MATH GRADE 8 UNIT 3

TRANSFORMATIONS

ANSWERS
FOR EXERCISES

LESSON 2: WHAT CHANGES?

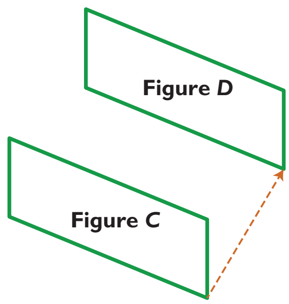
ANSWERS

ANSWERS

8.G.1 1. (B) reflection

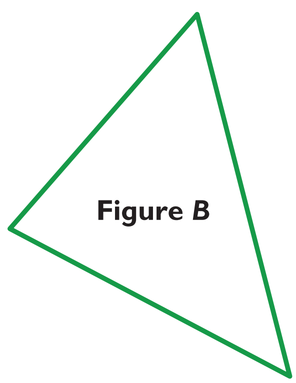
8.G.1 2. (C) rotation

8.G.1 3. (B)



8.G.1 4.
- (A) Line lengths stay the same in a translation.
- (B) A reflection does not modify parallel lines.
- (D) Rotations, translations, and reflections do not affect angle measures or side lengths of figures.

8.G.1 5. (D)



8.G.1 6. Reflections do not alter the shape of the figure. The side lengths, angles measures, and parallel sides should remain exactly the same as in the original figure. Here, the shape is no longer the same as the original. It is still a parallelogram, but its side lengths and angle measures are no longer the same as those of the original. Therefore, it cannot be a reflection of the original parallelogram.

LESSON 2: WHAT CHANGES?

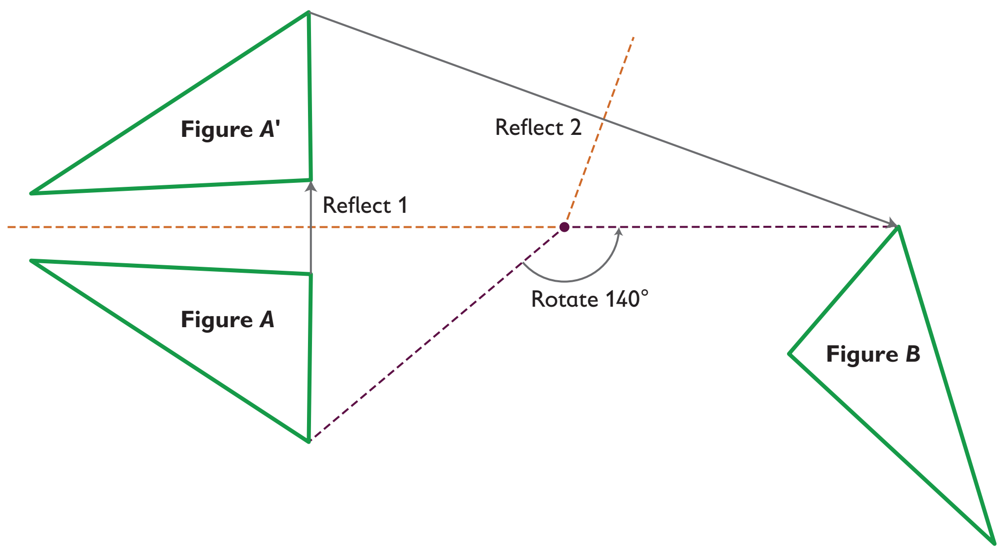
ANSWERS

8.G.1 7.

- **B** Line segments are taken to line segments of the same measure.
- **D** The angles are taken to angles of the same measure.
- **E** Parallel lines are taken to parallel lines.

Challenge Problem

8.G.1 8. Yes, it is possible to obtain Figure *B* from two reflections instead of a single rotation. Here is an example.



PEARSON

LESSON 3: TRANSLATIONS

ANSWERS

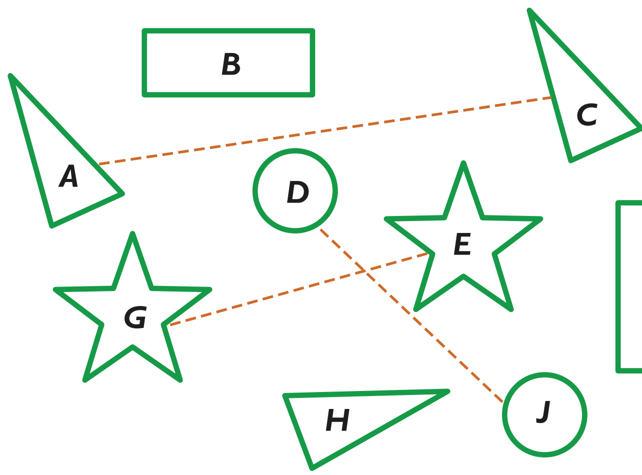
ANSWERS

8.G.1 **1.** There appear to be three congruent triangles: ΔA, ΔC, and ΔH.
However, only ΔA and ΔC are translations. ΔH is rotated and reflected.

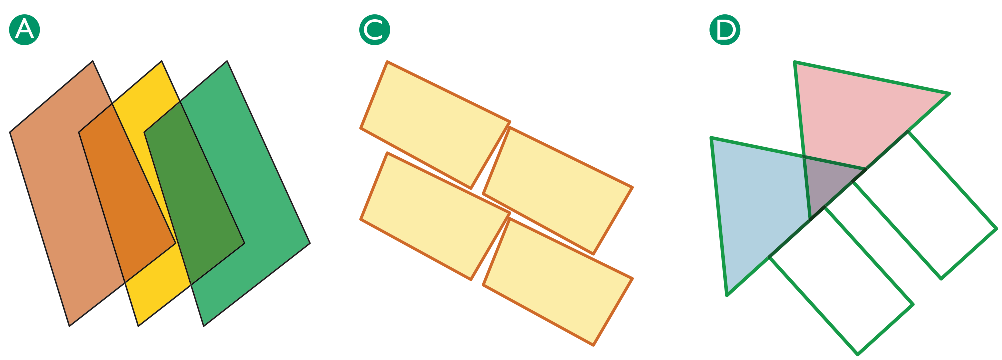
There appear to be two congruent rectangles: *B* and *F*. They are not translations.
They appear to be rotated by 90° (or –90°).

