MATH GRADE 8 UNIT 2

ROOTS AND EXPONENTS

ANSWERS
FOR EXERCISES

LESSON 2: EXPONENTS

ANSWERS

ANSWERS

8.EE.1 **1.** $3^4 = 81$

8.EE.1 **2.** $\left(\frac{1}{2}\right)^4 = \frac{1}{16}$

8.EE.1 **3.** $5^2 = 25$

8.EE.1 **4.** $4^3 \cdot 6^2 = 2{,}304$

8.EE.1 **5.**

$$\left(\frac{1}{3}\right)^2 \cdot 3^3 = 3$$

$$\frac{1}{3} \cdot \frac{1}{3} \cdot 3 \cdot 3 \cdot 3 = \frac{1}{9} \cdot 27 = 3$$

8.EE.1 **6.** $\left(\frac{1}{4}\right)^4 \cdot 5^2 = \frac{25}{256}$

$$\frac{1}{4} \cdot \frac{1}{4} \cdot \frac{1}{4} \cdot \frac{1}{4} \cdot 5 \cdot 5 = \frac{1}{256} \cdot 25 = \frac{25}{256}$$

8.EE.1 **7.** $4^2 \cdot 4^2 \cdot 4^2 \cdot 4^2 = 65{,}536$

8.EE.1 **8.**

a. $2^{10} = 2^9 \cdot 2 = 512 \cdot 2 = 1{,}024$
$2^{11} = 2^{10} \cdot 2 = 1{,}024 \cdot 2 = 2{,}048$
$2^{12} = 2^{11} \cdot 2 = 2{,}048 \cdot 2 = 4{,}096$
$2^{13} = 2^{12} \cdot 2 = 4{,}096 \cdot 2 = 8{,}192$

b. Here are two examples.

Example 1
Powers of 2 can be used to describe population growth.
For example, if a bacterium divides every hour, then after 1 hr the population is 2.
After 2 hr, it is 4. After 3 hr, it is 8.
In general, the population after n hours is $2n$.

Example 2
I've seen the powers of 2 in the way that gossip spreads at school. One person tells 2 friends. Those people each tell 2 more friends, which is 4 people. Those 4 people each tell 2 friends, which is 8 people, and so on.

1 person who knows the gossip becomes 2 people who know.
2 people who know the gossip become 4 people.
4 people become 8 people.

LESSON 2: EXPONENTS

ANSWERS

Challenge Problem

8.EE.1 **9.**

a. To find the volume of a cube, you multiply a side length by a side length by a side length (all side lengths are the same). For a cube with side length *s*:

$V = s \bullet s \bullet s$ This multiplication is the same as raising *s* to the power of 3.

b. Raising a number to the power of 3 is called cubing because it is how you calculate the volume of a cube.

c. The volume of a cube is s^3.
$V = 3^3$
$= 27 \text{ m}^3$

d. The unit is meters cubed: m^3. To find the unit of the volume, the unit of the length is cubed as well: $\text{m} \bullet \text{m} \bullet \text{m} = \text{m}^3$.

LESSON 3: SQUARE AND CUBE ROOTS

ANSWERS

ANSWERS

8.EE.2 **1.** (B) 3

8.EE.2 **2.** (B) 10

8.EE.2 **3.** (A) 6

8.EE.2 **4.** 12 units

8.EE.2 **5.** 0.5 units

8.EE.2 **6.** (D) 5 and −5 are both solutions.

8.EE.2 **7.** $x = 60$
$x = -60$

8.EE.2 **8.** (A) no solution

8.EE.2 **9.** $x = 9$
$x = -9$

8.EE.2 **10.** Side length = 5 units
Volume = 125 cubic units

Challenge Problem

8.EE.2 **11.** The number is 15,625.

To determine the number, you must take the inverse of each function.

First, you apply the inverse of the cube root, which is the exponent 3.
$5^3 = 125$

Then, you apply the inverse of the square root, which is the exponent 2.
$125^2 = 125 \cdot 125 = 15{,}625$

LESSON 4: SIMPLIFYING EXPRESSIONS

ANSWERS

ANSWERS

8.EE.1 **1.** Ⓑ 7,776
Ⓔ 6^5

8.EE.1 **2.** Ⓐ 0.3^4

8.EE.1 **3.** $n \cdot (4 \cdot 4 \cdot 4) = 64n$

8.EE.1 **4.** $g + g + g + g + g + g + g = 7g$

8.EE.1 **5.** Jacob added each of the factors instead of multiplying them. His simplification should look like this:

$$13^5 \div 13^2 = \frac{13 \cdot 13 \cdot 13 \cdot 13 \cdot 13}{13 \cdot 13} = 13 \cdot 13 \cdot 13 = 13^3 = 2,197$$

8.EE.2 **6.** Pedra's solution is correct.

