## EXERCISES

1. Write three things you already know about transforming and moving figures.

Share your work with a classmate.
Does your classmate understand what you wrote?
2. Write your wonderings about transformations.

Share your wonderings with a classmate.
3. Consider everyday life situations.

Think of a situation from your life that involved moving or altering one or more figures. Describe this situation and the kinds of movements or changes that were made to the figure(s).
4. Write a goal stating what you plan to accomplish in this unit.

## EXERCISES

1. Figure $B$ is a transformation of the original Figure $A$. Select which transformation was performed.


Figure $B$ is a $\qquad$ of Figure $A$.
(A) translation

B reflection
C rotation
2. Figure $B$ is a transformation of the original Figure $A$. Select which transformation was performed.


Figure $B$ is a $\qquad$ of Figure $A$.
(A) translation

B reflection
C rotation
3. Which transformation shows a translation?

4. Select the statements about transformations that are true.

There may be more than one true statement.
(A) Line lengths stay the same in a translation.

B A reflection does not modify parallel lines.
C A rotation changes the angle measures of a figure but does not change the side lengths.
(D) Rotations, translations, and reflections do not affect angle measures or side lengths of figures.
(E) The reflection of a parallelogram is no longer a parallelogram.
5.


Which figure could be a reflection of Figure $A$ ?


B

D

6. Look at this parallelogram and line of reflection.


Explain why transforming the parallelogram across the line of reflection cannot result in this parallelogram.

7. When a figure is transformed with a reflection, rotation, or translation, which statements are true about the transformation? There may be more than one true statement.

A The angle measures double in size.
B Line segments are taken to line segments of the same measure.
C The figures remain in the same position.
(D) The angles are taken to angles of the same measure.

E Parallel lines are taken to parallel lines.

## Challenge Problem

8. Sketch a triangle. Label it Figure A. Then rotate Figure A. Label the result Figure B.

Do you think it is possible to reproduce Figure $B$ with two reflections instead of $a$ single rotation?
Explain how you know. If you think it is possible, draw the intermediate figure (Figure $A^{\prime}$ ) that is the result of the first reflection and show the two lines of reflection.

## EXERCISES

1. Describe the translations you see in this image.

2. Which images show translations? There may be more than one correct image.


D


B


E

3. Translate quadrilateral $A B C D 6$ units right and 4 units down.

Choose and draw a quadrilateral into the grid in the correct position to represent the translated quadrilateral.

(A)

B

C

D

4. $\triangle A B C$ is translated 2 units up and 2 units left. The result is $\triangle A^{\prime} B^{\prime} C^{\prime}$. Which statements are true about the translation?
There may be more than one true statement.
(A) $A B=A^{\prime} B^{\prime}$
(B) $B C=2 B^{\prime} C^{\prime}$

C The distance between $A$ and $A^{\prime}$ is the same as the distance between $B$ and $B^{\prime}$ or $C$ and $C^{\prime}$
(D) 2CA $=C^{\prime} A^{\prime}$
(E) $\angle A^{\prime} B^{\prime} C^{\prime}$ is twice $\angle A B C$.
5. Quadrilateral $A^{\prime} B^{\prime} C^{\prime} D^{\prime}$ is a translation of quadrilateral $A B C D$.

What translation was applied to quadrilateral $A B C D$ ?


Quadrilateral ABCD was translated
$\qquad$ units(s) $\qquad$ and $\qquad$ unit(s)
(A) 2 units up and 1 unit right

B 1 unit down and 5 unit left
C 2 units down and 3 unit right
D 3 units up and 2 unit left
(E) 7 units up and 1 unit right
6. Quadrilateral $A^{\prime} B^{\prime} C^{\prime} D^{\prime}$ is a translation of quadrilateral $A B C D$. What translation was applied to quadrilateral $A B C D$ ?

7. List the vertices that result from the translation of $\triangle A B C 3$ units left and 4 units up. Plot the translated triangle.

8. Triangle $A^{\prime} B^{\prime} C^{\prime}$ is a translation of $\triangle A B C$. Line $a$ is the line containing $\overline{A B}$. Line $b$ is the line containing $\overline{A^{\prime} B^{\prime}}$.

