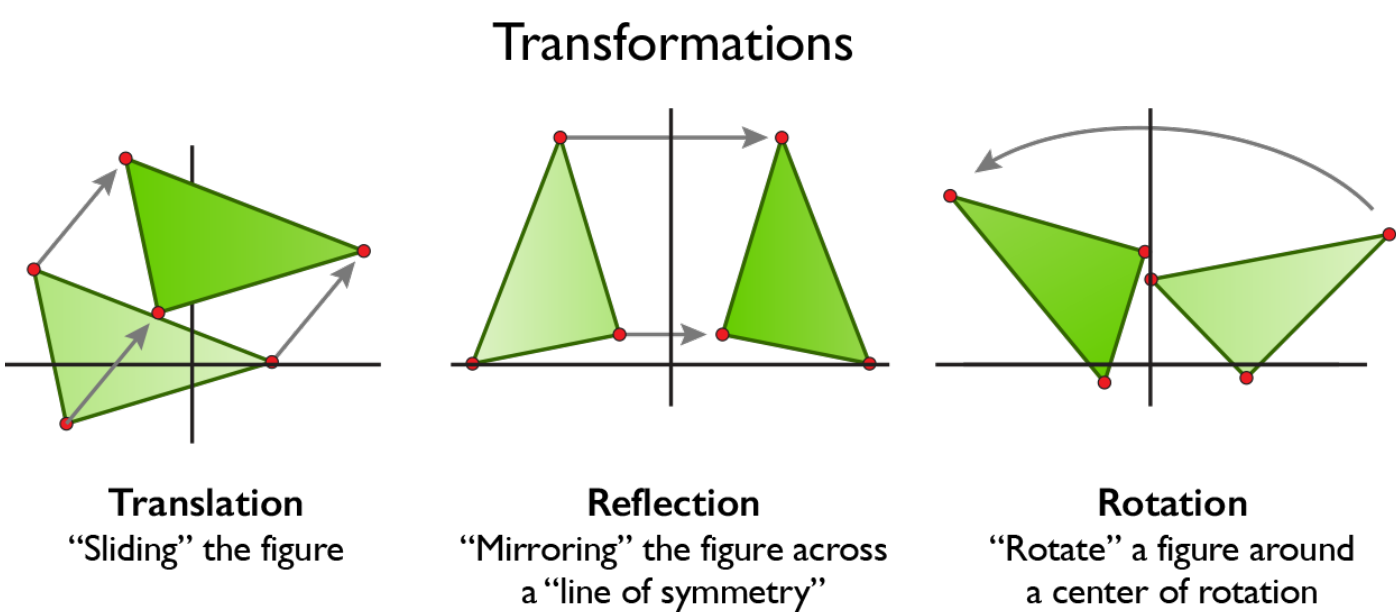
Transformations

The formal, mathematical way to describe the movement of a figure is through a *geometric transformation*. The type of geometric transformation called a *rigid motion* moves a figure as a whole without any distortion.

In other words, rigid motion transformations do not change the size or the shape of a figure. As such, transformed figures are always *congruent* to the original figure. In fact, the mathematical definition of *congruency* for two figures is that there is a sequence of rigid motion transformations that move one figure to coincide with the other.

The relationship of a figure in one location in the plane to an identical figure in any other location in the plane can be described in terms of three simple transformations:

