LESSON I: POWERS OF 2

EXERCISES

EXERCISES

- 1. Write what you already know about rational numbers.
 - Share your summary with a classmate.
 - Did you write about the same concepts?
- 2. In this lesson, you looked at a pattern based on cutting pieces of paper in twos over and over again. You found that the pattern increased by a factor of 2 each time you cut: 1, 2, 4, 8, 16, 32,
 - Consider how the situation would change if each time you cut the paper, you cut into threes instead of twos.
 - Write your wonderings about exponents.
- 3. Think about your work in the math units you have completed this year so far. Describe something you are especially proud of and that you would like to repeat in this unit.
 - For example, consider how you participated in discussions, ways you worked with partners or helped out a classmate, a new math concept you learned, questions you shared that helped you better understand a topic, a challenge or exercise that you enjoyed completing, and so on.
- 4. Write a goal stating what you plan to accomplish in this unit. Write your goals for all units in the same place so you can review your past goals as you write new goals for this unit.

LESSON 2: EXPONENTS

EXERCISES

EXERCISES

- 1. Evaluate the expression. 3⁴ = _____
- 3. Evaluate the expression. $5^2 =$
- 4. Evaluate the expression. $4^3 \cdot 6^2 =$
- 5. Evaluate the expression. Show your work. $\left(\frac{1}{3}\right)^2 \cdot 3^3$
- 6. Evaluate the expression. Show your work. $\left(\frac{1}{4}\right)^4 \cdot 5^2$
- 7. Evaluate the expression. $4^2 \cdot 4^2 \cdot 4^2 \cdot 4^2 =$
- 8. Exponents (especially powers of 2) are often used to express the memory capacity of computers. For example, 512 megabytes (MB) of RAM could be expressed as 2°.
 - a. Write the next four powers of 2 after 29 as whole numbers.
 - b. Describe another situation in which you have seen powers of 2 used.

Challenge Problem

- 9. Volume is expressed as a cubic measurement, such as cubic centimeters or cm³.
 - a. How is a cubic measurement related to raising a number to the power of 3?
 - b. Why do you think raising to the power of 3 is called cubing a number?
 - c. Consider a cube with edge length 3 m. What is the volume of the cube?
 - d. What unit is used for the volume of the cube in part c? Explain why the unit makes sense.

LESSON 3: SQUARE AND CUBE ROOTS

EXERCISES

EXERCISES

1	What	ic t	·h_	cuba	root	of	272
	vvnat	is t	.ne	cube	root	OI	Z/:

△ –3

B 3

9

27

2. What is
$$\sqrt{100}$$
?

A ±10

B 10

● ±25

D 25

3. What is
$$\sqrt[3]{216}$$
?

A 6

B ±6

● −6

There is no cube root.

5. If a square has an area of 0.25 square unit, what is the side length of the square? The side length is _____ units.

6. Solve.

 $x^2 = 25$

- A The equation has no solutions.
- **B** −5 is the only solution.
- **6** 5 is the only solution.
- 5 and -5 are both solutions.
- 7. Solve.

$$x^2 = 3.600$$

- 8. The equation $x^2 = -16$ has _____.
 - A no solution
 - B one solution
 - two solutions
 - three solutions

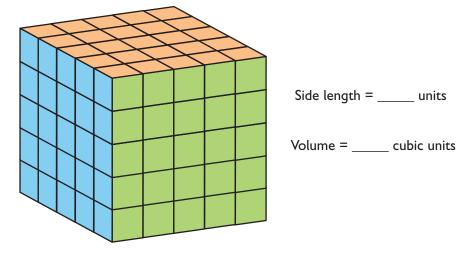
LESSON 3: SQUARE AND CUBE ROOTS

EXERCISES

9. Solve.

$$x^2 = 81$$

10. Find the volume and side length of the large cube.



Challenge Problem

11. If you take the square root of a number and then take the cube root of the result, you get 5.

What is the number? Explain.

LESSON 4: SIMPLIFYING EXPRESSIONS

EXERCISES

EXERCISES

1. Which expressions are equivalent to this repeated factor? There may be more than one equivalent expression.

- **B** 7,776
- \bigcirc 6³ + 6²
- **5** × 6
- **6**⁵
- 2. Which of these expressions show the simplification of this expression?

$$0.3 \cdot 0.3 \cdot 0.3 \cdot 0.3$$

- \triangle 0.3⁴
- **B** 3⁴
- 0.0003⁴
- **(**0.3)
- 3. Simplify the expression.