There appear to be two congruent stars: *E* and *G*.
They are translations of one another.

There appear to be two congruent circles: *D* and *J*.
They appear to be translations of one another. However, because circles do not have corners, even if they had been rotated or reflected, they would still look identical.



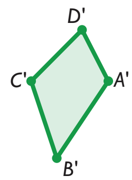
8.G.1 **2.**

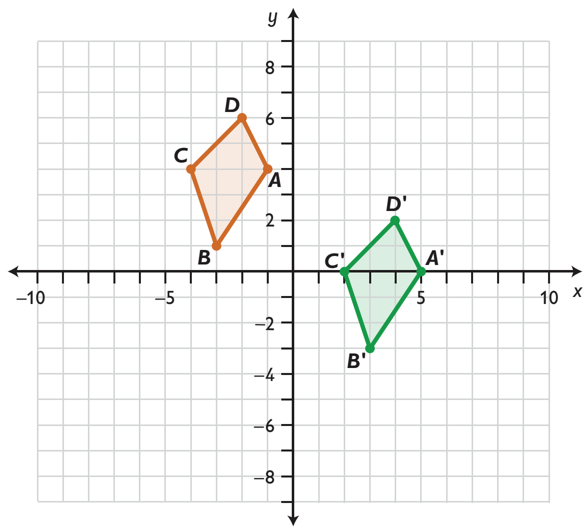


LESSON 3: TRANSLATIONS

ANSWERS

8.G.1 3. Ⓐ





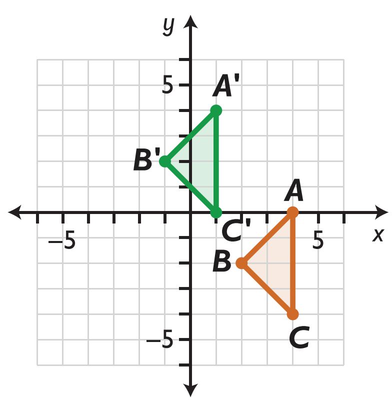
8.G.1 4. Ⓐ $AB = A'B'$

Ⓒ The distance between A and A' is the same as the distance between B and B' or C and C'

8.G.1 5. Ⓐ 2 units up and 1 unit right

8.G.1 6. Quadrilateral $ABCD$ was translated +12 units horizontally (i.e., 12 units right) and –6 units vertically (i.e., 6 units down).

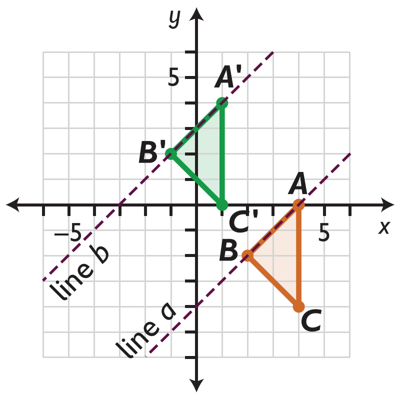
8.G.3 7. $A'(1, 4)$, $B'(-1, 2)$, $C'(1, 0)$



  PEARSON

LESSON 3: TRANSLATIONS ANSWERS

8.G.1 **8.** Lines *a* and *b* are parallel. During a translation, all the points are translated in exactly the same direction and by the same quantities (same vertical and horizontal shifts). Therefore, point *A*' and point *A* are at exactly the same distance from one another as point *B*' and point *B* are from one another. When all the points of two lines are equidistant from one another, then the two lines are parallel.



Challenge Problem

8.G.2
8.G.3 **9.** The sum of the second and third translations, from *A*'*B*'*C*' to *A*'''*B*'''*C*''', is (2 – 1) units up and (4 + 2) units right, which is 1 unit up and 6 units right. Since the fourth translation must return the figure to its *A*'*B*'*C*' position, you need to translate triangle *A*'''*B*'''*C*''' 1 unit down and 6 units left.

You can confirm this answer using the coordinates of vertex *A* and its translated vertices: 1 unit right corresponds to an increase of 1 of the *x*-coordinate and 1 unit up corresponds to an increase of 1 of the *y*-coordinate.

A(5, 2)	2 left, 3 up	*A*'(3, 5)
A'(3, 5)	4 right, 2 up	*A*''(7, 7)
A''(7, 7)	2 right, 1 down	*A*'''(9, 6)
A'''(9, 6)	6 left, 1 down	*A*'(3, 5)

LESSON 4: REFLECTIONS

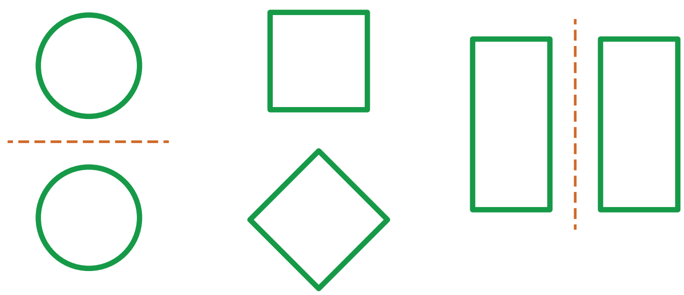
ANSWERS

ANSWERS

8.G.1 **1.** The two circles could be reflections of one another. However, since circles do not have corners, it is hard to tell. The circles could be translations of one another.

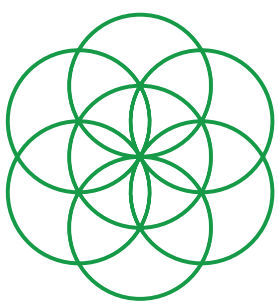
The two squares are not reflections. They appear to be rotated by 45° (or –45°).

The two rectangles could be horizontal reflections of one another. Or, they could be translations of one another.