- Translations
- Reflections
- Rotations

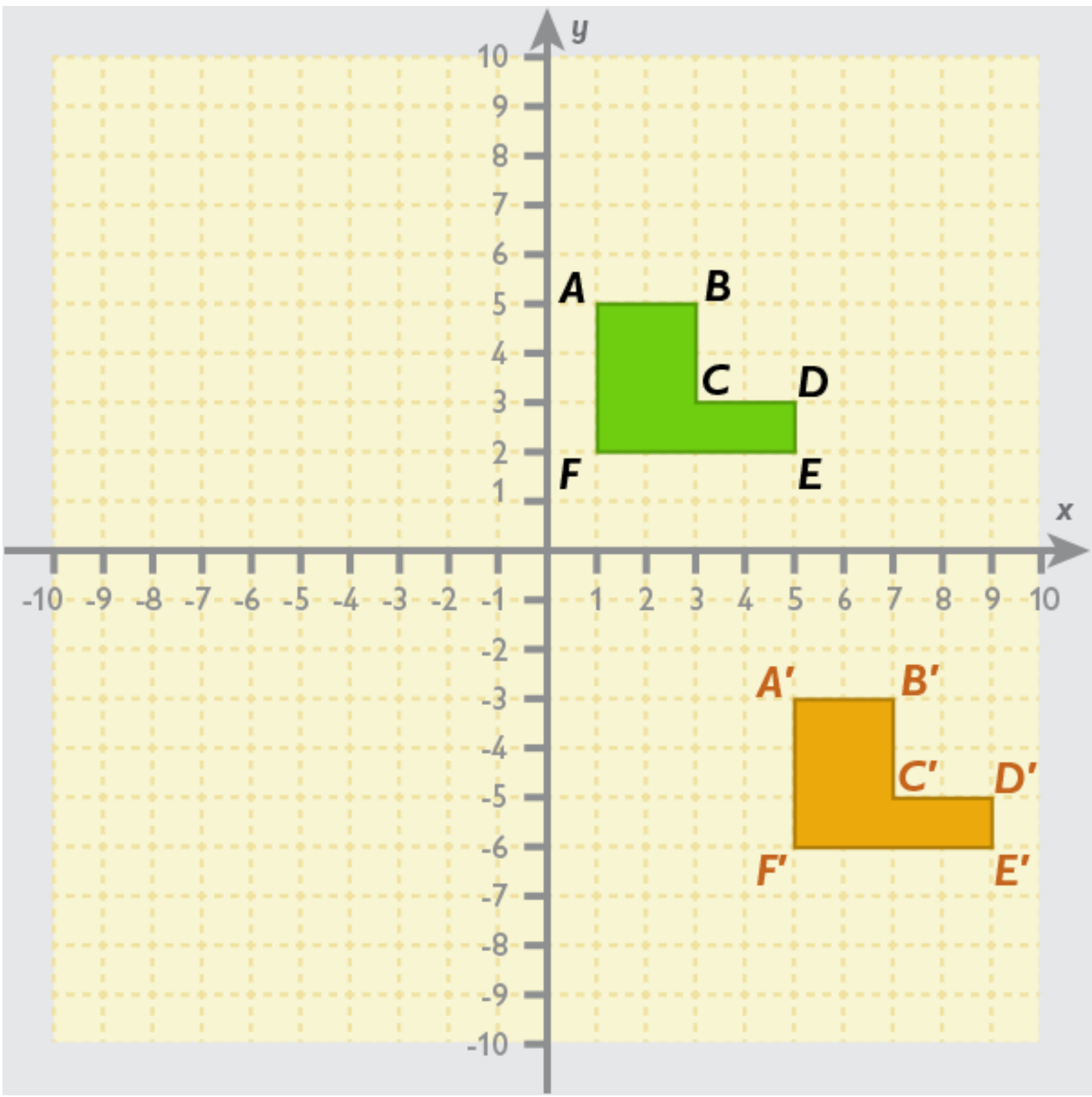


Transformations do not change the size or the shape of a figure—they merely move the figure in some way. A transformed figure is thus congruent with the original figure.

