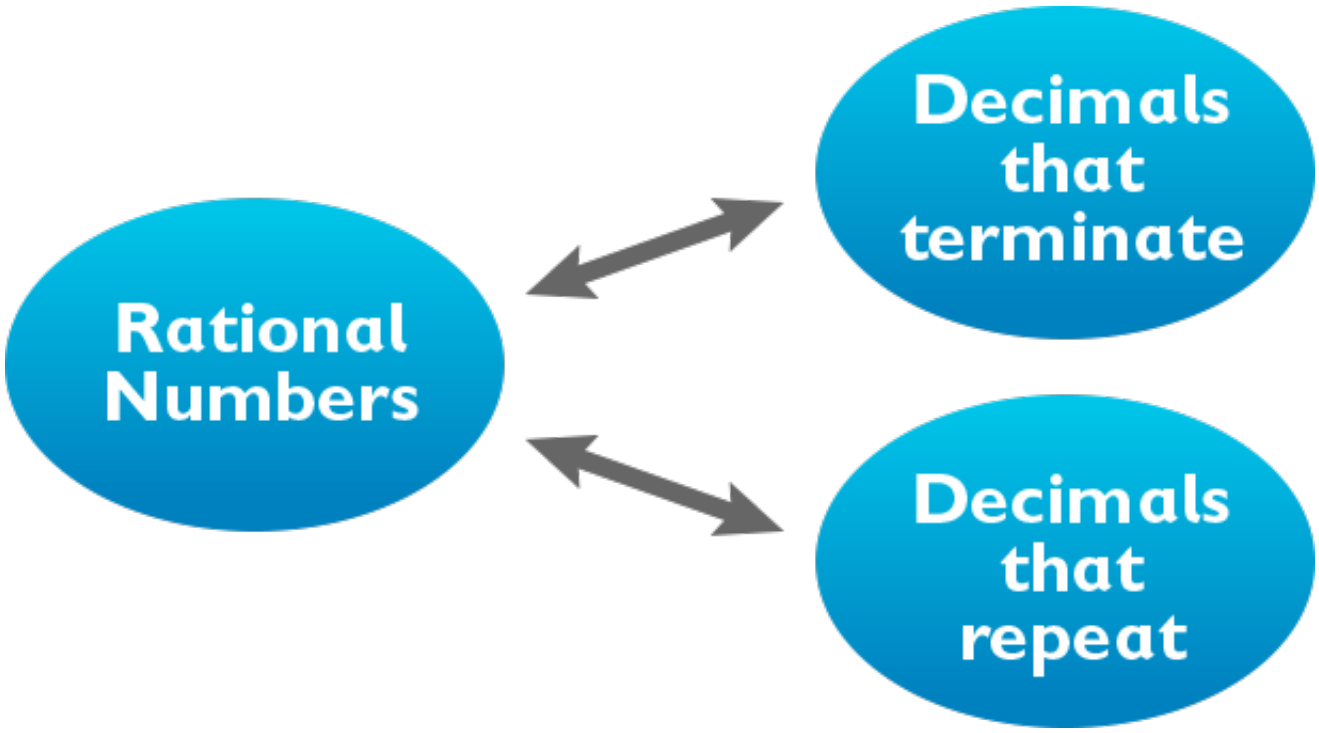
2.15 (or 215100)

−0.37 (or −37100)

20% (or 0.2 or 210)

Terminating and Repeating Decimals

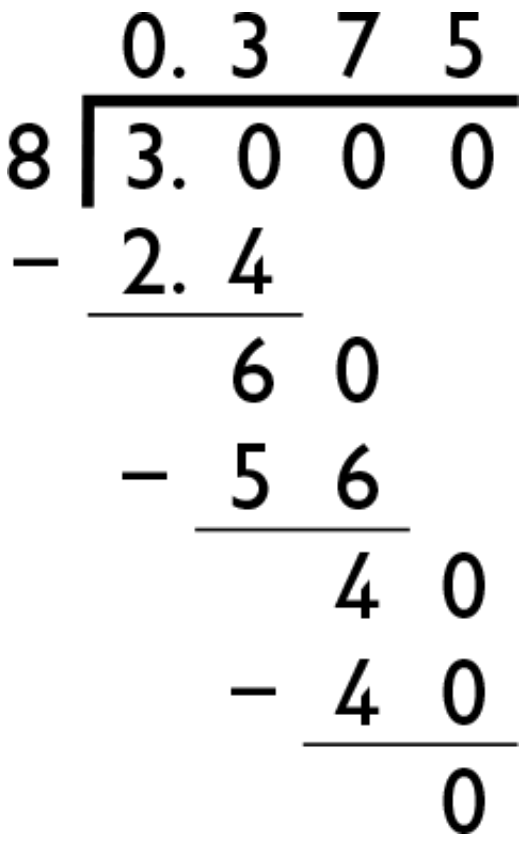
When you use long division to divide one integer by a nonzero integer, one of two interesting things can happen. The number may be represented by a *terminating* decimal, or the number may be represented by a *repeating* decimal.