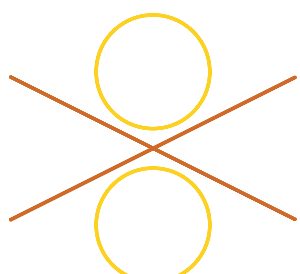


8.G.1 **2.**

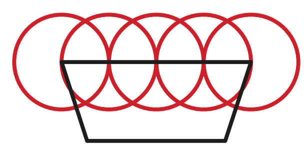
A



B



C

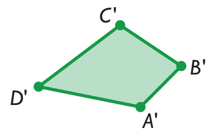


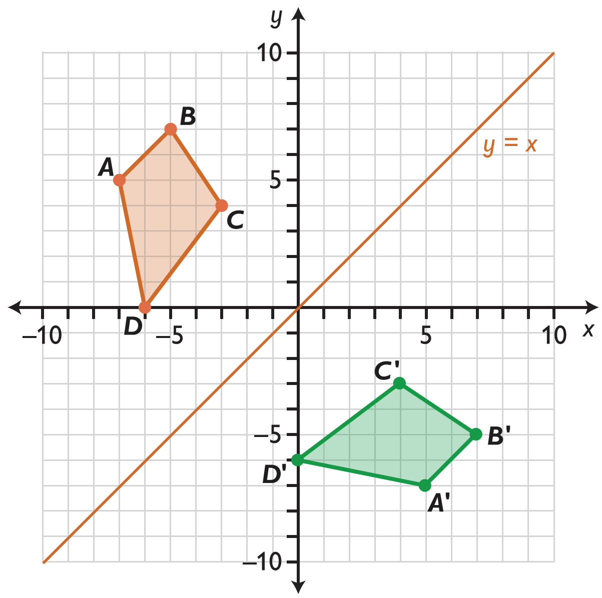
D

LESSON 4: REFLECTIONS

ANSWERS

8.G.1 3. C





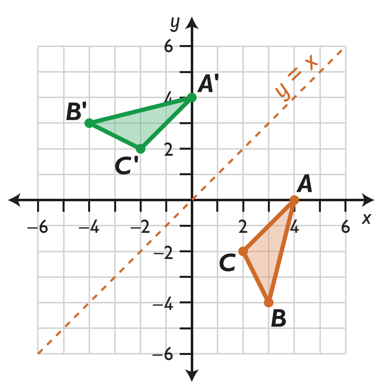
8.G.1 4. A The side lengths of ΔABC are the same as those of $\Delta A'B'C'$.

E $\angle A'B'C' = \angle ABC$.

8.G.1 5. $y = 0$

8.G.1 6. Quadrilateral *ABCD* was reflected across the line $x = 0$.

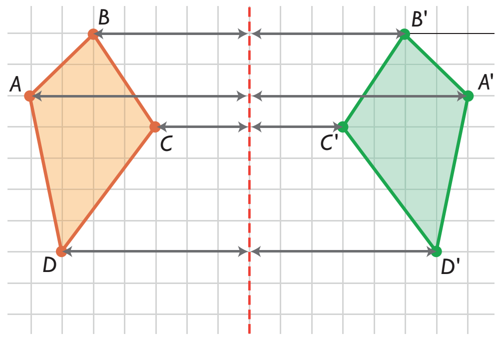
8.G.3 7. $A'(0, 4)$, $B'(–4, 3)$, $C'(–2, 2)$



LESSON 4: REFLECTIONS

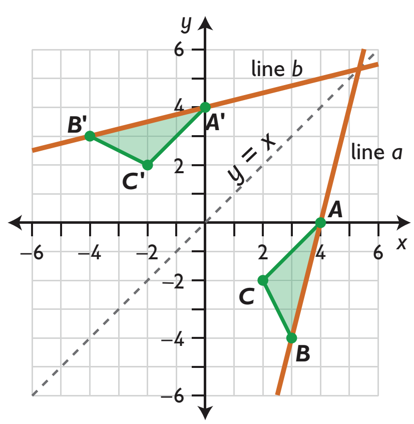
ANSWERS

8.G.2 **8.** In a reflection, all the reflected points are located at the same distance from the line of reflection as the corresponding points of the original figure.



This means that all the points on the reflected figure are at the same relative distance from one another as they are in the original figure. For example, points *A*' and *B*' are at the same distance from one another as points *A* and *B* are in the original figure. Therefore, all the corresponding side lengths are the same, and the reflected figure is congruent with the original figure.

8.G.1 **9.** Lines *a* and *b* intersect at the line of reflection, $y = x$.
During a reflection across the line $y = x$, the coordinates of the points become inverted: If $A(x, y)$, then $A'(y, x)$. The intersection of lines *a* and *b* occurs when a point on line *a* has the same coordinates as its reflection on line *b*. The only time this happens is when $x = y$, which is the line of reflection $y = x$.



LESSON 4: REFLECTIONS

ANSWERS

Challenge Problem

8.G.2
8.G.3

10. Each time a figure is reflected, the orientation of its vertices becomes inverted. For example, suppose vertices *A*, *B*, and *C* are arranged in clockwise order around the original figure. After the first reflection, the vertices will be arranged counterclockwise around the reflected figure. After the second reflection, the orientation will be clockwise again, and so on. Every sequential even reflection will have the same orientation as the original figure. Every sequential odd reflection will have the opposite orientation as the original figure.

Therefore, it would be impossible to find a third consecutive reflection that results in a figure with the same orientation as the original ΔABC.

Although it is not possible to revert to the original ΔABC with a single reflection, it would be possible to do with a single –90° rotation around the origin, (0, 0). Alternatively, two more reflections could be performed, for example, a third reflection across the line $y = -x$ and fourth one across the line $x = 0$.

LESSON 5: ROTATIONS

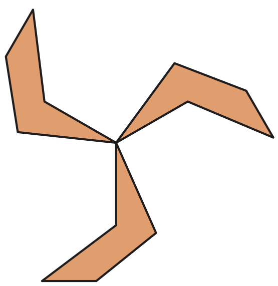
ANSWERS

ANSWERS

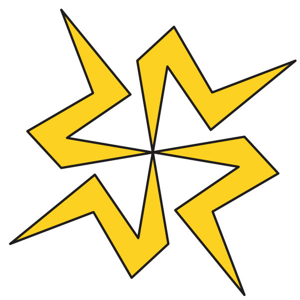
8.G.1 **1.** The darker and lighter bird-like figures are each rotated around the center of the image at angles of either plus or minus 60°.