Translations

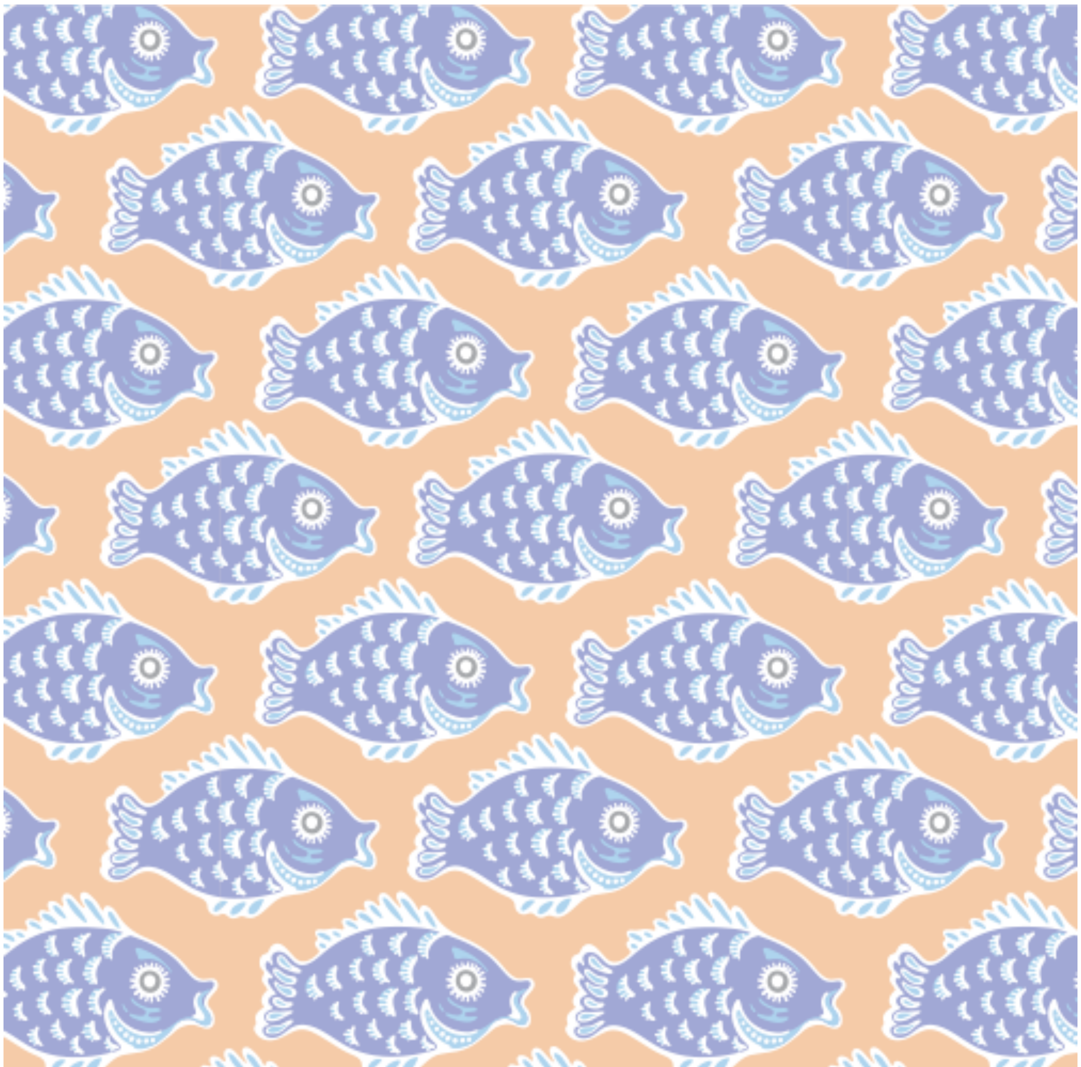
A *translation* is a type of transformation that moves every point in the plane a fixed distance in a fixed direction.

If you think of a figure as being a shape cut out of cardboard, a translation slides the whole figure without rotating it and without turning it over. Here is an example of a translation on the coordinate plane:



Note the convention of labeling the vertices of the transformed figure with a prime symbol (‘). Figure *A’B’C’D’E’F’* is a translation of figure *ABCDEF* two units left and one unit up.

You can find examples of translations in the real world. For example, look at this image:



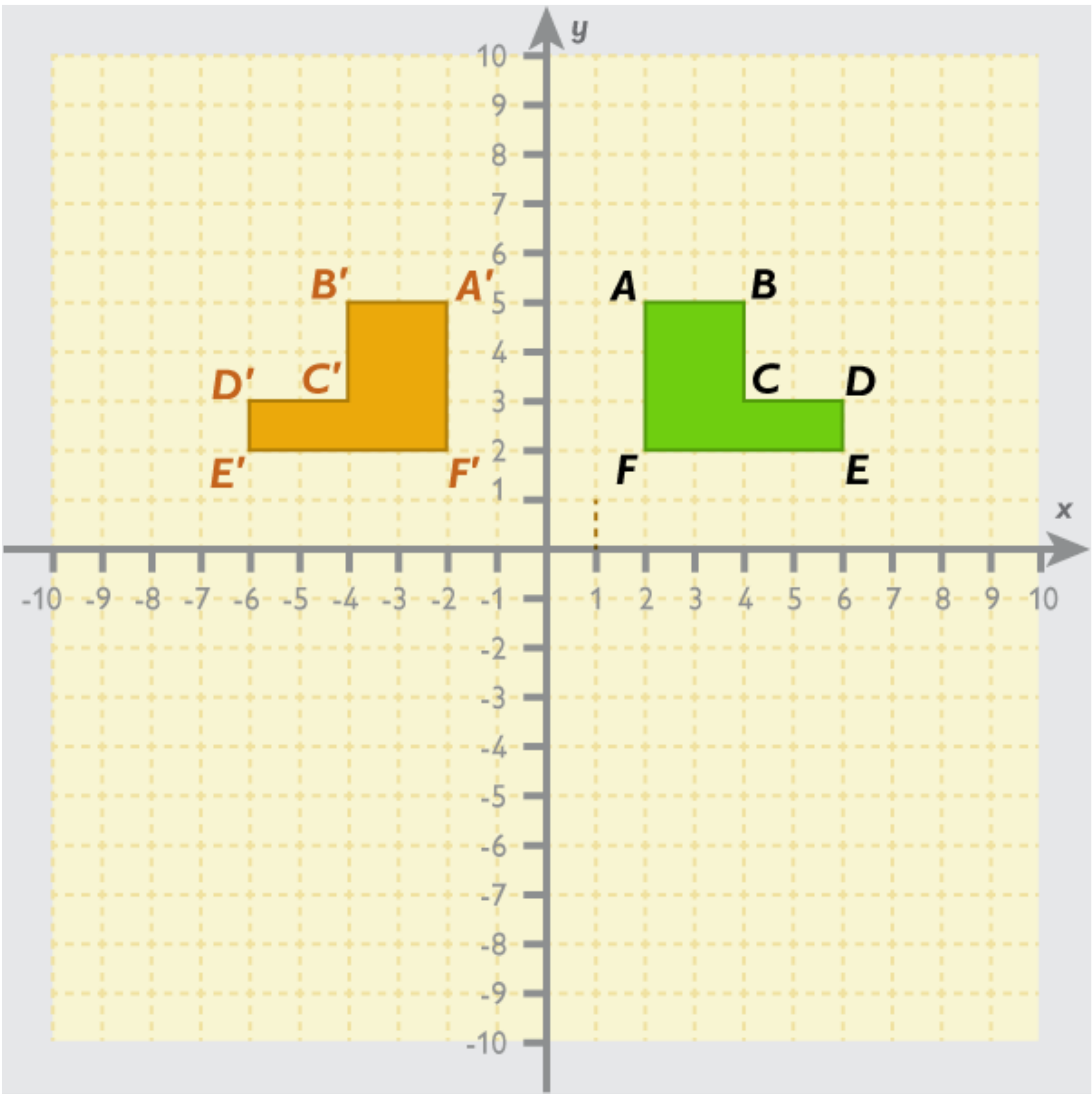
Each fish has been translated up and to the right to make a pattern.

Reflections

A *reflection* is a type of transformation that reflects every point in the plane across a fixed line ℓ in the plane. This line is called the *line of reflection*.

If you think of a figure as being a shape cut out of cardboard, a reflection flips the whole figure over with respect to the line of reflection. The line of reflection may pass through the figure, or it may lie completely outside the figure.

Here is an example of a reflection on the coordinate plane:



The line of reflection is the y-axis. Figure *A'B'C'D'E'F'* is a reflection of figure *ABCDEF* over the y-axis.