Terminating Decimals

A *terminating decimal* stops after a finite number of digits. Any further digits are zeros.

For example, when you carry out long division for 3 ÷ 8, you get a terminating decimal 0.375. The long division stops after these three digits because the third place divides exactly, with no remainder.



Here are some other examples of terminating decimals:

45= 4 ÷ 5 = 0.8

−3751,000 = −375 ÷ 1,000 = −0.375

Using place value, terminating decimals can be expressed as fractions. Then the fractions can be reduced to the simplest form of the fraction. For example:

0.656 = 6561,000 = 82•8125•8 = 82125

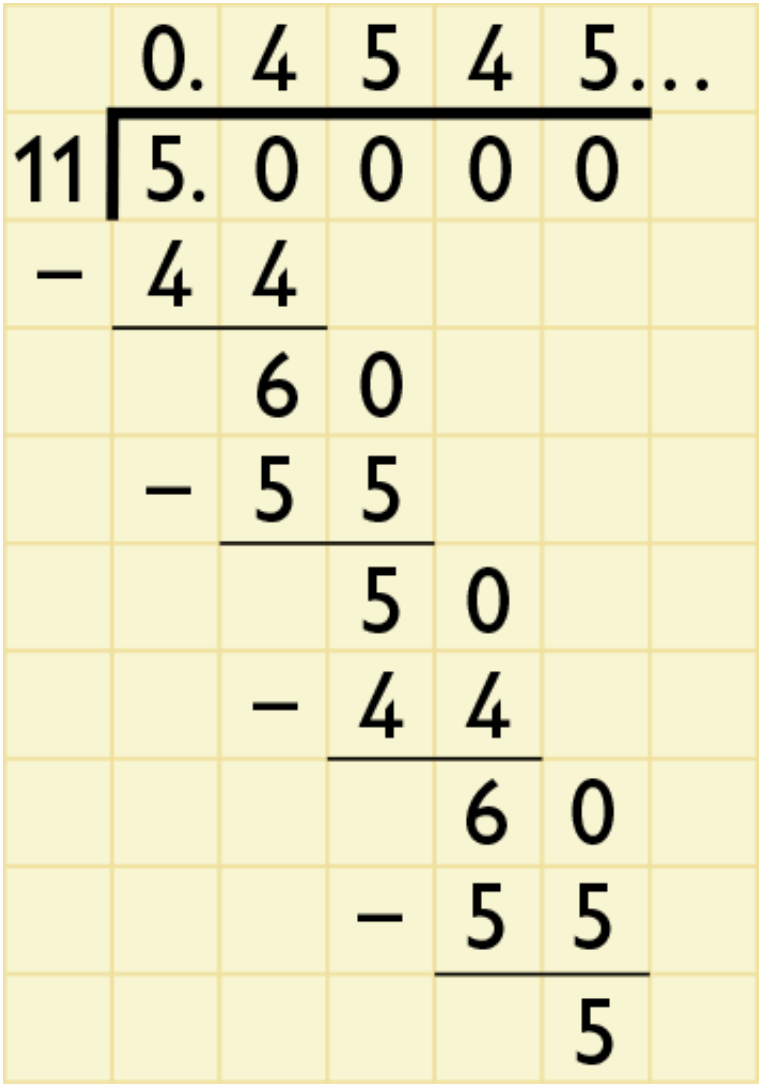
2.14 = 214100 = 2750

Repeating Decimals

When the quotient $a \div b$ is represented as a decimal with a finite number of digits that repeat infinitely, it is called a *repeating decimal*. For example, 511 is a repeating decimal.

$\frac{5}{11} = 0.454545\ldots$

The decimal never terminates because there is never a 0 remainder. There is a repeating pattern: the 45 repeats infinitely.



The repeating pattern is often indicated with a line over the top of the digits that repeat. The following numbers are examples of repeating decimals:

$\frac{1}{3} = 0.333333\ldots = 0.\overline{3}$

$\frac{7}{12} = 0.583333\ldots = 0.58\overline{3}$

$\frac{1}{11} = 0.090909\ldots = 0.\overline{09}$

Converting a Decimal to a Fraction

A terminating decimal, such as 0.375, can be converted to a fraction. The place value of the last digit in the decimal is the denominator of the fraction.

For example, 0.375 can be written as 3751,000. Then 3751,000 can be reduced:

$$3751{,}000 = 75 \cdot 5200 \cdot 5 = 75200 = 3 \cdot 258 \cdot 25 = 38$$