8.G.1 **2.**

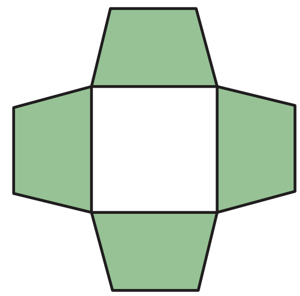
A



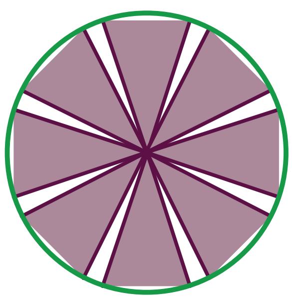
B



D

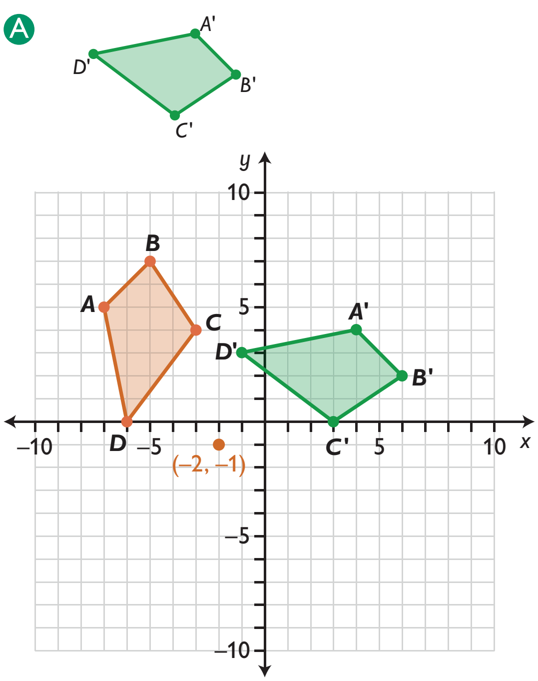


E



8.G.1 **3.**

A



LESSON 5: ROTATIONS

ANSWERS

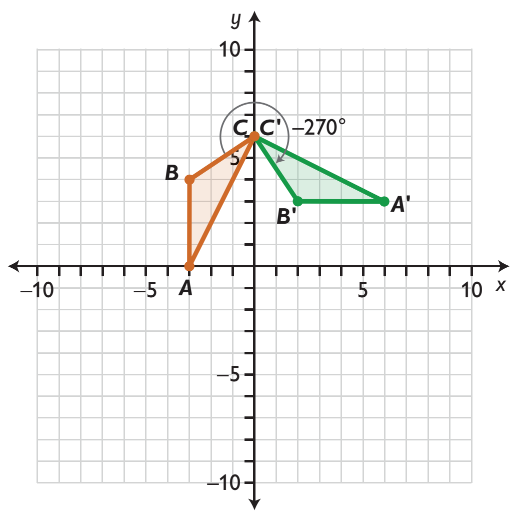
8.G.1 **4.**

- **A** The side lengths of ΔABC are the same as those of $\Delta A'B'C'$.
- **D** The distances between point (0, 2) and vertices *A*, *B*, and *C* are the same distances as those between point (0, 2) and *A'*, *B'*, and *C'*, respectively.
- **E** The angle measures of $\Delta A'B'C'$ are the same as those of ΔABC.

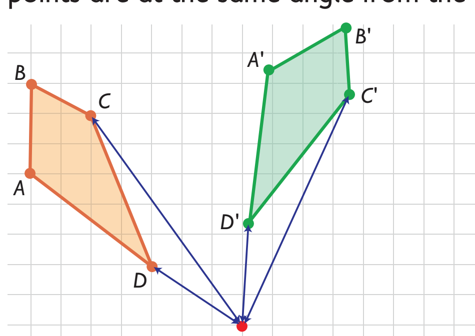
8.G.1 **5.** 180° (or –180°) around (0, –6)

8.G.1 **6.** Quadrilateral *ABCD* was rotated –90° around (0, –2).

8.G.3 **7.** *A'*(6, 3), *B'*(2, 3), *C'*(0, 6)



8.G.2 **8.** In a rotation, all the rotated points are located at the same distance from the point of rotation as the corresponding points on the original figure. In addition, all the rotated points are at the same angle from the original point relative to the point of rotation.

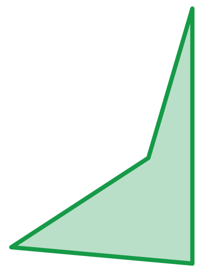


This means that all the points on the rotated figure are at the same relative distance from one another as they are in the original figure. For example, points *A'* and *B'* are the same distance from one another as points *A* and *B* are in the original figure. Therefore, all the side lengths and angle measures are the same, and the rotated figure is congruent with the original figure.

LESSON 5: ROTATIONS

ANSWERS

8.G.1 **9.** D



Challenge Problem

8.G.2
8.G.3 **10.** The fourth rotation should be either –237° or 123°.

Since the point of rotation does not change in the successive rotations, all the related points are on the same circle. For example, points *A*, *A*', *A*'', and *A*''' are all on a circle centered on the point of rotation; *B*, *B*', *B*'', and *B*''' are also on a circle. Both circles have the same center, but different radii.

A full circle has an angle measure of 360°. So, to travel along the circle from *A* all the way back to *A* requires a rotation of 360° around the center of the circle.

The first three rotations total an angle measure of –123°. Therefore, in order to go back to *A*, the fourth rotation must either go backward 123° (which is 123° in the counterclockwise direction) or complete the circle in the same clockwise direction: $-360° - (-123°) = -237°$.

LESSON 6: COMBINING TRANSFORMATIONS

ANSWERS

ANSWERS

8.G.1 1. C Reflect Figure *W*.