4. Simplify the expression.

$$g + g + g + g + g + g + g =$$

5. Consider Jacob's work simplifying an expression.

1. Simplify this expression.

$$\frac{13^5}{13^2} = \frac{13 + 13 + 13 + 13 + 13}{13 + 13} = 13 + 13 + 13 = 39$$

Explain whether Jacob's solution is correct. If necessary, correct any mistakes.

LESSON 4: SIMPLIFYING EXPRESSIONS

EXERCISES

6. Consider Pedra's work simplifying an expression.

Name: <u>Pedra</u>

2. Simplify this expression.

 $\sqrt[3]{343}$

$$\sqrt[3]{34\cdot 3} = \sqrt[3]{7 \cdot 7 \cdot 7} = 7$$

Explain whether Pedra's solution is correct. If necessary, correct any mistakes.

7. Write the expression as repeated factors.

$$(8^6 \cdot 2) \div (8^2 \cdot 2^3)$$

8. Write the expression as repeated factors.

$$\left(\frac{3}{7}\right)^7$$

Challenge Problem

9. Consider Talisha's work evaluating an expression.

Name: <u>Talisha</u>

3. Evaluate without using a calculator.

$$\left(\sqrt[3]{6}\right)^3$$

$$\left(\sqrt[3]{6}\right)^3 = \left(\sqrt[3]{2 \cdot 2 \cdot 2}\right)^3 = 2^3 = 8$$

Explain whether Talisha's solution is correct. If Talisha's work is incorrect, explain how she should have evaluated the expression.

LESSON 5: MULTIPLYING AND DIVIDING

EXERCISES

EXERCISES

- 1. Which of these expressions is equivalent to $\frac{3^9}{3^3}$?
 - \triangle 3³
- **B** 36
- **3**12
- **▶** 3²⁷
- 2. What is this expression simplified to its simplest exponential form? $\frac{6^3}{6^7}$
 - A 1⁷⁻³
- **B** $\frac{1}{6^4}$
- $\mathbf{6}^{\frac{3}{7}}$
- **●** 6⁴
- 3. What is this expression simplified to its simplest exponential form?

- A 27⁶⁴
- B 27¹²
- **3**64
- 3¹²
- 4. Simplify the expression to its simplest exponential form. Show your work.

$$\frac{11^6}{11^7}$$

5. Simplify the expression to its simplest exponential form. Show your work.

$$5^2 \cdot 5^2 \cdot 5^2 \cdot 5^2$$

- 6. Choose all the expressions that are equivalent to $2^2 \cdot 2^5$? There may be more than one equivalent expression.
 - \triangle 2² 2² 2³
 - **B** 4¹⁰
 - **6** 4²⁺⁵
 - 27
 - **3**10
- 7. Simplify the expression to its simplest exponential form. Show your work.

$$\frac{3^4}{3^2} \cdot \frac{3^3}{3^4}$$

LESSON 5: MULTIPLYING AND DIVIDING

EXERCISES

8. Simplify the expression to its simplest exponential form. Show your work.

$$\frac{8^4}{8^2} \cdot \frac{8^3}{8^5}$$

Challenge Problem

9. Find the value of p in this equation. Show your work.

$$500 \bullet 10^{98} + 10^{10^2} = p \bullet 10^{100}$$

LESSON 6: PROPERTIES OF EXPONENTS

EXERCISES

EXERCISES

- 1. Consider this equation. $(4^7)^3 = 4^x$. What is the value of x?

 - **A** x = 4 **B** x = 10 **c** x = 21
- x = 28
- 2. Consider this equation. $13^7 \cdot 13^9 = 13^9$. What is the value of y?
 - \triangle y = 2
- **B** y = 13 **Q** y = 16
- y = 63
- 3. Choose all the expressions that are equivalent to $5^2 \cdot 5^6$. There may be more than one equivalent expression.
 - \triangle (5 5)⁴
 - **B** $2 \cdot (5^2)^2$
 - $(5^2)^2 \cdot (5^2)^2$
 - **5**8
 - $(5^4)^2$
- 4. Write four different expressions that are equivalent to this expression. $\left(6^4\right)^2$
- 5. Write four different expressions that are equivalent to this expression. $(9 \cdot 2)^4$
- 6. Use the properties of exponents to show that this equality is true. $25^5 = 5^{10}$
- 7. Use the properties of exponents to show that this equality is true.