$$\sqrt[3]{343} = \sqrt[3]{7 \cdot 7 \cdot 7} = 7$$

8.EE.1 **7.**

$$(8^6 \cdot 2) \div (8^2 \cdot 2^3) = \frac{8 \cdot 8 \cdot 8 \cdot 8 \cdot 8 \cdot 8 \cdot 2}{8 \cdot 8 \cdot 2 \cdot 2 \cdot 2}$$

8.EE.1 **8.**

$$\left(\frac{3}{7}\right)^7 = \frac{3}{7} \cdot \frac{3}{7} \cdot \frac{3}{7} \cdot \frac{3}{7} \cdot \frac{3}{7} \cdot \frac{3}{7} \cdot \frac{3}{7}$$

Challenge Problem

8.EE.2 **9.** Talisha should have known that the result of a cube root cubed is the original number. Then, she wouldn't have had to try to figure out the cube root of 6 at all.

The correct evaluation is 6.

LESSON 5: MULTIPLYING AND DIVIDING

ANSWERS

ANSWERS

8.EE.1 **1.** B 3^6

8.EE.1 **2.** B $\frac{1}{6^4}$

8.EE.1 **3.** D 3^{12}

8.EE.1 **4.** $\frac{11^6}{11^7} = 11^6 \div 11^7 = 11^{6-7} = 11^{-1}$

8.EE.1 **5.** $5^2 \bullet 5^2 \bullet 5^2 \bullet 5^2 = 5^{2+2+2+2} = 5^8$

8.EE.1 **6.** A $2^2 \bullet 2^2 \bullet 2^3$

D 2^7

8.EE.1 **7.** Here are two examples.

$$\frac{3^4}{3^2} \bullet \frac{3^3}{3^4} = \frac{3^{3+4}}{3^{2+4}} = \frac{3^7}{3^6} = 3^{7-6} = 3^1$$

OR

$$\frac{\cancel{3}^4}{3^2} \bullet \frac{3^3}{\cancel{3}^4} = \frac{3^3}{3^2} = 3^{3-2} = 3^1$$

8.EE.1 **8.** Here are two examples.

$$\frac{8^4}{8^2} \bullet \frac{8^3}{8^5} = \frac{8^{4+3}}{8^{2+5}} = \frac{8^7}{8^7} = 1$$

OR

$$\frac{8^4}{8^2} \bullet \frac{8^3}{8^5} = 8^{(4+3)-(2+5)} = 8^{7-7} = 8^0 = 1$$

LESSON 5: MULTIPLYING AND DIVIDING

ANSWERS

Challenge Problem

8.EE.1 **9.**

$$\begin{aligned}
500 \cdot 10^{98} + 10^{10^2} &= p \cdot 10^{100} \\
(5 \cdot 10^2) \cdot 10^{98} + 10^{10^2} &= p \cdot 10^{100} \\
5 \cdot (10^2 \cdot 10^{98}) + 10^{100} &= p \cdot 10^{100} \\
5 \cdot (10^{2+98}) + 10^{100} &= p \cdot 10^{100} \\
5 \cdot 10^{100} + 10^{100} &= p \cdot 10^{100} \\
5 \cdot 10^{100} + 1 \cdot 10^{100} &= p \cdot 10^{100} \\
6 \cdot 10^{100} &= p \cdot 10^{100} \\
\frac{1}{\cancel{10^{100}}} \cdot \left(6 \cdot \cancel{10^{100}}\right) &= \left(p \cdot \cancel{10^{100}}\right) \cdot \frac{1}{\cancel{10^{100}}} \\
6 &= p
\end{aligned}$$

LESSON 6: PROPERTIES OF EXPONENTS

ANSWERS

ANSWERS

8.EE.1 **1.** Ⓒ $x = 21$

8.EE.1 **2.** Ⓒ $y = 16$

8.EE.1 **3.** Ⓐ $(5 \cdot 5)^4$

Ⓒ $(5^2)^2 \cdot (5^2)^2$

Ⓓ 5^8

Ⓔ $(5^4)^2$

8.EE.1 **4.** Ask a classmate to check your expressions.

Here is one example.

6^8

$6^4 \cdot 6^4$

$\left(6^2\right)^4$

$\left(3^2 \cdot 2^2\right)^4$

8.EE.1 **5.** Ask a classmate to check your expressions.

Here is one example.