8.G.1 2. C

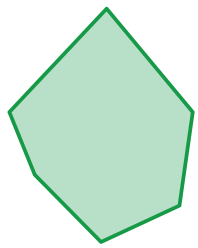
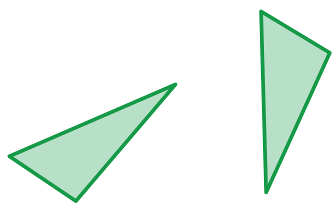


Figure *A*'

8.G.1 3. C



8.G.2 4.

a. The design has a line of reflection down the center. The smaller circles are rotated ±30° around a center point (the midpoint of the bottom of the headdress). The rotations can also be thought of as translations. The rectangles can be thought of as translations.

b. Design with four figures: The main design, an abstract figure repeated four times, is rotated ±90° around the center of the pot.

Design with rectangles and circles: This design, a pie-shaped wedge repeated six times, is rotated ±60° around the center of the bowl and then colored with different designs. You could also think of a larger wedge that is rotated ±120°. Within each design, there is a line of reflection that divides the design in half and passes through the center of the bowl. The circles could also be translations.

Flower design: This design, a petal shape repeated eight times, is rotated ±45° around the center of the bowl. Within each design, there is a line of reflection that divides the design in half and passes through the center of the bowl.

8.G.2 5.

A A reflection across the line $x = 0$ followed by a reflection across the line $y = x$

C A reflection across the line $y = x$ followed by a reflection across the line $y = 0$'

E A rotation of –90° around the origin (0, 0)

LESSON 6: COMBINING TRANSFORMATIONS ANSWERS

8.G.2 6. **Transformation from ΔABC to $\Delta A'B'C'$**

The distance between each vertex and its related transformed vertex is the same for the three vertices.

$AA' = BB' = CC'$

This means that all the points were moved in the same direction and in the same quantities. This kind of transformation is a translation.

If you compare the coordinates of the six vertices, you will see that the vertices were shifted 4 units left (–4 units along the x-axis) and 4 units up (+4 units along the y-axis).

$A(4, 0)$	$(4 - 4, 0 + 4)$	$A'(0, 4)$
$B(7, 1)$	$(7 - 4, 1 + 4)$	$B'(3, 5)$
$C(7, 4)$	$(7 - 4, 4 + 4)$	$C'(3, 8)$

Transformation from $A'B'C'$ to $A''B''C''$

This transformation cannot be a translation because the distance between the related vertices is not the same.

$A'A'' \neq B'B'' \neq C'C''$

The transformation is also not a reflection because the lines passing through the related vertices are not parallel. $\overline{A'A''}$ is not parallel to $\overline{B'B''}$. $\overline{B'B''}$ is not parallel to $\overline{C'C''}$. $\overline{C'C''}$ is not parallel to $\overline{A'A''}$.

The transformation appears to be a –90° rotation around (0, 0).

8.G.2
8.G.3 7. There are an infinite number of possible combinations of transformations. Here are several examples.

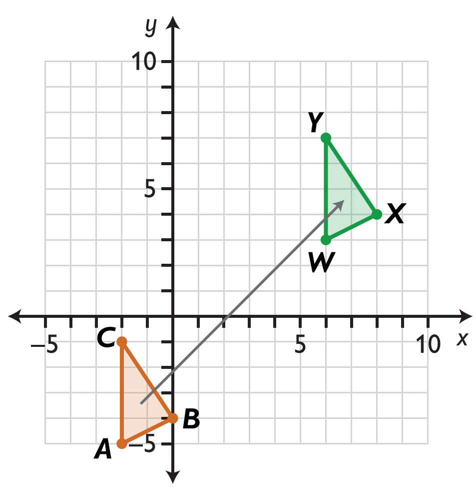
Set 1

The distance between each vertex and its related transformed vertex is the same for the three vertices.

$AW = BX = CY$

Therefore, a single translation can transform ΔABC into ΔWXY.

This translation is 8 units right and 8 units up.

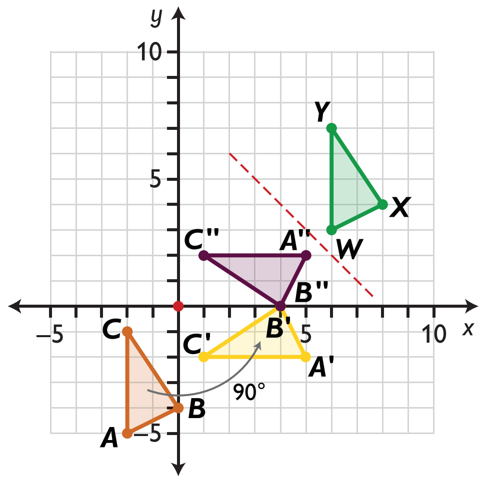


LESSON 6: COMBINING TRANSFORMATIONS — ANSWERS

8.G.2
8.G.3

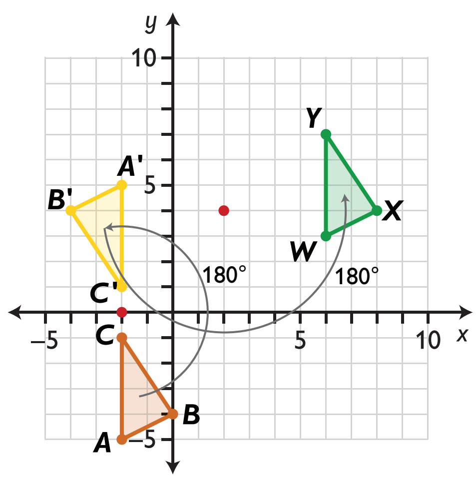
7. **Set 2**

You could perform a rotation of 90° around (0, 0), a reflection across the line $y = 0$, and then a second reflection across the line equidistant between each of the related vertices. (Draw the line segments connecting the related vertices: $\overline{A''W}$, $\overline{B''X}$, and $\overline{C''Y}$. Find the midpoint of each segment and draw the line passing through the three midpoints.)



Set 3

You could perform a rotation of 180° around (–2, 0) followed by another rotation of 180° around (2, 4).



8.G.2

8. a. You need to make 12 rotations to get back to the original triangle. $\frac{360°}{30°} = 12$, so this is the number of 30° rotations it would take to create a 360° rotation.

b. A rotation of 180° around the origin can replace six rotations of 30° around the origin. 6 • 30° = 180°, so repeating a rotation of 30° six times is equivalent to one rotation of 180°.