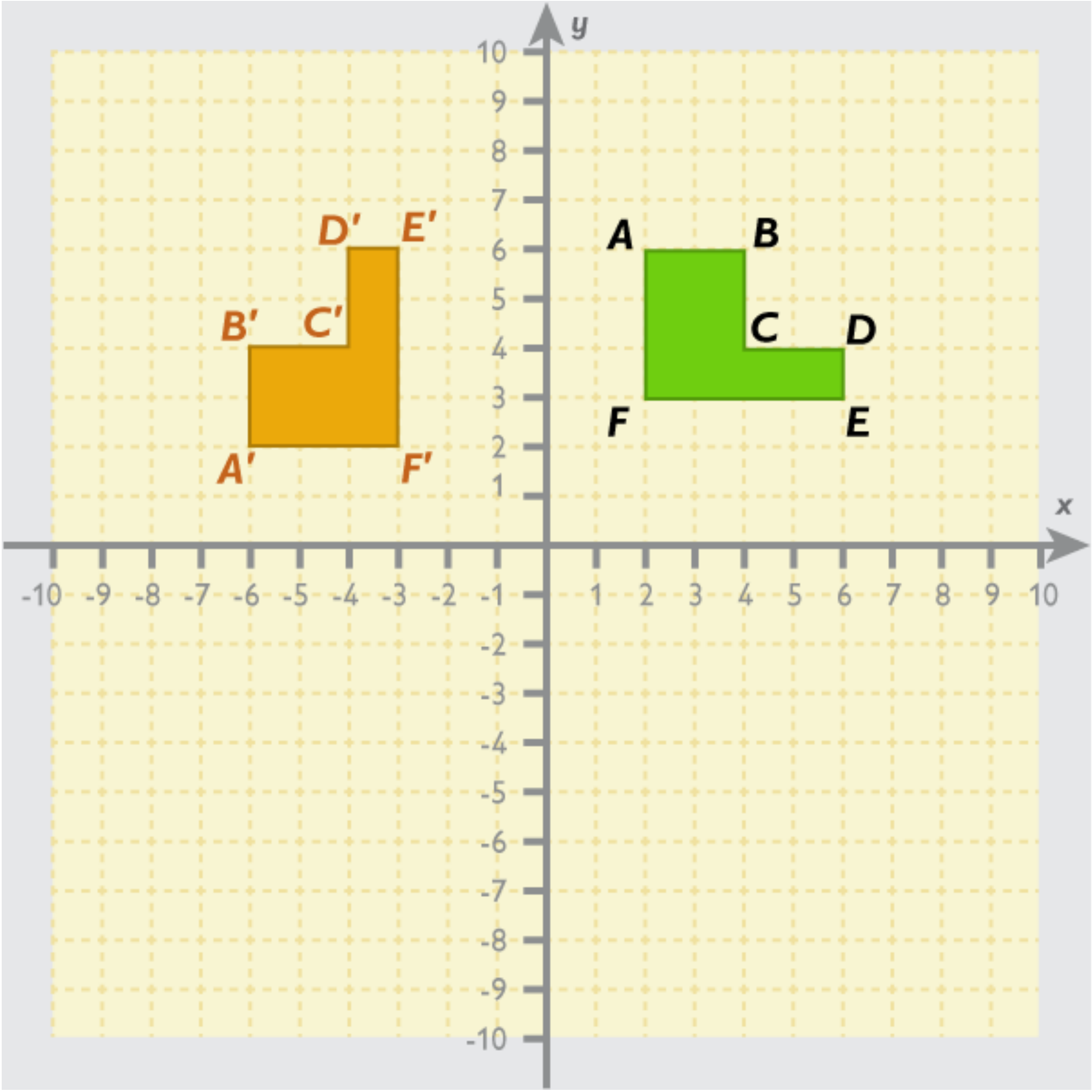
You can find examples of reflections in the real world. For example, look at this photograph of the Taj Mahal, a famous building in India:

Rotations

A *rotation* is a type of transformation that rotates every point in the plane through a fixed angle with respect to a fixed center point.

If you think of a figure as being a shape cut out of cardboard, a rotation turns the whole figure around the center point. The center point may be inside the figure or outside the figure.

Here is an example of a rotation on the coordinate plane:



The center of rotation is the origin, (0, 0). Figure *A'B'C'D'E'F'* is a rotation of figure *ABCDEF* counterclockwise 90° about the origin.

You can find examples of rotations in the real world. For example, look at this photograph:

The piece of shrimp on top is a rotation of the piece of shrimp on the bottom. The angle of rotation is 180°.

Properties of Rigid Motion Transformations

To show how rigid motions affect geometric figures, you can examine their effect on the components that make up figures: lines, line segments, and angles.

Here are the important properties of all rigid motion transformations (i.e., translations, rotations, and reflections):

- A line segment is transformed to a line segment of the same length.

- A line is transformed to a line.
- An angle is transformed to an angle of the same measure.
- Parallel lines are transformed to parallel lines.