18^4

$9^4 \cdot 2^4$

$3^8 \cdot 4^2$

$81^2 \cdot 16$

8.EE.1 **6.** $25^5 = \left(5^2\right)^5$ Multiplication and definition of exponents: $25 = 5 \cdot 5 = 5^2$

$= 5^{10}$ Exponent as a product: $\left(b^m\right)^n = b^{m \cdot n}$

8.EE.1 **7.** $\left(2^4\right)^3 = 2^{12} = 2^{6 \cdot 2} = \left(2^6\right)^2$

$\left(2^4\right)^3 = 2^{12}$ Exponent as a product: $\left(b^m\right)^n = b^{m \cdot n}$

$2^{6 \cdot 2} =$ Multiplication

$\left(2^6\right)^2 =$ Exponent as a product: $b^{m \cdot n} = \left(b^m\right)^n$

LESSON 6: PROPERTIES OF EXPONENTS ANSWERS

8.EE.1 **8.** The expression -2^4 means the negative of 2^4, which is -16.
The expression $(-2)^2$ is the square of -2, which is 4.
So $(-2)^2 \cdot (-2)^2 = 4 \cdot 4 = 16$.
$-16 \neq 16$

Challenge Problem

8.EE.1 **9.**

a. They are equal.

$$\left(2^2\right)^2 = 16 \text{ and } 2^{\left(2^2\right)} = 16$$

b. $3^{\left(3^3\right)}$ is greater.

$$\left(3^3\right)^3 = 27^3 = 19{,}683$$

$$3^{\left(3^3\right)} = 3^{27} \approx 7{,}625{,}597{,}485{,}000$$

LESSON 7: SCIENTIFIC NOTATION

ANSWERS

ANSWERS

8.EE.3	1.	D $5.001 \cdot 10^{-25}$
8.EE.3	2.	B $4.099 \cdot 10^{11}$
8.EE.4	3.	The half-life of helium is $3.04 \cdot 10^{-21}$ sec.
8.EE.4	4.	In 1 cup of water, there are approximately $7.5 \cdot 10^{24}$ molecules.
8.EE.4	5.	$1.7562 \cdot 10^{10}$ mi
8.EE.3 8.EE.4	6.	$4.44 \cdot 10^{12}$
8.EE.3 8.EE.4	7.	$7.5 \cdot 10^{-43}$
8.EE.3 8.EE.4	8.	$8.068 \cdot 10^{-8} = 0.00000008068$
8.EE.3 8.EE.4	9.	$1.00001 \cdot 10^{13} = 10{,}000{,}100{,}000{,}000$
8.EE.3	10.	$4.5 \cdot 10^{20} > 4.5 \cdot 10^{-20}$
8.EE.3	11.	$72{,}000{,}000{,}000 < 2.5 \cdot 10^{11}$
8.EE.3	12.	$3.2 \cdot 10^{-8} < 1.2 \cdot 10^{-7}$
8.EE.3	13.	2.14×10^{31}

LESSON 7: SCIENTIFIC NOTATION

ANSWERS

Challenge Problem

8.EE.3
8.EE.4

14.

a. The length of an *Amoeba proteus* is $5.00 \cdot 10^{-4}$ m.

1 μm = 10^{-6} m, which can be rewritten as 10^{6} μm = 1 m and used to convert the length. Then it can be rewritten in scientific notation:

$$\begin{aligned} 500\ \mu\text{m} &= 500 \cdot 10^{-6}\ \text{m} \\ &= (5.00 \cdot 10^{2}) \cdot 10^{-6}\ \text{m} \\ &= 5.00 \cdot 10^{-4}\ \text{m} \end{aligned}$$

b. The diameter of a glucose molecule is $9.00 \cdot 10^{-4}$ μm.

1 μm = 10^{-6} m can be rewritten as 10^{6} μm = 1 m, and 1 pm = 10^{-12} m can be rewritten as 10^{12} pm = 1 m. Setting these expressions equal to each other gives:

$$\begin{aligned} 10^{12}\ \text{pm} &= 10^{6}\ \mu\text{m} \\ (10^{-6})10^{12}\ \text{pm} &= (10^{-6})10^{6}\ \mu\text{m} \\ 10^{6}\ \text{pm} &= 1\ \mu\text{m} \end{aligned}$$

This equality can be used to convert the length, and then it can be rewritten in scientific notation.

$$\begin{aligned} 900\ \text{pm} &= 900 \cdot 10^{-6}\mu\text{m} \\ &= (9.00 \cdot 10^{2}) \cdot 10^{-6}\mu\text{m} \\ &= 9.00 \cdot 10^{-4}\mu\text{m} \end{aligned}$$

c. The diameter of a measles virus is $2.20 \cdot 10^{-7}$ m.