LESSON 6: COMBINING TRANSFORMATIONS ANSWERS

Challenge Problem

8.G.2 **9.** If a single translation of $A'''B'''C'''D'''$ will generate a figure that overlaps exactly with the original parallelogram *ABCD,* then you can assume that the sides of both figures are oriented in such a way that all the related pairs of parallel sides between the two figures are parallel.

$\overline{AD} \parallel \overline{BC} \parallel \overline{A'D'} \parallel \overline{B'C'}$

$\overline{AB} \parallel \overline{CD} \parallel \overline{A'B'} \parallel \overline{C'D'}$

The first transformation is a translation. It does not alter the orientation of the line segments. The parallelism is maintained. The second transformation is a reflection. During the reflection, the parallelism is not maintained. The only type of transformation that can revert the parallelism back is a horizontal or vertical reflection.

Since A'''' overlaps with C, the reflection has to be a horizontal reflection across a line $x = a$. With the information provided, it is not possible to know exactly the value of a. The reflection should be across any line of reflection of the form $x = a$.

To determine the exact value of a, you would need to know the coordinates of the vertices of *ABCD* and either the coordinates of $A'''B'''C'''D'''$ or the characteristics of the single translation that generates parallelogram $A''''B''''C''''D''''$.

LESSON 7: DILATIONS AND SCALE FACTOR

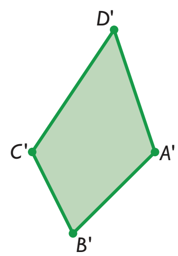
ANSWERS

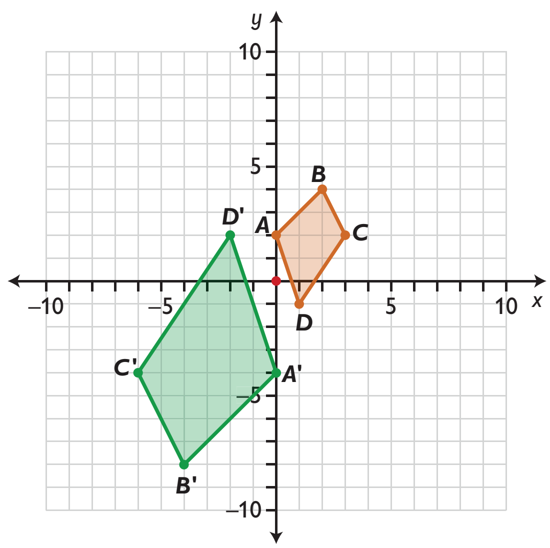
ANSWERS

8.G.3 **1.** Scale factor: 1.5

8.G.3 **2.** $A'(6, 8)$; $B'(14, 8)$; $C'(18, 16)$; $D'(6,16)$

8.G.3 **3.** Ⓑ





8.G.3 **4.** Ⓑ If the scale factor is –2, then $BC = 2B'C'$.

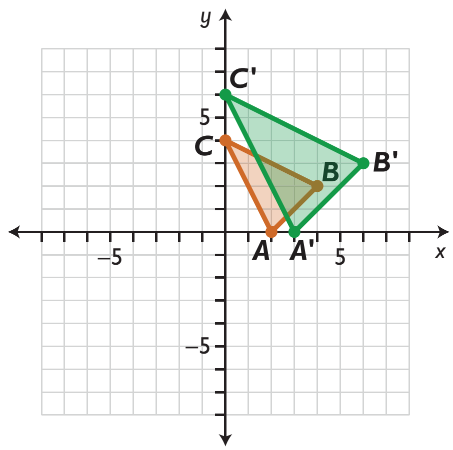
Ⓔ ΔABC and $\Delta A'B'C'$ are congruent when the scale factor is 1 or –1.

8.G.3 **5.** Ⓔ 4

8.G.3 **6.** Quadrilateral *ABCD* was dilated by a factor of $-\frac{1}{3}$, with the origin (0, 0) as the center of dilation.

LESSON 7: DILATIONS AND SCALE FACTOR — ANSWERS

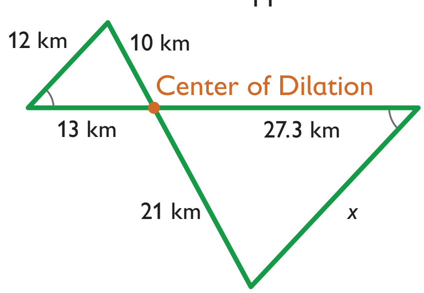
8.G.3 **7.** $A'(3, 0), B'(6, 3), C'(0, 6)$



8.G.4
8.G.3 **8.** In a dilation, the distance between the dilated point and the center of dilation is the distance between the original point and the center of dilation multiplied by the scale factor. For example, if A is a point and A' a dilation point of A across the center of dilation O by a scale factor k, then $\overline{A'O} = k\overline{AO}$.

Since all the points in the original figure are dilated according to the same scale factor, the overall shape of the figure remains the same, meaning that the dilated figure is always similar to the original figure. However, the dilated figure is usually not congruent because it is either larger or smaller (proportionally to the scale factor). The only instance when the original figure and its dilation are congruent is when the scale factor is either 1 or –1. In these two cases, the dimensions remain unchanged. When the scale factor is 1, the dilation overlaps exactly the original figure; when it is –1, the dilated figure is congruent but rotated 180° around the center of dilation.

8.G.4 **9.** The center of dilation is the point where the two lines intersect. This point is the vertex of the original figure and its dilated counterpart on the dilated figure. Here, the center of dilation happens to overlap with one of the vertices.



To find the scale factor, you divide the corresponding side lengths: $\frac{27.3}{13} = 2.1$ and $\frac{21}{10} = 2.1$. This value is the magnitude of the scale factor. You know the scale factor must be negative because of the orientation of the triangles, so the scale factor must be –2.1.

LESSON 7: DILATIONS AND SCALE FACTOR ANSWERS

8.G.4 **10.** In a dilation, all the sides of the dilated figure are proportional to the sides of the original figure. The proportionality factor is the scale factor of the dilation.