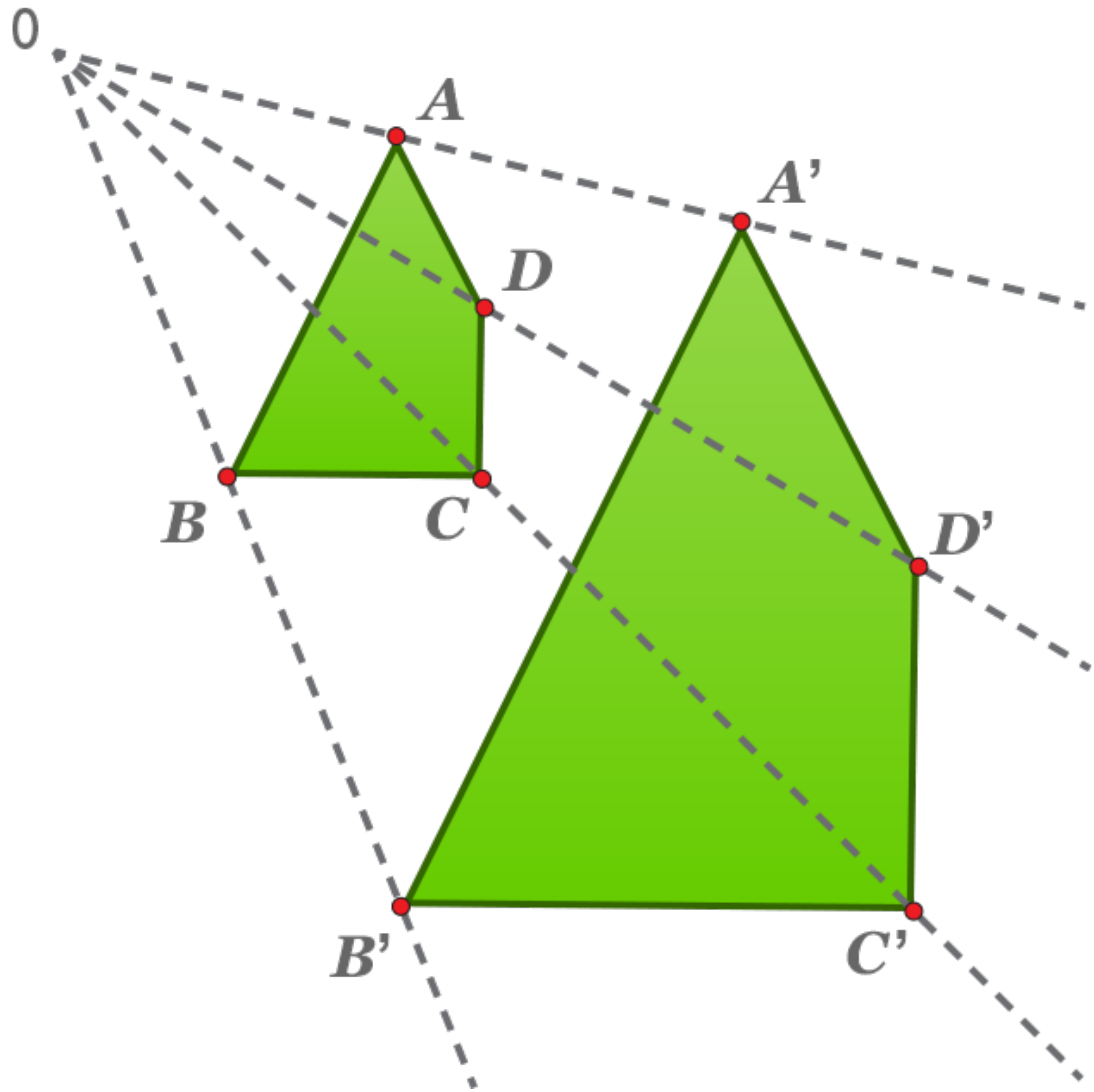
These properties make sense given that a rigid motion transformation does not distort a figure in any way.

Dilations

In mathematics, the concept of enlarging or shrinking is made precise through a type of transformation called a *dilation*.