1 nm = 10^{-9} m, which can be rewritten as 10^{9} nm = 1 m and used to convert the length. Then it can be rewritten in scientific notation:

$$\begin{aligned} 220\ \text{nm} &= 220 \cdot 10^{-9}\text{m} \\ &= (2.20 \cdot 10^{2}) \cdot 10^{-9}\text{m} \\ &= 2.20 \cdot 10^{-7}\text{m} \end{aligned}$$

LESSON 8: ZERO AND NEGATIVE EXPONENTS — ANSWERS

ANSWERS

8.EE.1 **1.** true

8.EE.1 **2.** false

8.EE.1 **3.** $10^{-7} = 10{,}000{,}000^{-1} = \dfrac{1}{10{,}000{,}000}$

8.EE.1 **4.** $3^{-3} = 27^{-1} = \dfrac{1}{27}$

8.EE.1 **5.** $4^{-8} \bullet 4^{8} = 1$

8.EE.1 **6.** $8^{-1} = \dfrac{1}{8}$

8.EE.1 **7.** $\left(2^{-3}\right)^{-2} = 64$

8.EE.1 **8.** $\left(3^{-2}\right)^{-2} = 81$

8.EE.1 **9.**
- B 4^{-4}
- D $\dfrac{1}{4^4}$
- E $\dfrac{4^3}{4^7}$

8.EE.1 **10.**
- A $\left(\dfrac{1}{5^2}\right)^4$
- C $5^{-4} \bullet 5^{-4}$
- E 5^{-8}

8.EE.1 **11.** Here is one example.

$$\left(\frac{2^3 \bullet 7^8}{2^5 \bullet 7^{-2}}\right)^{-1} = \left[\left(\frac{2^3}{2^5}\right) \bullet \left(\frac{7^8}{7^{-2}}\right)\right]^{-1} = \left[\left(2^{3-5}\right) \bullet \left(7^{8-(-2)}\right)\right]^{-1} = \left[\left(2^{-2}\right) \bullet \left(7^{10}\right)\right]^{-1} = \left[\frac{7^{10}}{2^2}\right]^{-1} = \frac{2^2}{7^{10}}$$

LESSON 8: ZERO AND NEGATIVE EXPONENTS ANSWERS

8.EE.1 **12.**

a. 256

b. 4^{-2}

c. 0.03125

Challenge Problem

8.EE.1 **13.**

a. All answers in which $a < 0$, $n < 0$ and even, and $b > 0$ are correct.

$$(-2)^{-2} = \frac{1}{4}$$

b. All answers in which $a > 0$, $n > 0$, and $b > 0$ are correct.

Possible answer: $2^2 = 4$

c. All answers in which $a < 0$, $n > 0$, n is odd, and $b < 0$ are correct.

$$(-2)^3 = -8$$

d. This situation is not possible because a positive number raised to any power is positive.

LESSON 9: PUTTING IT TOGETHER I

ANSWERS

ANSWERS

8.EE.2
8.EE.3

3. Here is one example.

EXPONENTIAL NOTATION				
Exponential Notation	**Evaluation Using Repeated Factor Form**	**Exponent in Words**	**Related Root**	**Root in Words**
6^2	$6^2 = 6 \cdot 6$ $= 36$	six squared (or six to the second power)	$\sqrt{36} = 6$	square root of thirty-six
7^3	$7^3 = 7 \cdot 7 \cdot 7$ $= 343$	seven cubed (or seven to the third power)	$\sqrt[3]{343} = 7$	cube root of three hundred forty-three
2^4	$2^4 = 2 \cdot 2 \cdot 2 \cdot 2$ $= 16$	two to the fourth power	$\sqrt[4]{16} = 2$	fourth root of sixteen
x^5	$x^5 = x \cdot x \cdot x \cdot x \cdot x$	x to the fifth power	$\sqrt[5]{x^5} = x$	fifth root of x to the fifth

SCIENTIFIC NOTATION			
Standard Form/ Decimal Notation	**Scientific Notation**	**Repeated Factor Form (Expanded Form)**	**Scientific Notation in Words**
271,000,000	2.71×10^8	$2.71 \cdot 10 \cdot 10 \cdot 10 \cdot 10 \cdot 10 \cdot 10 \cdot 10 \cdot 10$	two and seventy-one hundredths times ten to the eighth power
0.0000000271	2.71×10^{-8}	$2.71 \cdot \frac{1}{10} \cdot \frac{1}{10} \cdot \frac{1}{10} \cdot \frac{1}{10} \cdot \frac{1}{10} \cdot \frac{1}{10} \cdot \frac{1}{10} \cdot \frac{1}{10}$	two and seventy-one hundredths times ten to the negative eighth power

LESSON 9: PUTTING IT TOGETHER I

ANSWERS

8.EE.1

4.