Since the proportionality factor is the same for all the sides, the ratio of two equivalent sides will be the same as the ratio of two other equivalent sides. In this situation:

$$\frac{x}{12} = \frac{27.3}{13} = \frac{21}{10} = 2.1$$

$x = 2.1 \bullet 12 = 25.2$

Therefore, $x = 25.2$ km.

Challenge Problem

8.G.4 **11.** Call the unknown scale factor x. ΔPQR is a dilation of ΔDEF by a scale factor of –3. You also know that to go from ΔDEF to ΔPQR you had to perform four dilations (dilation 2 through dilation 5). The overall dilation factor is the product of each successive dilation factor.

$3 \bullet 0.5 \bullet x \bullet (-4) = -6x$

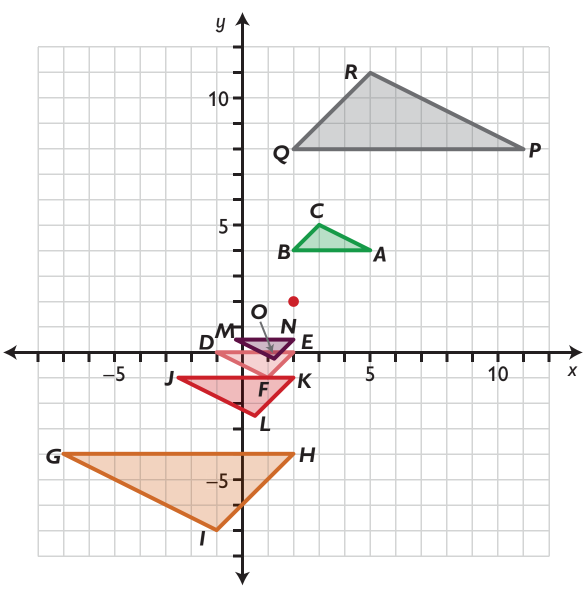
Therefore:

$-6x = -3$

$x = 0.5$

The scale factor for dilation 4 is 0.5.

Here is an example using a ΔABC with coordinates $A(5, 4)$, $B(2, 4)$, and $C(3, 5)$.



LESSON 8: ON A COORDINATE PLANE

ANSWERS

ANSWERS

8.G.3 **1.** $D(-1, 5)$, $E(1, 8)$, $F(3, 6)$
$G(1, -5)$, $H(-1, -8)$, $I(-3, -6)$
$J(1, 9)$, $K(-1, 12)$, $L(-3, 10)$

8.G.4 **2.** (D) Dilation by a scale factor of 0.5 with center (0,10)

8.G.4 **3.**
a. (B) a 90° rotation around the origin
c. (A) a reflection across the line $x = 2$

8.G.3 **4.** (B) $A'''(1, -1)$, $B'''(4, -3)$, $C'''(4, -6)$, $D'''(2, -4)$

8.G.3 **5.** When reflecting over the line $y = x$, the coordinates are switched.
$A(4, -1)$ becomes $A'(-1, 4)$.
$B(10, 1)$ becomes $B'(1, 10)$.
$C(7, 4)$ becomes $C'(4, 7)$.

When reflecting over the line $y = 0$, the sign of the y-coordinate is reversed (positive becomes negative, negative becomes positive).
$A'(-1, 4)$ becomes $A''(-1, -4)$.
$B'(1, 10)$ becomes $B''(1, -10)$.
$C'(4, 7)$ becomes $C''(4, -7)$.

Therefore, the coordinates of the final triangle are $A''(-1, -4)$, $B''(1, -10)$, and $C''(4, -7)$.

8.G.2 **6.**
(A) Rotation around the origin by 180°
(D) Dilation by a scale factor of –1, with the origin as the center of dilation

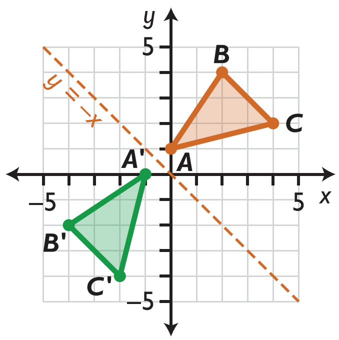
8.G.3 **7.**

Rule for Coordinates	Transformation
$(x, y) \rightarrow (y, -x)$	Rotation of 270° (or –90°) around the origin
$(x, y) \rightarrow (x, -y)$	Reflection across the x-axis (across the line $y = 0$)
$(x, y) \rightarrow (x + 2, y - 3)$	Translation 2 units horizontally and –3 units vertically
$(x, y) \rightarrow (-x, y)$	Reflection across the y-axis (across the line $x = 0$)
$(x, y) \rightarrow (3x, 3y)$	Dilation with a scale factor of 3 and center of dilation at (0, 0)

LESSON 8: ON A COORDINATE PLANE

ANSWERS

8.G.3 **8.** This transformation reflects a figure over the line $y = -x$.
Here is a possible graph of this transformation using ΔABC as the original figure.



Challenge Problem

8.G.2 **9.** Since none of the transformations can be used more than once, reflections cannot be used in this problem. Reflections alter the directionality of a figure (i.e., if the vertices of ΔABC can be read A, B, and C clockwise, the vertices of reflected $\Delta A'B'C'$ will be A, C, and B clockwise). As a result, for reflected figures to overlap exactly, an even number of reflections must be applied.

This leaves you with translations, rotations, and dilations. Since you need to apply three consecutive transformations and each can only be used once, you will need to use one of each: one rotation, one dilation, and one translation, not necessarily in that order.

Since the final figure must overlap with the original one, the dilation can only have a scale factor of –1 (a scale factor of 1 is not allowed, because it results in the same the figure).

A dilation of –1 rotates the figure by 180°. A translation does not rotate the figure, so you need to use the actual rotation transformation to cancel out the rotation of the dilation. Therefore, the rotation must be 180°.

If both the dilation and rotation use the origin as the center of dilation and center of rotation, respectively, the resulting figure would overlap exactly with the original figure. However, since a translation of 0 units vertically and horizontally is also not allowed, then at least one of the rotation and dilation transformations must use a point that is not the origin—for example, a rotation around (1, 1) and/or a dilation centered on (3, 5). The figures resulting from both these transformations will need to be translated in order to fit the requirement for a translation.