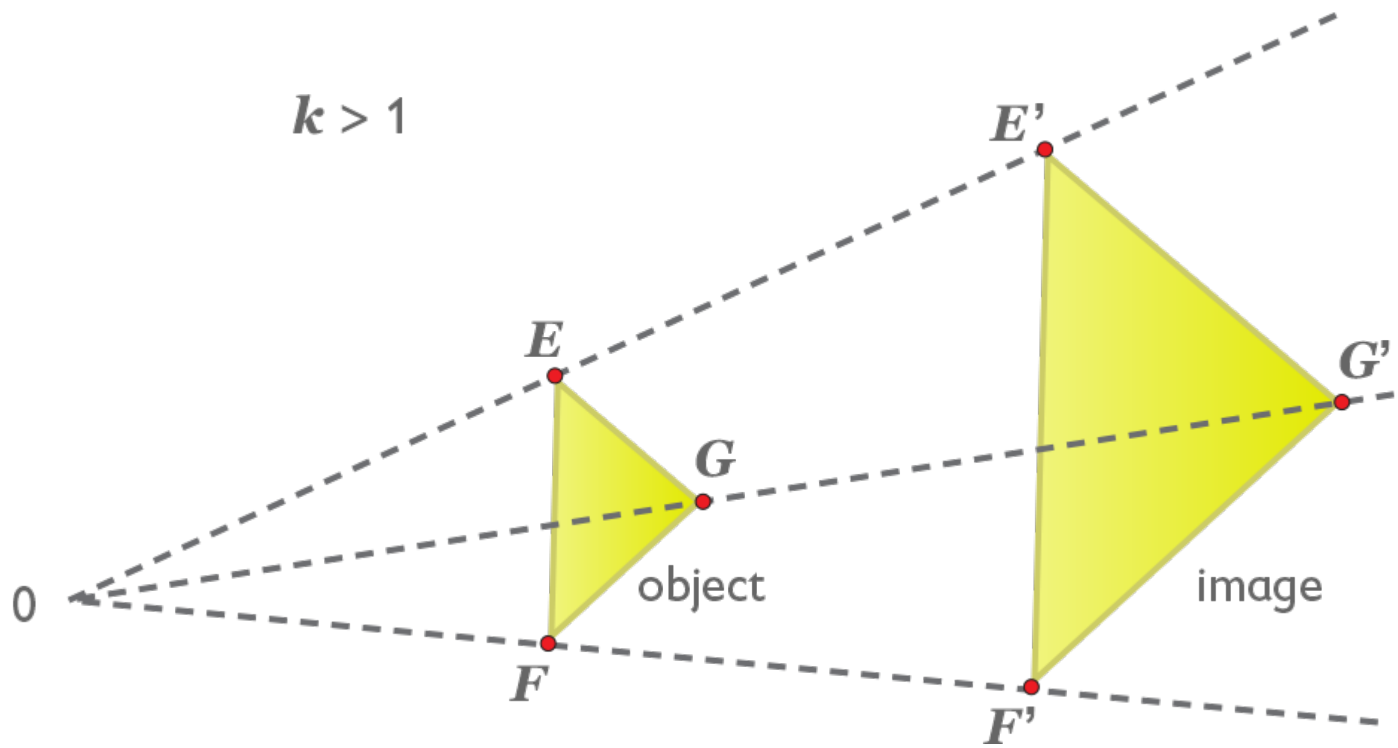
A *dilation* is a transformation that enlarges or shrinks a figure. A *dilated figure* is a figure that is enlarged (or reduced) by multiplying the lengths of the original figure by a scale factor. In order to dilate a figure, you must know the scale factor and the center of dilation. Figures are similar under dilation and congruent in the case of a scale factor of 1.

Consider this diagram:

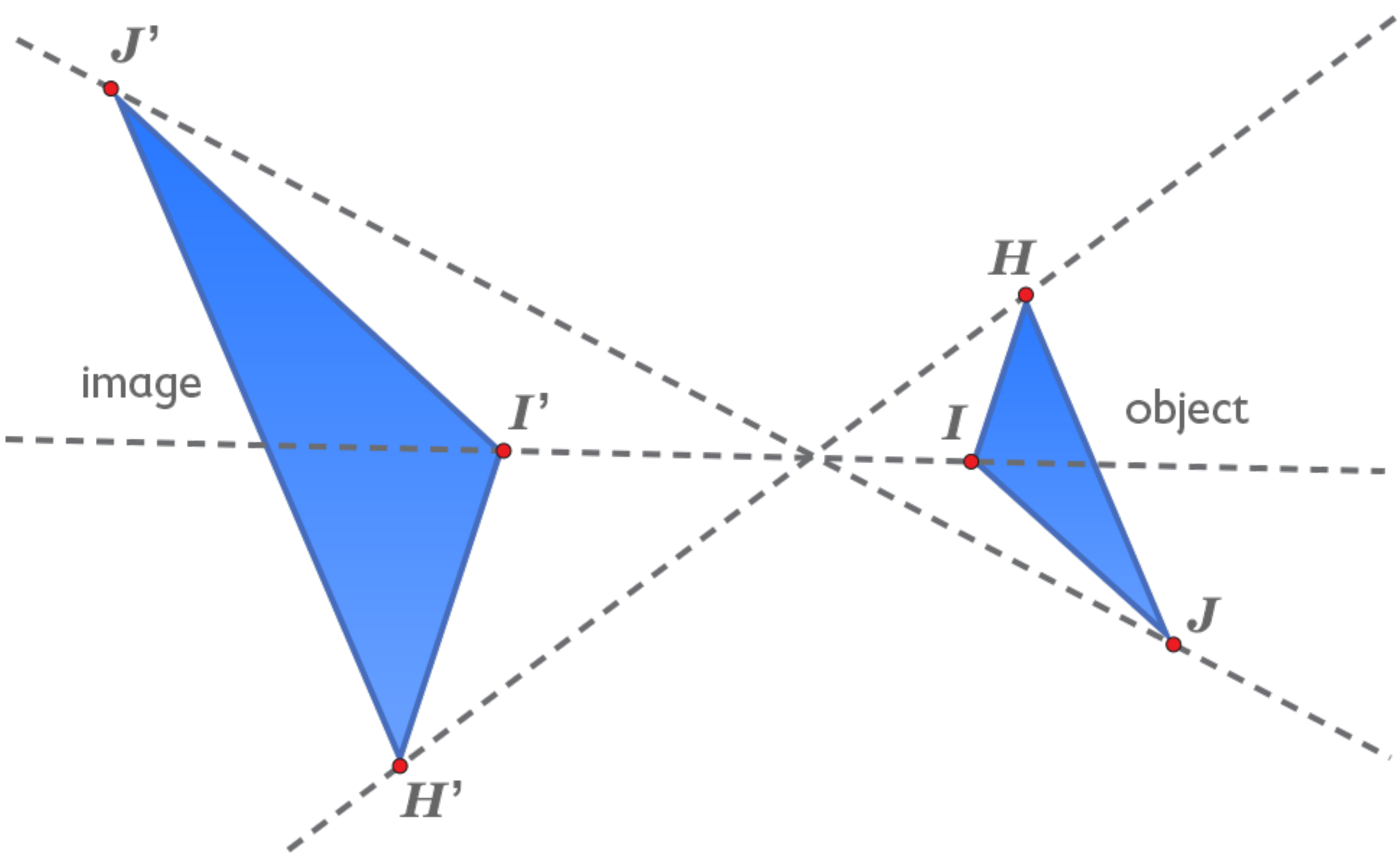


This diagram shows two similar quadrilaterals that have been moved into positions of similarity, with the center of dilation at point *O*.

A scale factor greater than 1 will produce an enlargement of the original figure, while a scale factor, *k*, less than 1 and greater than 0, will produce a reduction of the original figure:



A negative scale factor, *k*, will move the dilated figure to the opposite side of the center of dilation:



Dilations can change the size of a figure. That is their purpose. However, they do not change the shape of a figure. The test for whether a shape is changed lies in its angle measures. If the measure of all angles remains the same, the shape is unchanged. Figures that are transformed through dilation are *similar figures*, because they satisfy the similarity criteria that the angles are the same measure.

Properties of Dilation Transformations

To show how dilations affect geometric figures, you can examine their effect on lines, line segments, and angles.

Here are the important properties of dilation transformations. Except for the first item, they are the same as the properties of rigid motions. (See the Properties of Rigid Motion Transformations section.)

- A line segment of length n is transformed to a line segment with length $|k|n$, where k is the scale factor of the dilation. That is, lengths are scaled up or down by the scale factor of the dilation.
- A line is transformed to a line.
- An angle is transformed to an angle of the same measure.
- Parallel lines are transformed to parallel lines.