What can you say about lines $a$ and $b$ ?

## Challenge Problem

9. $\triangle A B C$ is shown in a coordinate plane. The coordinates of vertex $A$ are $(5,2)$. You perform three successive translations.

- First translation, $\triangle A B C$ to $\triangle A^{\prime} B^{\prime} C^{\prime}$ :Translate 3 units up and 2 units left.
- Second translation, $\Delta A^{\prime} B^{\prime} C^{\prime}$ to $\triangle A^{\prime \prime} B^{\prime \prime} C^{\prime \prime}:$ Translate 2 units up and 4 units right.
- Third translation, $\Delta A^{\prime \prime} B^{\prime \prime} C^{\prime \prime}$ to $\Delta A^{\prime \prime} B^{\prime \prime} C^{\prime \prime} C^{\prime \prime}$ :Translate 1 unit down and 2 units right.

What should the fourth translation be so that the resulting $\Delta A$ "'" $B$ "" $C$ "" overlaps exactly with the triangle resulting from the first translation, $\Delta A^{\prime} B^{\prime} C$ ? Explain how you know.

## EXERCISES

1. Describe which sets of these congruent figures are reflections.

2. Which images show reflections? There may be more than one correct image.
(A)


C

D

E

3. Reflect quadrilateral $A B C D$ across the line $y=x$.

Choose and draw one of these quadrilaterals into the grid in the correct position to represent the reflected quadrilateral.

(A)

B

C

D

4. $\triangle A^{\prime} B^{\prime} C^{\prime}$ is a reflection of $\triangle A B C$ across the line $y=0$. Which statements are true about the reflection? There may be more than one true statement.
A The side lengths of $\triangle A B C$ are the same as those of $\triangle A^{\prime} B^{\prime} C^{\prime}$.
B $B C=2 B^{\prime} C^{\prime}$
C The distance between $A$ and $A^{\prime}$ is the same as the distance between $B$ and $B^{\prime}$ or $C$ and $C^{\prime}$
(D) $2 C A=C^{\prime} A^{\prime}$
(E) $\angle A^{\prime} B^{\prime} C^{\prime}=\angle A B C$.
5. Quadrilateral $A^{\prime} B^{\prime} C^{\prime} D^{\prime}$ is a reflection of quadrilateral $A B C D$. What reflection was applied to quadrilateral ABCD?


Quadrilateral ABCD was reflected across the line $\qquad$ .
(A) $y=x$
(B) $y=0$

C $x=0$
(D) $y=-x$
6. Quadrilateral $A^{\prime} B^{\prime} C^{\prime} D^{\prime}$ is a reflection of quadrilateral $A B C D$. What reflection was applied to quadrilateral $A B C D$ ?

7. List the vertices that result from the reflection of $\triangle A B C$ across the line $y=x$. Plot the reflected triangle.

8. Explain why reflected figures are always congruent to their original figures.
9. $\triangle A^{\prime} B^{\prime} C^{\prime}$ is a reflection of $\triangle A B C$ across the line $y=x$.

Line $a$ is the line containing $\overline{A B}$. Line $b$ is the line containing $\overline{A^{\prime} B^{\prime}}$.
What can you say about lines $a$ and $b$ ?

## Challenge Problem

10. $\triangle A B C$ is shown in a coordinate plane. The coordinates of vertex $A$ are $(5,2)$.

You perform two successive reflections.

- First reflection, $\triangle A B C$ to $\triangle A^{\prime} B^{\prime} C^{\prime}$ : across the line $y=0$
- Second reflection, $\Delta A^{\prime} B^{\prime} C^{\prime}$ to $\Delta A^{\prime \prime} B^{\prime \prime} C^{\prime \prime}$ : across the line $y=x$

Is it possible that a third reflection could result in $\Delta A$ "' $B$ "' $C$ "' that overlaps exactly with the original $\triangle A B C$ ? Explain how you know. If you think that it is possible, describe the reflection. If you think it is not possible, describe what kind of transformation you would need to perform instead.