Property	Definition	Example(s)	Evaluation
$a^n \cdot a^m = a^{n+m}$	If the bases are the same: When multiplying like bases, add the exponents.	$3^2 \cdot 3^5 =$ $3^{2+5} = 3^7$	$3^2 \cdot 3^5 = (3 \cdot 3) \cdot (3 \cdot 3 \cdot 3 \cdot 3 \cdot 3)$ $= 3 \cdot 3 \cdot 3 \cdot 3 \cdot 3 \cdot 3 \cdot 3$ $= 2{,}187$
$a^n \div a^m = a^{n-m}$ $\frac{a^n}{a^m} = a^{n-m}$	If the bases are the same: When dividing like bases, subtract the exponents.	$\frac{7^6}{7^4} = 7^{6-4} = 7^2$	$\frac{7^6}{7^4} = \frac{7 \cdot 7 \cdot 7 \cdot 7 \cdot 7 \cdot 7}{7 \cdot 7 \cdot 7 \cdot 7}$ $= \frac{\cancel{7} \cdot \cancel{7} \cdot \cancel{7} \cdot \cancel{7} \cdot 7 \cdot 7}{\cancel{7} \cdot \cancel{7} \cdot \cancel{7} \cdot \cancel{7}}$ $= 7 \cdot 7$ $= 49$
$(a^n)^m = a^{n \cdot m}$	When raising a base to two exponents, multiply the exponents.	$(8^3)^2 = 8^{3 \cdot 2} = 8^6$	$(8^3)^2 = (8 \cdot 8 \cdot 8)^2$ $= (8 \cdot 8 \cdot 8) \cdot (8 \cdot 8 \cdot 8)$ $= 8 \cdot 8 \cdot 8 \cdot 8 \cdot 8 \cdot 8$ $= 262{,}144$
$(ab)^n = a^n \cdot b^n$	When you raise a product to a power, you raise each factor to the power.	$(4 \cdot 3)^4 = 4^4 \cdot 3^4$	$(4 \cdot 3)^4 = 4^4 \cdot 3^4$ $= 256 \cdot 81$ $= 20{,}736$
$a^0 = 1$	If the base is not 0: A number raised to the power of 0 is always equal to 1.	$15^0 = 1$	$15^0 = 1$
$a^{-n} = \frac{1}{a^n}$ $\frac{1}{a^{-n}} = a^n$	If the base is not 0: A number raised to a negative power is equal to the reciprocal raised to the opposite power.	$9^{-4} = \frac{1}{9^4}$ $\frac{1}{9^{-4}} = 9^4$	$9^{-4} = \frac{1}{9^4}$ $= \frac{1}{9 \cdot 9 \cdot 9 \cdot 9}$ $= \frac{1}{6{,}561}$ $\frac{1}{9^{-4}} = 9^4$ $= 9 \cdot 9 \cdot 9 \cdot 9$ $= 6{,}561$

LESSON 13: CALCULATING WITH NOTATION ANSWERS

ANSWERS

8.EE.3 **1.** (C) $12 \cdot 10^4$

8.EE.3 **2.** (D) $2.3 \cdot 10^2$

8.EE.3 **3.** (C) 8,000.005

8.EE.3 **4.** (A) $8.985 \cdot 10^8$

8.EE.3
8.EE.4 **5.**

a. In the United Kingdom, about $1 \cdot 10^{11}$ text messages were sent; in the Netherlands, about $1 \cdot 10^{10}$ text messages were sent.

b. Here are two examples.

The number of text messages sent in the United Kingdom was about 10 times the number sent in the Netherlands.

$$\frac{1 \cdot 10^{11}}{1 \cdot 10^{10}} = 10$$

OR

The number of text messages sent in the Netherlands was about $\frac{1}{10}$ the number sent in the United Kingdom.

$$\frac{1 \cdot 10^{10}}{1 \cdot 10^{11}} = \frac{1}{10}$$

8.EE.3
8.EE.4 **6.**

a. The U.S. population was about $3 \cdot 10^8$; the Australian population was about $2 \cdot 10^7$.

b. Here are two examples.

The U.S. population was about 15 times the Australian population.

$$\frac{3 \cdot 10^8}{2 \cdot 10^7} = \frac{3}{2} \cdot 10 = 15$$

OR

The Australian population was about $\frac{1}{15}$ the U.S. population.

$$\frac{2 \cdot 10^7}{3 \cdot 10^8} = \frac{2}{3} \cdot \frac{1}{10} = \frac{1}{15}$$

LESSON 13: CALCULATING WITH NOTATION ANSWERS

8.EE.3
8.EE.4

7. a. A fluorine ion is about $4 \cdot 10^{-11}$ m in diameter. A grain of sand is about $2 \cdot 10^{-5}$ m in diameter.

b. Here are two examples.

The diameter of a grain of sand is about 500,000 times the diameter of a fluorine ion.

$$\frac{2 \cdot 10^{-5}}{4 \cdot 10^{-11}} = \frac{1}{2} \cdot 10^{6} = 0.5 \cdot 1{,}000{,}000 = 500{,}000$$

OR

The diameter of a fluorine ion is about $\frac{1}{500{,}000}$ the diameter of a grain of sand.

$$\frac{4 \cdot 10^{-11}}{2 \cdot 10^{-5}} = 2 \cdot 10^{-6} = \frac{2}{1{,}000{,}000} = \frac{1}{500{,}000}$$

LESSON 13: CALCULATING WITH NOTATION

ANSWERS

8.EE.3
8.EE.4

8. a.