$$\left(2^4\right)^3 = \left(2^6\right)^2$$

8. Explain why -2^4 is not equivalent to this expression.

$$(-2)^2 \cdot (-2)^2$$

LESSON 6: PROPERTIES OF EXPONENTS

EXERCISES

Challenge Problem

- 9. Which expression is greater? Explain. a. $(2^2)^2$ or $2^{(2^2)}$ b. $(3^3)^3$ or $3^{(3^3)}$

LESSON 7: SCIENTIFIC NOTATION

EXERCISES

EXERCISES

1. Which of these numbers is written in scientific notation?

A 27.2 • 10⁸

B 1.4 • 5⁶

○ 0.907 • 10⁻¹²

5.001 • 10⁻²⁵

2. Which of these numbers is the scientific notation form of 409.9×10^{9} ?

 \triangle 4.099 • 10⁷

B 4.099 • 10¹¹

0.000004099

0.0000004099

3. The half-life of a helium-7 atom is about 0.000000000000000000304 sec.

Write this measurement in scientific notation.

4. The number of water molecules in 1 cup of water is approximately 7,500,000,000,000,000,000,000.

Write this statement using scientific notation.

5. The circumference of Neptune's orbit around the sun is about 17,562,000,000 mi.

Write the measurement in scientific notation.

mi

- 6. Write 0.00444 10¹⁵ in scientific notation.
- 7. Write $0.75 \cdot 10^{-42}$ in scientific notation.
- 8. Write the number as a decimal number.

8.068 • 10-8 =

9. Write the number as a decimal number.

1.00001 • 10¹³ =

10. Compare the two expressions. Choose <, =, or >.

 $4.5 \cdot 10^{20}$ _____ $4.5 \cdot 10^{-20}$

LESSON 7: SCIENTIFIC NOTATION

EXERCISES

11. Compare the two expressions. Choose <, =, or >.

12. Compare the two expressions. Choose <, =, or >.

$$3.2 \cdot 10^{-8}$$
 _____ $1.2 \cdot 10^{-7}$

13. Find the number that is half of this number.

$$4.28 \times 10^{31}$$

Write your answer in scientific notation.

Challenge Problem

14. The micrometer, nanometer, and picometer are units used to measure very small objects.

Unit	Length	
micrometer (µm)	10 ⁻⁶ m	(1 millionth of a meter)
nanometer (nm)	10 ⁻⁹ m	(1 billionth of a meter)
picometer (pm)	10 ⁻¹² m	(1 trillionth of a meter)

- a. The length of an Amoeba proteus is 500 μ m. What is this length in meters? Give your answer in scientific notation.
- b. The diameter of a glucose molecule is 900 pm. What is the diameter of a glucose molecule in micrometers?
- c. A measles virus has a diameter of 220 nm. What is the diameter of a measles virus in meters?

LESSON 8: ZERO AND NEGATIVE EXPONENTS

EXERCISES

EXERCISES

1. Determine whether this equation is true or false.

$$7^{-2} = \frac{1}{49}$$

The equation is _____.

2. Determine whether this equation is true or false.

$$1^0 = 0$$

The equation is _____.

3. Find the value of the expression.

4. Find the value of the expression.

5. Find the value of the expression.

6. Find the value of the expression.

7. Find the value of the expression.

$$(2^{-3})^{-2} = \underline{\hspace{1cm}}$$

LESSON 8: ZERO AND NEGATIVE EXPONENTS

EXERCISES

8. Find the value of the expression.

$$(3^{-2})^{-2} =$$

- 9. Which expressions are equivalent to $4^{-7} \cdot 4^{3}$? There may be more than one equivalent expression.
 - A 4⁻²¹
 - **B** 4⁻⁴
 - **○** 8⁻⁴

 - $\frac{4^3}{4^7}$
- 10. Which expressions are equivalent to $(5^{-2})^4$? There may be more than one equivalent expression.