## EXERCISES

1. Explain what this image illustrates in terms of rotation. Identify the center of rotation and angle of rotation.

2. Which images show rotations? There may be more than one correct image.

B


C

D

E

3. Rotate quadrilateral $A B C D-90^{\circ}$ around the point $(-2,-1)$.

Choose and draw one of these quadrilaterals into the grid in the correct position to represent the rotated quadrilateral.

(A)

B

C

D

4. $\Delta A^{\prime} B^{\prime} C^{\prime}$ is a $45^{\circ}$ rotation of triangle $\triangle A B C$ around point $(0,2)$.Which statements are true of the rotation? There may be more than one true statement.

A The side lengths of $\triangle A B C$ are the same as those of $\triangle A^{\prime} B^{\prime} C^{\prime}$.
B The sum of the angle measures of the three angles of $\Delta A^{\prime} B^{\prime} C^{\prime}$ is $45^{\circ}$ greater than the sum of the angle measures of $\triangle A B C$.
(C) The distance between $A$ and $A^{\prime}$ is the same as the distance between $B$ and $B^{\prime}$ or $C$ and $C$ '.
(D) The distances between point $(0,2)$ and vertices $A, B$, and $C$ are the same distances as those between point $(0,2)$ and $A^{\prime}, B^{\prime}$, and $C^{\prime}$, respectively.
(E) The angle measures of $\triangle A^{\prime} B^{\prime} C^{\prime}$ are the same as those of $\triangle A B C$.
5. Quadrilateral $A^{\prime} B^{\prime} C^{\prime} D^{\prime}$ is a rotation of quadrilateral $A B C D$. What rotation was applied to quadrilateral $A B C D$ ?


Quadrilateral ABCD was rotated $\qquad$ .
(A) $-90^{\circ}$ around $(0,0)$
(B) $180^{\circ}$ around $(0,-6)$

C $-45^{\circ}$ around $(0,6)$
(D) $30^{\circ}$ around $(-8,0)$ ]
6. Quadrilateral $A^{\prime} B^{\prime} C^{\prime} D^{\prime}$ is a rotation of quadrilateral $A B C D$. What rotation was applied to quadrilateral $A B C D$ ?

7. List the vertices that result from the $-270^{\circ}$ rotation of $\triangle A B C$ around point $(0,6)$. Plot the rotated triangle.

8. Explain why rotated figures are always congruent to their original figures.
9. What is the resulting figure after rotating Figure $\mathrm{A} 90^{\circ}$ ?


A

B

©

D


## Challenge Problem

10. $\triangle A B C$ is shown in a coordinate plane.

You perform three successive rotations around the same point.

- First rotation, $\triangle A B C$ to $\triangle A^{\prime} B^{\prime} C^{\prime}:-90^{\circ}$
- Second rotation, $\Delta A^{\prime} B^{\prime} C^{\prime}$ to $\triangle A^{\prime \prime} B^{\prime \prime} C^{\prime \prime}:-45^{\circ}$
- Third rotation, $\Delta A^{\prime \prime} B^{\prime \prime} C^{\prime \prime}$ to $\Delta A^{\prime \prime \prime} B^{\prime \prime \prime} C^{\prime \prime \prime}: 12^{\circ}$

What should the fourth rotation be so that the resulting $\Delta A$ "'" $B$ "'" $C$ "" overlaps exactly with the original triangle? Explain how you know.

## LESSON 6: COMBINING TRANSFORMATIONS

## EXERCISES

1. What single transformation can be used to change Figure $W$ to Figure $Y$ ?


Figure W


Figure $Y$

A It cannot be done with only one transformation.
B Translate Figure W horizontally.
C Reflect Figure W.
(D) Rotate Figure W.
2.


Figure $A$
Figure $A$ was rotated by $90^{\circ}$, then reflected vertically.Which figure is the result?
(A)

Figure $A^{\prime}$
B

C

Figure $A^{\prime}$
D

Figure $A^{\prime}$

## LESSON 6: COMBINING TRANSFORMATIONS

## EXERCISES

3. Which set of triangles shows a combination of one rotation and one reflection?
A

B

C

D

4. Look at these examples of Hopi art. The Hopi are a federally recognized tribe of Native American people who primarily live on the Hopi reservation in northeastern Arizona. According to the 2000 Census, the Hopi reservation has a population of 6,946 people.
Describe the transformations you can see in the Hopi art.
a. Recreated Hopi headdress design.

b. Recreated Hopi pottery designs.