Country	**Area** (km^2)	**Area** (km^2)
Russia	17,075,200	2×10^7
United States	9,826,630	1×10^7
Kenya	582,650	6×10^5
Uruguay	176,220	2×10^5
Haiti	27,750	3×10^4
Singapore	693	7×10^2
Monaco	2	2×10^0

b. Here are four example statements.

The area of Russia is about twice the area of the United States.

$$\frac{2 \bullet 10^7}{1 \bullet 10^7} = \frac{2}{1} = 2$$

The area of the United States is about 50 times the area of Uruguay.

$$\frac{1 \bullet 10^7}{2 \bullet 10^5} = \frac{1}{2} \bullet 10^2 = 0.5 \bullet 100 = 50$$

The area of Haiti is about $\frac{1}{20}$ the area of Kenya.

$$\frac{3 \bullet 10^4}{6 \bullet 10^5} = \frac{1}{2} \bullet \frac{1}{10} = \frac{1}{20}$$

The area of Singapore is about 350 times the area of Monaco.

$$\frac{7 \bullet 10^2}{2 \bullet 10^0} = \frac{7}{2} \bullet 10^2 = 3.5 \bullet 10^2 = 350$$

Challenge Problem

8.EE.3
8.EE.4

9. a. $111{,}041{,}000 = 1.11041 \bullet 10^8$

b. About 37,000,000 people watched the Academy Awards.

Rounding $1.11041 \bullet 10^8$ to the hundredths place gives $1.11 \bullet 10^8$

Multiply this number by one-third to estimate the number of Academy Award watchers:

$$\frac{1}{3} \bullet 1.11 \bullet 10^8 = \frac{1.11}{3} \bullet 10^8 = 37{,}000{,}000$$

LESSON 14: RATIONAL NUMBERS

ANSWERS

ANSWERS

8.NS.1 **1.** B $0.\overline{3}$

8.NS.1 **2.** B 0.5625

8.NS.1 **3.** A $0.8\overline{3}$

8.NS.1 **4.** C $\frac{7}{12}$

8.NS.1 **5.** A $\frac{7}{8}$

8.NS.1 **6.** a. $\frac{1}{11} = 0.\overline{09}$

$\frac{2}{11} = 0.\overline{18}$

$\frac{3}{11} = 0.\overline{27}$

$\frac{4}{11} = 0.\overline{36}$

$\frac{5}{11} = 0.\overline{45}$

b. The repeating digits of each decimal are equal to 9 times the numerator of the fraction.

c. $\frac{7}{11} = 0.\overline{63}$

$\frac{10}{11} = 0.\overline{90}$

$7 \bullet 9 = 63$, so the repeating decimal in $\frac{7}{11}$ is 63.

$10 \bullet 9 = 90$, so the repeating decimal in $\frac{10}{11}$ is 90.

LESSON 14: RATIONAL NUMBERS ANSWERS

8.NS.1 **7.** Since 14 is a multiple of 7, $\frac{7}{14}$ can be reduced to $\frac{1}{2}$ or 0.5, which is a terminating decimal.

Looking at the long division algorithm can help you understand why $\frac{1}{14}$ is a repeating decimal:

$$
\begin{array}{r}
0.07142857 \\
14\overline{)1.00000000} \\
\underline{98} \\
20 \\
\underline{14} \\
60 \\
\underline{56} \\
40 \\
\underline{28} \\
120 \\
\underline{112} \\
80 \\
\underline{70} \\
100 \\
\underline{98} \\
2
\end{array}
$$

The dividends created by the remainders for the first few steps are 20, 60, 40, 120, 80, and 100. None of these numbers are multiples of 14, so they all result in a remainder. The last remainder shown in this example is 2, which creates a dividend of 20. The dividends will repeat themselves again and will continue in this manner infinitely.

8.NS.1 **8.** $\frac{99}{1,000}$

8.NS.1 **9.** $\frac{3}{11}$

$$x = 0.27272727\ldots$$
$$100x = 27.272727\ldots$$

$$
\begin{array}{r}
100x = 27.272727\ldots \\
\underline{-x = 0.272727\ldots} \\
99x = 27
\end{array}
$$

$$x = \frac{27}{99} = \frac{3}{11}$$

LESSON 14: RATIONAL NUMBERS

ANSWERS

8.NS.1 **10.** $\frac{8}{15}$

$x = 0.533333...$

$10x = 5.333333...$

$100x = 53.333333...$

$$\begin{array}{r} 100x = 53.333333... \\ -10x = 5.333333... \\ \hline 90x = 48 \end{array}$$