 - **B** 25⁴
 - **6** 5⁻⁴ 5⁻⁴
 - **5**²
 - **ⓑ** 5⁻⁸
- 11. Show that the following equation is true.

$$\left(\frac{2^3 \cdot 7^8}{2^5 \cdot 7^{-2}}\right)^{-1} = \frac{2^2}{7^{10}}$$

LESSON 8: ZERO AND NEGATIVE EXPONENTS

EXERCISES

12. Look at this table of powers of 2.

210 = 1,024	2³ = 8	2-4 = 0.0625
2° = 512	$2^2 = 4$	2-5 =
28 =	$2^1 = 2$	2 ⁻⁶ = 0.015625
2 ⁷ = 128	2º = 1	2 ⁻⁷ = 0.0078125
26 = 64	$2^{-1} = 0.5$	2-8 = 0.00390625
25 = 32	2-2 = 0.25	2-9 = 0.001953125
24 = 16	$2^{-3} = 0.125$	$2^{-10} = 0.0009765625$

- a. What is the value of 28?
- b. What is the value of 2^{-4} as a power of 4?
- c. What is the value of 2^{-5} ?

Challenge Problem

- 13. Write an equation of the form $a^n = b$ that satisfies the given criteria. If it is not possible, explain why.
 - a. a is negative, n is negative, and b is positive.
 - b. a is positive, n is positive, and b is positive.
 - c. a is negative, n is positive, and b is negative.
 - d. a is positive, n is negative, and b is negative.

LESSON 9: PUTTING IT TOGETHER I

EXERCISES

EXERCISES

1. Read your notes and think about your work with exponents, square roots, and cube roots in this unit.

Write about something that was surprising, unexpected, or especially interesting to you from the topics in the unit.

2. Is there anything that still confuses you about exponents or roots?

Make a plan for understanding the things that still confuse you. Who will you ask for help and what help will you ask for?

3. Consider what you have learned in this unit about expressions: how they are expressed in words and how they are written in repeated factor form, exponential notation, decimal notation, and scientific notation.

Create a graphic organizer to show how to understand, evaluate, and simplify numbers with roots, exponents, and scientific notation. Organize your visual in a way that will allow you to use it as a reference throughout the rest of the school year.

Use an organizer similar to the chart shown, or create your own way to organize this information.

EXPONENTIAL NOTATION					
Exponential Notation	Evaluation Using Repeated Factor Form	Exponent in Words	Related Root	Root in Words	

SCIENTIFIC NOTATION				
Standard Form/ Decimal Notation Scientific Repeated Factor Form (Expanded Form) Scientific Notation in Words				

LESSON 9: PUTTING IT TOGETHER I

EXERCISES

4. Consider the properties of exponents you explored in this unit.

Create a graphic organizer to show the properties of exponents. Organize your visual in a way that will allow you to use it as a reference throughout the rest of the school year.

Use an organizer similar to the chart shown, or create your own way to organize this information.

Property	Definition	Example(s)	Evaluation
$a^n \cdot a^m = a^{n+m}$			
$a^n \div a^m = a^{n-m}$			
$\frac{a^n}{a^m}=a^{n-m}$			
$(a^n)^m = a^{n \cdot m}$			
$(ab)^n = a^n \bullet b^n$			
$a^0 = 1$			
$a^{-n} = \frac{1}{a^n}$			
$\frac{1}{a^{-n}}=a^n$			

5. Complete any exercises from this unit that you have not finished.

LESSON 13: CALCULATING WITH NOTATION

EXERCISES

EXERCISES

- 1. Simplify. (3 10⁶)(4 10⁻²)
 - **A** 12 10⁻⁴
- **B** 7 10⁻⁸
- 12 10⁴
- 7 10⁻¹²

- 2. Simplify. $\frac{6.9 \cdot 10^5}{3 \cdot 10^3}$
 - \triangle 2.07 10³
- B 2.3 10⁸
- \bigcirc 2.07 10²
- \bullet 2.3 10²

- 3. Simplify. $(5 \cdot 10^{-3}) + (8 \cdot 10^{3})$
 - A 13 10°
- **B** 1.3 10³
- 8,000.005
- 0.013

- 4. Simplify. (9 108) (1.5 106)
 - A 8.985 10⁸
- \bigcirc 7.5 10²
- **8.985 10**²
- 8.85 10⁸
- 5. In 2010 in the United Kingdom, 129,000,000,000 text messages were sent. That same year in the Netherlands, 11,000,000,000 text messages were sent.
 - a. Write an estimate of the number of texts from each country in scientific notation as a single digit times a power of 10.
 - b. Use your numbers written in scientific notation to help you write a statement comparing the number of text messages sent in the two countries.
- 6. In 2010, the United States population was 309,975,000; the population of Australia was 22,421,417.
 - a. Write an estimate of each population in scientific notation as a single digit times a power of 10.
 - b. Use your numbers written in scientific notation to help you write a statement comparing the populations of the two countries.
- 7. The diameter of a fluorine ion is 0.000000000038 m. The diameter of a small grain of sand is about 0.00002 m.
 - a. Write an estimate of each diameter in scientific notation as a single digit times a power of 10.
 - b. Use your numbers written in scientific notation to help you write a statement comparing the two diameters.