## LESSON 6: COMBINING TRANSFORMATIONS

## EXERCISES

5. A rectangle was rotated $270^{\circ}$ around the origin, $(0,0)$.

Which other transformations would give the same result?
There may be more than one correct transformation.
A A reflection across the line $x=0$ followed by a reflection across the line $y=x$
(B) A rotation of $90^{\circ}$ around the origin $(0,0)$

C A reflection across the line $y=x$ followed by a reflection across the line $y=0$
(D) A translation to the next clockwise quadrant at the same relative coordinates
(E) A rotation of $-90^{\circ}$ around the origin $(0,0)$
6.

$\triangle A B C$ was transformed to $\triangle A^{\prime} B^{\prime} C^{\prime}$, which was then transformed to $\Delta A^{\prime \prime} B^{\prime \prime} C^{\prime \prime}$. Describe the two sequential transformations that were applied.
7.


Describe two possible sets of translations, reflections, and/or rotations that transform $\triangle A B C$ into $\triangle W X Y$.
8. Graph a triangle in a coordinate plane. Rotate this triangle $30^{\circ}$ around the origin. Repeat this transformation until your final image is the same as the original.
a. How many rotations did you have to make?
b. Which single transformation can replace six rotations of $30^{\circ}$ around the origin?

## Challenge Problem

9. $A B C D$ is a parallelogram (not a rectangle).

A series of three successive transformations were performed. Jacob has only been given the instructions for the first and second transformations; he does not have the directions for the last transformation.

- First transformation ( $A B C D$ to $A^{\prime} B^{\prime} C^{\prime} D^{\prime}$ ): a translation 3 units down and 1 unit right
- Second transformation ( $A^{\prime} B^{\prime} C^{\prime} D^{\prime}$ to $\left.A^{\prime \prime} B^{\prime \prime} C^{\prime \prime} D^{\prime}\right)$ : a vertical reflection across the line $y=-1$
- Third transformation ( $A$ " $B^{\prime \prime} C^{\prime \prime} D^{\prime}$ to $A$ "' $B^{\prime \prime} C^{\prime \prime}{ }^{\prime \prime} D^{\prime \prime}$ ): ?

Suppose Jacob knows that a single translation of A"'B"'C"'D"' will generate a parallelogram.

A""B"'"C"'D"' that will overlap exactly with the original parallelogram ABCD (and that $A^{\prime \prime "}$ overlaps with C). Do you think that Jacob has enough information to determine the missing transformation? If you think so, provide the details. If not, describe what additional information would be needed.

## EXERCISES

1. $\Delta K L P$ has vertices $K(-4,2), L(2,6)$, and $P(4,-6)$. It is dilated to form $\Delta K^{\prime} L^{\prime} P$ with the origin as the center of dilation.


If the coordinates of $K^{\prime}$ are $(-6,3)$, what scale factor was used to form $\Delta K^{\prime} L^{\prime} P$ '? Scale factor: $\qquad$
2. The coordinates of quadrilateral $A B C D$ are $A(3,4), B(7,4), C(9,8)$, and $D(3,8)$.


Without graphing, determine the coordinates of the dilation quadrilateral $A^{\prime} B^{\prime} C^{\prime} D^{\prime}$, where the center of dilation is the origin $(0,0)$ and the scale factor is 2 .
$A^{\prime}($ $\qquad$ , _-_) ); $B^{\prime}($ $\qquad$ ,__) ); C' $\qquad$ , -_); $D^{\prime}($ $\qquad$ __) _)
3. Dilate the original quadrilateral using a scale factor of -2 and using the point $(0,0)$ as the center of dilation.
Choose and draw one of these quadrilaterals into the grid in the correct position to represent the dilated quadrilateral.