$x = \frac{48}{90} = \frac{8}{15}$

Challenge Problem

8.NS.1 **11.** a. $1 = 3 \bullet \frac{1}{3} = 3 \bullet 0.\overline{3} = 0.\overline{9}$

b. $$\begin{array}{r} 10x = 9.\overline{9} \\ -\ x = 0.\overline{9} \\ \hline 9x = 9 \\ x = 1 \end{array}$$

LESSON 15: ESTIMATING SQUARE ROOTS — ANSWERS

ANSWERS

8.NS.2 **1.** (A) 3.16

8.NS.2 **2.** (A) $\sqrt[3]{27}, \sqrt{10}, \frac{10}{3}$

8.NS.2 **3.** (C) 2.22, 2.23, $\sqrt{5}$

8.NS.2 **4.** $\sqrt{3} \approx 1.732$

$\sqrt{1} < \sqrt{3} < \sqrt{4}$, which means $1 < \sqrt{3} < 2$.

$1.7^2 = 2.89$ and $1.8^2 = 3.24$, which means $1.7 < \sqrt{3} < 1.8$.

$1.73^2 = 2.9929$ and $1.74^2 = 3.0276$, which means $1.73 < \sqrt{3} < 1.74$.

$1.732^2 = 2.999824$ and $1.733^2 = 3.003289$, which means $1.732 < \sqrt{3} < 1.733$.

$$\begin{array}{r} 3.000000 \\ -\ 2.999824 \\ \hline 0.000176 \end{array} \qquad \begin{array}{r} 3.003289 \\ -\ 3.000000 \\ \hline 0.003289 \end{array}$$

Since 1.732^2 is closer to 3 than 1.733^2 is, 1.732 is a better estimate for $\sqrt{3}$.

8.NS.2 **5.**

a. The square root value is between 2 and 3, but is closer to 3.

b. $\sqrt{7} \approx 2.646$

$\sqrt{4} < \sqrt{7} < \sqrt{9}$, which means $2 < \sqrt{7} < 3$.

$2.6^2 = 6.76$ and $2.7^2 = 7.29$, which means $2.6 < \sqrt{7} < 2.7$.

$2.64^2 = 6.9696$ and $2.65^2 = 7.0225$, which means $2.64 < \sqrt{7} < 2.65$.

$2.645^2 = 6.996025$ and $2.646^2 = 7.001316$, which means $2.645 < \sqrt{7} < 2.646$.

$$\begin{array}{r} 7.000000 \\ -\ 6.996025 \\ \hline 0.003975 \end{array} \qquad \begin{array}{r} 7.001316 \\ -\ 7.000000 \\ \hline 0.001316 \end{array}$$

Since 2.6462 is closer to 7 than 2.6452 is, 2.646 is a better estimate for $\sqrt{7}$.

LESSON 15: ESTIMATING SQUARE ROOTS

ANSWERS

8.NS.2 **6.**

$\sqrt{6} \approx 2.449$

$\sqrt{4} < \sqrt{6} < \sqrt{9}$, which means $2 < \sqrt{6} < 3$

$2.4^2 = 5.76$ and $2.5^2 = 6.25$, which means $2.4 < \sqrt{6} < 2.5$

$2.44^2 = 5.9536$ and $2.45^2 = 6.0025$, which means $2.44 < \sqrt{6} < 2.45$

$2.449^2 = 5.997601$ and $2.450^2 = 6.0025$, which means $2.449 < \sqrt{6} < 2.450$

$$\begin{array}{r} 6.000000 \\ -\ 5.997601 \\ \hline 0.002399 \end{array} \qquad \begin{array}{r} 6.0025 \\ -\ 6.0000 \\ \hline 0.0025 \end{array}$$

Since 2.449^2 is closer to 6 than 2.450^2 is, 2.449 is a better estimate for $\sqrt{6}$.

8.NS.2 **7.**

$\sqrt{11} \approx 3.317$

$\sqrt{9} < \sqrt{11} < \sqrt{16}$, which means $3 < \sqrt{11} < 4$

$3.3^2 = 10.89$ and $3.4^2 = 11.56$, which means $3.3 < \sqrt{11} < 3.4$

$3.31^2 = 10.9561$ and $3.32^2 = 11.0224$, which means $3.31 < \sqrt{11} < 3.32$

$3.316^2 = 10.995856$ and $3.317^2 = 11.002489$, which means $3.316 < \sqrt{11} < 3.317$

$$\begin{array}{r} 11.000000 \\ -\ 10.995856 \\ \hline 0.004144 \end{array} \qquad \begin{array}{r} 11.002489 \\ -\ 11.000000 \\ \hline 0.002489 \end{array}$$

Since 3.317^2 is closer to 11 than 3.316^2 is, 3.317 is a better estimate for $\sqrt{11}$.