LESSON 13: CALCULATING WITH NOTATION

EXERCISES

8. The table shows the land area (in square kilometers) of seven countries.

Country	Area (km²)	Area (km²)
Russia	17,075,200	
United States	9,826,630	
Kenya	582,650	
Uruguay	176,220	
Haiti	27,750	
Singapore	693	
Monaco	2	

- a. Complete the table by writing each number as a single digit times a power of 10.
- b. Write four statements comparing the areas of the countries in the table.

Challenge Problem

- 9. About 111,041,000 people in the United States tuned in to watch Super Bowl XLV in 2011. The 2011 Academy Awards were watched by about one-third as many people.
 - a. Rewrite the number of people who watched the Super Bowl in scientific notation.
 - b. Round the first factor to the hundredths place. Then use that number to estimate how many people watched the Academy Awards.

LESSON 14: RATIONAL NUMBERS

EXERCISES

EXERCISES

- 1. Write $\frac{1}{3}$ in decimal form.
 - **A** 0.3
- \mathbf{B} $0.\overline{3}$
- 0.33
- 0.34
- 2. Which of these decimals is equivalent to $\frac{90}{160}$?
 - \triangle 0.562
- **B** 0.5625
- 0.05625
- 0.5625
- 3. Which of these decimals is equivalent to $\frac{5}{6}$?
 - 0.83
- B 0.83
- 1.83
- 1.83
- 4. Which of these fractions has a decimal form that repeats?

 - **A** $\frac{1}{5}$ **B** $\frac{3}{8}$ **C** $\frac{7}{12}$
- 5. Which of these rational numbers has a decimal form that terminates?
 - $\triangle \frac{7}{8}$
- **B** $\frac{7}{11}$ **C** $\frac{5}{12}$
- 6. a. Use your calculator to write each fraction as a decimal.

$$\frac{5}{11} =$$

- b. Describe the pattern in your results from part a.
- c. Predict the decimal forms of $\frac{7}{11}$ and $\frac{10}{11}$. Then check your prediction.

LESSON 14: RATIONAL NUMBERS

EXERCISES

- 7. Explain why the decimal form of $\frac{1}{14}$ repeats, but the decimal form of $\frac{7}{14}$ terminates.
- 8. What is the fraction form of 0.099?
- 9. What is the fraction form of $0.\overline{27}$?
- 10. What is the fraction form of $0.5\overline{3}$?

Challenge Problem

- 11. Did you know that the repeating decimal $0.\overline{9}$ is equal to 1? You can show that this is true in two different ways.
 - a. Use the fact that $0.\overline{3} = \frac{1}{3}$ to help you show that $0.\overline{9} = 1$.
 - b. Use the method you learned for changing a repeating decimal to a fraction to show that $0.\overline{9} = 1$.

LESSON 15: ESTIMATING SQUARE ROOTS

EXERCISES

EXERCISES

- 1. Which of these numbers is the best estimate for the value of this square root? $\sqrt{10}$
 - **A** 3.16
- **B** 3.09
- **3**.2
- 3.25
- 2. Which of these lists of numbers is in order from least to greatest?
 - \triangle $\sqrt[3]{27}$, $\sqrt{10}$, $\frac{10}{3}$
 - **B** $\frac{10}{3}$, $\sqrt[3]{27}$, $\sqrt{10}$
 - $\sqrt{10}, \sqrt[3]{27}, \frac{10}{3}$
 - **(a)** $\sqrt[3]{27}$, $\frac{10}{3}$, $\sqrt{10}$
- 3. Which of these lists of numbers is in order from least to greatest?
 - \triangle 2.23, 2.24, $\sqrt{5}$
 - **B** 2.24, $\sqrt{5}$, 2.22
 - **3** 2.22, 2.23, $\sqrt{5}$
 - \bullet 2.22, $\sqrt{5}$, 2.23
- 4. Consider this square root.

 $\sqrt{3}$

Estimate the value of the square root to three decimal places.

Explain or show your method for estimating the square root.