(A)

B

C

D

4. $\triangle A^{\prime} B^{\prime} C^{\prime}$ is a dilation of $\triangle A B C$. Which statements are true of the dilation? There may be more than one true statement.
(A) The side lengths of $\triangle A B C$ are always different from those of $\triangle A^{\prime} B^{\prime} C^{\prime}$.

B If the scale factor is -2 , then $B C=2 B^{\prime} C^{\prime}$.
C The distance between $A$ and $A^{\prime}$ is the same as the distance between $B$ and $B^{\prime}$ or $C$ and $C$.
(D) If the scale factor is 2 , then $2 C A=C^{\prime} A^{\prime}$.
(E) $\triangle A B C$ and $\triangle A^{\prime} B^{\prime} C^{\prime}$ are congruent when the scale factor is 1 or -1 .
5. Quadrilateral $A^{\prime} B^{\prime} C^{\prime} D^{\prime}$ is a dilation of quadrilateral $A B C D$, with the origin as the center of dilation. What scale factor was applied to quadrilateral $A B C D$ ?


Quadrilateral ABCD was dilated by a scale factor of $\qquad$ .
(A) -4
(B) -3

C $-\frac{1}{4}$
D $\frac{1}{4}$
(E) 3
(F) 4
6. Quadrilateral $A^{\prime} B^{\prime} C^{\prime} D^{\prime}$ is a dilation of quadrilateral $A B C D$. What dilation was applied to quadrilateral $A B C D$ ?

7. List the vertices that result from the dilation of $\triangle A B C$ by a scale factor of 1.5 with the origin as the center of dilation. Plot the dilated triangle.

8. Explain why dilated figures are always similar to their original figures.

Can they sometimes be congruent? Explain why or why not.
9. In this figure, the smaller triangle is the original figure, and the larger triangle is a dilation of the smaller triangle.


Describe where the center of dilation is located and what the scale factor is. Explain your thinking.
10. In this figure, the smaller triangle is the original figure, and the larger triangle is a dilation of the smaller triangle.


What is the value of $x$ ? Explain your calculations.

## Challenge Problem

11. An acute triangle $A B C$ is transformed through a series of successive dilations, all of which have point $(2,2)$ as the center of dilation.

Dilation 1 ( $\triangle A B C$ to $\triangle D E F$ ): scale factor -1
Dilation $2(\triangle D E F$ to $\triangle G H I)$ : scale factor 3
Dilation 3 ( $\Delta G H I$ to $\Delta J K L$ ): scale factor 0.5
Dilation 4 ( $\Delta K L$ to $\triangle M N O$ ): scale factor unknown
Dilation 5 ( $\triangle M N O$ to $\triangle P Q R$ ): scale factor -4
If you know that $\triangle P Q R$ is a dilation of $\triangle D E F$ by a scale factor of -3 around the center of dilation $(2,2)$, what is the scale factor of dilation 4 ?

## EXERCISES

1. $\triangle A B C$ has vertices $A(0,1), B(2,4)$, and $C(4,2)$. Complete the coordinates of each vertex of these successive transformations.
a. $\triangle D E F$ is a translation of $\triangle A B C-1$ unit horizontally and 4 units vertically. D $\qquad$ , $\qquad$ ), E( $\qquad$ , _), $\qquad$ , _-)
b. $\triangle G H$ is a $180^{\circ}$ rotation of $\triangle D E F$ around the origin.

$\qquad$ ), H ( , __), $\qquad$ ,__)
c. $\Delta J K L$ is a reflection of $\Delta G H$ across the horizontal line $y=2$.
$J$ $\qquad$ , -_), K( $\qquad$ , ), $L($ $\qquad$ , __)
2. What transformation maps $\triangle A B C$ to $\triangle A^{\prime} B^{\prime} C^{\prime}$ ?

(A) Rotation around the origin by $180^{\circ}$

B Reflection across the line $x=0$
C Translation of 8 units horizontally
(D) Dilation by a scale factor of 0.5 with center $(0,10)$
3.