8.NS.2 **8.**

$\sqrt{48} \approx 6.928$

Since $\sqrt{49}$ is 7, you can estimate that $\sqrt{48}$ is almost 7.
$\sqrt{36} < \sqrt{48} < \sqrt{49}$, which means $6 < \sqrt{48} < 7$.

$6.9^2 = 47.61$ and $7.0^2 = 49$, which means $6.9 < \sqrt{48} < 7.0$.

$6.92^2 = 47.8864$ and $6.93^2 = 48.0249$, which means $6.92 < \sqrt{48} < 6.93$.

$6.928^2 = 47.997184$ and $6.929^2 = 48.011041$, which means $6.928 < \sqrt{48} < 6.929$.

$$\begin{array}{r} 48.000000 \\ -\ 47.997184 \\ \hline 0.002816 \end{array} \qquad \begin{array}{r} 48.011041 \\ -\ 48.000000 \\ \hline 0.011041 \end{array}$$

Since 6.928^2 is closer to 48 than 6.929^2 is, 6.928 is a better estimate for $\sqrt{48}$.

LESSON 15: ESTIMATING SQUARE ROOTS

ANSWERS

8.NS.2 **9.** $\sqrt{63} \approx 7.937$

Since $\sqrt{64}$ is 8, you can estimate that $\sqrt{63}$ is almost 8.

$\sqrt{49} < \sqrt{63} < \sqrt{64}$, which means $7 < \sqrt{63} < 8$

$7.9^2 = 62.41$ and $8.0^2 = 64$, which means $7.9 < \sqrt{63} < 8.0$

$7.93^2 = 62.8849$ and $7.94^2 = 63.0436$, which means $7.93 < \sqrt{63} < 7.94$

$7.937^2 = 62.995969$ and $7.938^2 = 63.011844$, which means $7.937 < \sqrt{63} < 7.938$

$$\begin{array}{r} 63.000000 \\ -\ 62.995969 \\ \hline 0.004031 \end{array} \qquad \begin{array}{r} 63.011844 \\ -\ 63.000000 \\ \hline 0.011844 \end{array}$$

Since 7.937^2 is closer to 63 than 7.938^2 is, 7.937 is a better estimate for $\sqrt{63}$.

The square root value is between 7 and 8, but is much closer to 8.

Challenge Problem

8.NS.2 **10.** Share your explanation with a classmate.

Does your classmate understand what you wrote?

A *rational number* is a number that can be represented as the quotient of two natural numbers. Rational numbers are whole numbers, negative numbers, fractions, and decimals that either end or repeat.

An *irrational number* is a real number that is not a rational number. That is, it cannot be expressed as the quotient of two natural numbers. An irrational number is a number for which the numbers on the right side of the decimal point go on forever and don't have any repeating pattern. Irrational numbers can be shown with symbols, like π or $\sqrt{2}$.

The numbers are different because a rational number can be expressed as a fraction containing two natural numbers but an irrational number can't.

LESSON 16: IRRATIONAL NUMBERS

ANSWERS

ANSWERS

8.NS.1 **1.** Ⓓ $\sqrt{13}$

8.NS.1 **2.** Ⓐ 9π

Ⓔ $\sqrt{2}$

8.NS.2 **3.** $\sqrt{70}$ is between 8 and 9.

$8^2 = 64$ and $9^2 = 81$

So, $8 < \sqrt{70} < 9$.

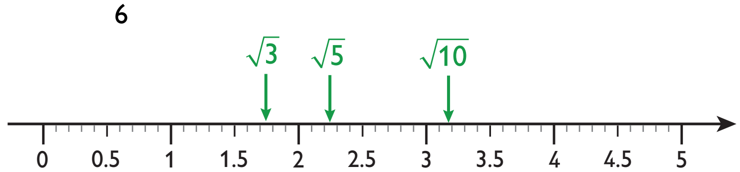
8.NS.2 **4.** $\sqrt{8} < 2.9$

8.NS.2 **5.** $1.\bar{1} > \frac{\pi}{3}$

8.NS.2 **6.** $2\pi > \sqrt{37}$

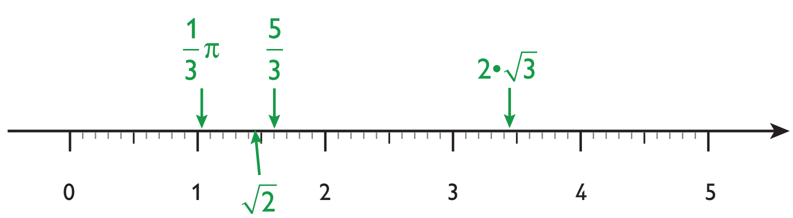
8.NS.2 **7.** $-\pi^2 < -\frac{59}{6}$

8.NS.2 **8.**



Challenge Problem

8.NS.2 **9.**



LESSON 17: PUTTING IT TOGETHER 2

ANSWERS

ANSWERS

8.NS.1 **4.** Here is one example.