5. Consider this square root.

 $\sqrt{7}$

- a. Between which two whole numbers is the square root?
- b. Estimate the value of the square root to three decimal places. Explain or show your method for estimating the square root.

LESSON 15: ESTIMATING SQUARE ROOTS

EXERCISES

6. Consider this square root.

 $\sqrt{6}$

Estimate the value of the square root to three decimal places. Explain or show your method for estimating the square root.

7. Consider this square root.

 $\sqrt{11}$

Estimate the value of the square root to three decimal places. Explain or show your method for estimating the square root.

8. Consider this square root.

 $\sqrt{48}$

Estimate the value of the square root to three decimal places. Explain or show your method for estimating the square root.

9. Consider this square root.

 $\sqrt{63}$

Estimate the value of the square root to three decimal places. Explain or show your method for estimating the square root.

Challenge Problem

Imagine you are explaining irrational and rational numbers to a classmate. Write how
you would define these types of numbers, making sure your classmate understands the
differences between them.

LESSON 16: IRRATIONAL NUMBERS

EXERCISES

EXERCISES

- 1. Which of these numbers is irrational?
- **B** $\frac{1}{3}$
- 0.45
- √13
- 2. Which of these numbers are irrational numbers? There may be more than one irrational number.
 - **Α** 9π
 - **B** 0.4321

 - $-\frac{17}{47}$
 - \bigcirc $\sqrt{2}$
- 3. Which two consecutive whole numbers is $\sqrt{70}$ between?
- 4. Compare the two expressions and choose <, >, or =.

5. Compare the two expressions and choose <, >, or =.

$$1.\bar{1} - \frac{\pi}{3}$$

6. Compare the two expressions and choose <, >, or =.

$$2\pi - \sqrt{37}$$

7. Compare the two expressions and choose <, >, or =.

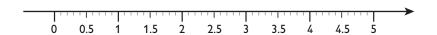
$$-\pi^2 - \frac{59}{6}$$

LESSON 16: IRRATIONAL NUMBERS

EXERCISES

8. Place each irrational number to its approximate location on the number line.

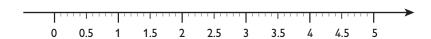
$$\sqrt{3}$$
 $\sqrt{5}$ $\sqrt{10}$



Challenge Problem

9. Place each expression to its approximate location on the number line.

$$\frac{1}{3}\pi \qquad \frac{5}{3} \qquad 2 \cdot \sqrt{3} \qquad \sqrt{2}$$



LESSON 17: PUTTING IT TOGETHER 2

EXERCISES

EXERCISES

1. Read your Self Check and think about your work in this unit.

Write three things you have learned during the unit.

Share your list with a classmate.

Does your classmate understand what you wrote?

Consider the topics from this unit: roots, estimating square roots, positive and negative
exponents, simplifying expressions with roots and exponents, scientific notation,
finding equivalent expressions, and irrational and rational numbers. Make a plan for
understanding the things that still confuse you.

For example, consider how you can research your questions and add more information to your notes. You might ask a classmate to help you review the related lessons or to look at the resources in the Concept Corner, or you could talk with your teacher to clarify any areas of confusion.

 In the second part of this unit, you used scientific notation to make calculations with very large and very small numbers; estimated square roots; converted decimals to fractions; and explored terminating decimals, repeating decimals, non-terminating decimals, and non-repeating decimals.

Select one of these topics and explain why it is useful in everyday life. If possible, include an example in your explanation.

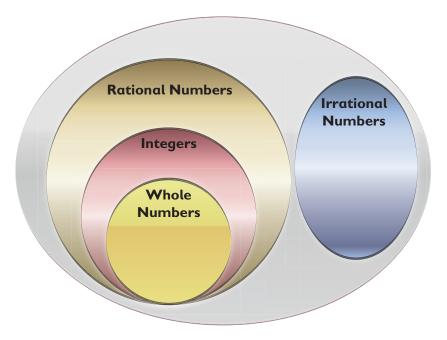
LESSON 17: PUTTING IT TOGETHER 2

EXERCISES

4. Consider the differences between rational and irrational numbers. Look back at your notes and exercises.

Create a graphic organizer about rational and irrational numbers that shows the structure of the number system and provides examples of different kinds of numbers. Organize your visual in a way that will allow you to use it as a reference throughout the rest of the school year.

Use a visual similar to the one shown, or create your own.



5. Look back at the exercises in this unit. Are there any you did not finish? If so, finish them now.