Select the transformations matching this graph.
a. $\triangle A B C$ to $\triangle A^{\prime} B^{\prime} C^{\prime}$ : $\qquad$
(A) a reflection across the line $x=2$
(B) a $90^{\circ}$ rotation around the origin

C a dilation by scale factor -1 with the origin as center of dilation
b. $A^{\prime} B^{\prime} C$ ' to $A^{\prime \prime} B^{\prime \prime} C^{\prime \prime}$ :

A a reflection across the line $x=2$
B a $90^{\circ}$ rotation around the origin
C a dilation by scale factor -1 with the origin as center of dilation
4. A quadrilateral $A B C D$ is transformed according to three consecutive transformations.

- First transformation ( $A B C D$ to $A^{\prime} B^{\prime} C^{\prime} D^{\prime}$ ): translation 2 units up and 3 units left
- Second transformation ( $A^{\prime} B^{\prime} C^{\prime} D^{\prime}$ to $\left.A^{\prime \prime} B^{\prime \prime} C^{\prime \prime} D^{\prime \prime}\right)$ : reflection across the line of reflection $y=x$
 origin $(0,0)$

If you know that the coordinates of the original quadrilateral are $A(4,-1), B(7,1)$, $C(7,4)$, and $D(5,2)$, what are the coordinates of $A$ "' $B^{\prime \prime} C^{\prime} C^{\prime \prime} D^{\prime \prime}$ ?
(A) $A^{\prime \prime \prime}(6,-4), B^{\prime \prime \prime}(9,-2), C^{\prime \prime \prime}(9,1), D^{\prime \prime \prime}(7,-1)$

B $A^{\prime \prime \prime}(1,-1), B^{\prime \prime \prime}(4,-3), C^{\prime \prime \prime}(4,-6), D^{\prime \prime}(2,-4)$
C $A^{\prime \prime \prime}(7,-1), B^{\prime \prime \prime}(10,-3), C^{\prime \prime \prime}(10,-6), D^{\prime \prime \prime}(8,-4)$
(D) A"'(-1,1), B"'(-4,3), C'"(-4,6), D"'(-2,4)
5. Consider a triangle with vertices $A(4,-1), B(10,1)$, and $C(7,4)$. Use this triangle as the original figure. Perform two sequential reflections: first reflect the original triangle across the line $y=x$, and then reflect the newly formed triangle across the line $y=0$. Give the coordinates of the final triangle.
6. Which transformations are equivalent (i.e., the final figure is the same)? There may be more than two equivalent transformations.
(A) Rotation around the origin by $180^{\circ}$

B Reflection across the line $y=0$
C Translation of $a$ units horizontally and $b$ units vertically, where point $(a, b)$ is the center of the original figure

D Dilation by a scale factor of -1 , with the origin as the center of dilation
(E) Reflection across the line $y=-x$
7. Complete the table by describing in words each transformation given as a coordinate rule.

| Rule for Coordinates | Transformation |
| :---: | :---: |
| $(x, y) \rightarrow(y,-x)$ |  |
| $(x, y) \rightarrow(x,-y)$ |  |
| $(x, y) \rightarrow(x+2, y-3)$ |  |
| $(x, y) \rightarrow(-x, y)$ |  |
| $(x, y) \rightarrow(3 x, 3 y)$ |  |

8. Describe this transformation in words.

$$
(x, y) \rightarrow(-y,-x)
$$

Support your description with a graph showing a figure and its transformation.

## Challenge Problem

9. Create a series of three consecutive transformations in which the final figure overlaps exactly with the original figure. You must follow these rules.

- Each type of transformation (i.e., translation, reflection, rotation, and dilation) can only be used once.
- With each transformation, the new figure must be in a different position.

Transformations that result in the same overlapping figure (e.g., a dilation by a scale factor 1) are not allowed.
Describe how you chose your transformations.

## EXERCISES

1. Which statements are true? There may be more than one true statement.

A A figure dilated by a negative scale factor is smaller than the original.
B A figure dilated by a scale factor that is greater than 1 is larger than the original.
C A figure dilated by a positive scale factor is larger than the original.
(D) A figure dilated by a scale factor that is less than -1 is larger than the original.

E A figure dilated by a scale factor less than 1 and greater than -1 (not 0 ) is smaller than the original.
2. $\triangle A B C$ and $\triangle X Y Z$ are similar.


What is the scale factor from
$\triangle A B C$ to $\triangle X Y Z$ ?
Scale factor: $\qquad$
3. The two quadrilaterals $A B C D$ and $R S T U$ are similar.


If you know that quadrilateral RSTU is a dilation of quadrilateral $A B C D$ by a scale factor -3 , what are the lengths of the sides of quadrilateral $A B C D$ ?
$A B=$ $\qquad$ units
$B C=$ $\qquad$ units
$C D=$ $\qquad$ units
$D A=$ $\qquad$ units
4. Look at these two leaves.


Original Image


Transformed Image

Describe possible transformations that would result in these images.
5. Look at these two leaves.


Original Image


Transformed Image

The original leaf measures 60 cm by 25 cm . The scale factor is 0.6 . What are the dimensions of the transformed image?
The transformed image is $\qquad$ cm by $\qquad$ cm.
6.


Show by experimenting how you can transform quadrilateral RSTU into quadrilateral $A B C D$ through a variety of transformations.
7. Geometric patterns are often found in nature. One example of such a pattern is the growth of the leaves of aloe plants, which grow according to what is known as a phyllotactic spiral.


Look at these three stages of growth of an aloe plant. One of the rows of leaves has been colored in red to show its evolution over time as the plant grows. The center of the plant's spiral is placed at the origin in the coordinate grid. The diameter of the circle is shown on the grid.



a. Explain the transformations between Stage 1 to Stage 2 and between Stage 2 to Stage 3.
b. Marshall says that at Stage 6 the spiral will have made a full rotation around the center. Do you agree or disagree? Explain your reasoning.

## Challenge Problem

8. Figure $B$ is the result of $a$ series of four consecutive transformations in the following sequence: one dilation, one rotation, one reflection, and one translation. You are given the following rules about the transformations.

- Figures $A$ and $B$ are congruent.
- None of the transformations applied resulted in a figure that was in the exact same position as its original.
- The rotation is either $-90^{\circ}, 90^{\circ}$, or $180^{\circ}$.
- The reflection is either horizontal or vertical.
- Assuming a rotation around an unmarked origin and a reflection across the axis of that unlabeled origin, the rule for coordinates following the rotation and the reflection would be $(x, y) \rightarrow(?, ?) \rightarrow(-x, y)$.
Given these rules, provide as much information about the four transformations as possible.

Figure $A$


Figure $B$


## EXERCISES

1. Two triangles that are similar but not congruent are a $\qquad$ of one another.
(A) reflection
(B) rotation

C dilation
(D) translation
2. How do you know these triangles are similar?

3. Suppose that $\triangle A B C$ is similar to $\triangle D E F$.


Determine the ratio of the corresponding side lengths between $\triangle A B C$ and $\triangle D E F$.
4. Suppose that $\triangle A B C$ is similar to $\triangle D E F$.


What is the length of $\overline{D F}$ ?
$\overline{D F}$ is $\qquad$ cm long.
5. $\triangle A B C$ and $\triangle D E F$ are similar. Find the missing side lengths and angle measures.


$$
\angle B=
$$

$\angle C=$ $\qquad$
$\angle D=$ $\qquad$ -
$\angle F=$
$A C=$ $\qquad$ units
$D E=$ $\qquad$ units
6. From the triangles shown, select pairs of similar triangles and express their relationship using the similarity symbol, $\sim$. Name the similarity condition that applies (SSS similarity, SAS similarity, or AA similarity).

7. In the figure shown, point $D$ is on $\overline{A B}$ of $\triangle A B C$ and $\angle A=\angle B C D$.

Explain why $\triangle A B C \sim \triangle C B D$.

8. Explain why the two triangles are similar. Express the similarity between triangles using the similarity symbol, $\sim$, and name the similarity condition you used to determine similarity (SSS similarity, SAS similarity, or AA similarity).

9. Explain why the two triangles are similar. Express the similarity between triangles using the similarity symbol, $\sim$, and name the similarity condition you used to determine similarity (SSS similarity, SAS similarity, or AA similarity).

10. State which triangles are similar and explain why. Express the similarity between triangles using the similarity symbol, $\sim$, and name the similarity condition you used to determine similarity (SSS similarity, SAS similarity, or AA similarity).


## Challenge Problem

11. Marshall is describing two triangles to Pedra. Pedra cannot see the triangles. Marshall describes the triangles as right triangles with the same hypotenuse length.
Pedra concludes from this information that the two triangles are congruent.
Do you agree? If so, explain why you think so. If not, provide the minimum amount of additional information needed to figure out whether the triangles are indeed congruent.

## EXERCISES

1. 



If these three triangles are similar, which equations are correct? There may be more than one correct equation.
(A) $\frac{a}{b}=\frac{a+e}{b+e}$
(B) $\frac{a+c}{b+d}=\frac{f}{e}$
(C) $\frac{d}{c}=\frac{a}{b}$
(D) $\frac{c+e}{d+f}=\frac{a}{b}$
(E) $\frac{b+d+f}{a+c+e}=\frac{d}{c}$
2. Which set of similar triangles makes this equation true?
$\frac{b}{a}=\frac{d}{c}=\frac{e}{f}$

A

B

C


D


## LESSON II: USING SIMILAR TRIANGLES

Look at this pair of similar triangles and answer questions 3-4.

3. What is the scale factor from $\triangle D E F$ to $\triangle A B C$ ? Show your work.
4. Find the missing angle measures and side lengths.

Use this image to answer questions 5-7.

5. The man in this picture is 6 ft tall and his shadow is 15 ft long. He is 40 ft away from the streetlight. How high is the streetlight? Explain how you know. Sketch and label a diagram to show your work.
6. The man in this picture is 5 ft tall. The streetlight is 20 ft high. How long is his shadow when he is 60 ft from the streetlight? Show your work.
7. The man in the picture is $m \mathrm{ft}$ tall. The streetlight is $p \mathrm{ft}$ high. The man is currently dft away from the streetlight. His shadow is $s \mathrm{ft}$ long.

How many additional feet does he have to walk for his shadow to be twice as long as it is now? Show your work.

## LESSON II: USING SIMILAR TRIANGLES

## EXERCISES

8. Just before sunset a man who is 1.7 m tall casts a shadow of 4.25 m . At the same time, a nearby tree casts a shadow of 87.5 m .

How tall is the tree?
The tree is $\qquad$ m tall.

Use this information to answer questions 9-10.
The Hangzhou Bay Bridge in China is one of the longest bridges in the world.


In the original design, the height of the A-shaped pylon above the 40 m wide road deck is 160 m .
9. If the width of the pylon's base, where it meets the water, is 60 m , approximately how high is the road deck above the water?

## Challenge Problem

10. Suppose, in another design for the bridge, the road deck is 40 m above the water. If the pylon in this design is similar to the A -shaped pylon in the original design, how wide would the base of this pylon be (where it meets the water)?

## EXERCISES

1. Read your Self Check and think about your work in this unit so far.

Write three things you have learned about transformations.
Share your work with a classmate.
Does your classmate understand what you wrote?
2. Use your notes from class and your thoughts about the unit to add to your math vocabulary list in your Notebook.

Include the vocabulary word or phrase, a definition, and one or more examples. When appropriate, your example should include a diagram, a picture, or a step-bystep problem-solving approach.

| Word or <br> Phrase | Definition | Examples |
| :--- | :--- | :--- |
| transformation | a process by which a figure <br> is converted into another <br> that is similar but may <br> have a different orientation, <br> rotation, scale, or position | $\triangle A B C$ has been transformed into a <br> smaller $\triangle D E F$. |

Add these words to your vocabulary list.

- translation
- reflection
- rotation
- dilation
- triangle similarity
- triangle congruence

3. Review the notes you took during the lessons about transformations. Add any additional ideas you have about this topic to your notes.
If you are still confused about anything, make sure to research your questions and add more information to your notes. Ask for help from a classmate, review the related lessons, look at the resources in the Concept Corner, talk with your teacher, and so on, to help you clear up any confusion.
4. Complete any exercises from this unit you have not finished.