In summary, the transformations must be a translation, a 180° rotation, and a dilation by scale factor –1. The center of dilation and center of rotation cannot both be the origin. The exact translation depends on the centers of dilation and rotation chosen.

LESSON 9: SCALE FACTOR

ANSWERS

ANSWERS

8.G.3 **1.**

- **B** A figure dilated by a scale factor that is greater than 1 is larger than the original.
- **D** A figure dilated by a scale factor that is less than –1 is larger than the original.
- **E** A figure dilated by a scale factor less than 1 and greater than –1 (not 0) is smaller than the original.

8.G.4 **2.** Scale factore: 2

8.G.4 **3.** *AB* = 19 units

BC = 15 units

CD = 24 units

DA = 21 units

8.G.4 **4.** A reflection across a vertical line and a dilation by a scale factor between 0 and 1 could result in the image. Since the image is smaller than the original, you know that there must be a dilation to make the image smaller. The orientation is different between the two in a way that can only be created with a reflection.

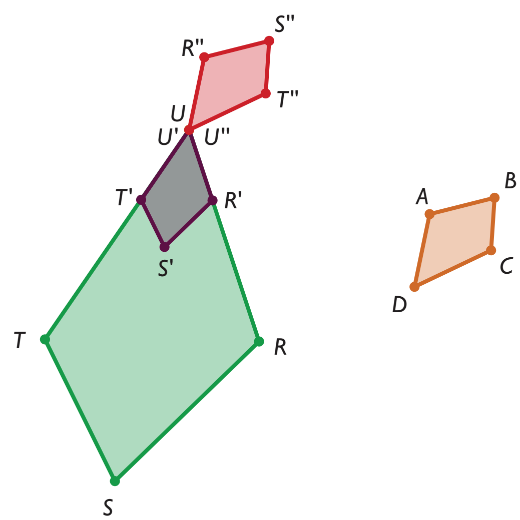
8.G.3 **5.** The transformed image is 36 cm by 15 cm.

8.G.4 **6.** Here is a possible set of transformations that will transform quadrilateral *RSTU* into quadrilateral *ABCD*.

First a dilation of $\frac{1}{3}$, using point *U* as the center of dilation, leads to figure *R'S'T'U'*.

Then a rotation of 150° around point U leads to figure *R''S''T''U''*.

Finally, a translation takes *R''S''T''U''* to *ABCD*.



LESSON 9: SCALE FACTOR ANSWERS

8.G.4 7.

a. The change from Stage 1 to Stage 2 is a 45° rotation around the origin (0, 0) and a dilation of scale factor 1.25, with the center of dilation at the origin. From Stage 2 to Stage 3 the rotation is the same but the dilation has a scale factor of only 1.2.

b. Marshall is incorrect. A full rotation will be completed at Stage 8. Each stage rotates the figure 45°. A complete rotation is 360°.

$$\frac{360}{45} = 8$$

Therefore, it will take 8 rotations of 45° to make a complete rotation.

Challenge Problem

8.G.4 8.

The two figures are congruent. This means that the scale factor of the dilation has to be either –1 or 1. Since the second rule states that none of the transformations were in the same position as its original, only scale factor –1 is possible.

You know that the coordinates following the rotation and the reflection follow the rule $(x, y) \rightarrow (?, ?) \rightarrow (-x, y)$.

Here is a table summarizing the various possible combinations of rules, knowing that the rotation is either –90°, 90°, or 180° and that the reflection can only be vertical or horizontal.

Rotation	Reflection
–90°: $(x, y) \rightarrow (y, -x)$	vertical: $(y, -x) \rightarrow (y, x)$
–90°: $(x, y) \rightarrow (y, -x)$	horizontal: $(y, -x) \rightarrow (-y, -x)$
90°: $(x, y) \rightarrow (-y, x)$	vertical: $(-y, x) \rightarrow (-y, -x)$
90°: $(x, y) \rightarrow (-y, x)$	horizontal: $(-y, x) \rightarrow (y, x)$
180°: $(x, y) \rightarrow (-x, -y)$	vertical: $(-x, -y) \rightarrow (-x, y)$
180°: $(x, y) \rightarrow (-x, -y)$	horizontal: $(-x, -y) \rightarrow (x, -y)$

The only coordinate rules that match the given rule are a rotation of 180° and a vertical reflection.

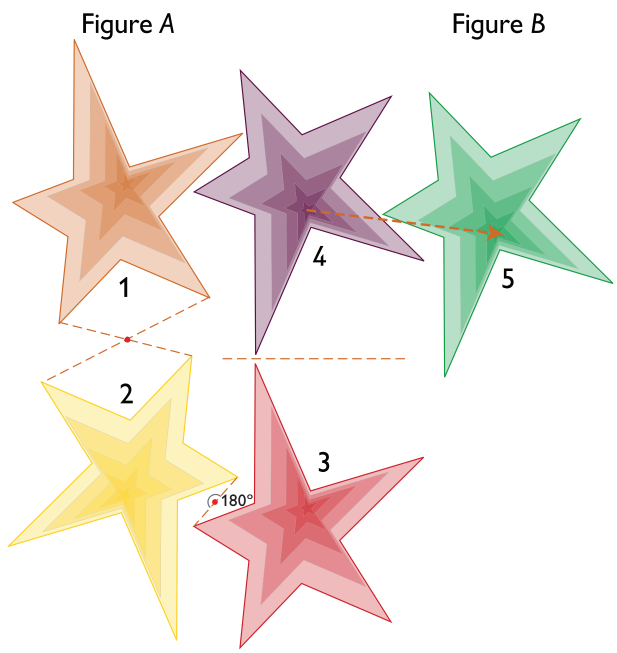
No precise information is available about the translation. The translation will depend on the position of the center of dilation, the center of rotation, and the line of reflection used.

LESSON 9: SCALE FACTOR

ANSWERS

8.G.4 **8.** Here is a possible set of transformations.

- dilation scale factor –1: star 2
- rotation 180°: star 3
- vertical reflection: star 4
- translation: star 5



LESSON 10: SIMILAR TRIANGLES

ANSWERS

ANSWERS

8.G.4 **1.** (C) dilation

8.G.5 **2.** The triangles are similar because two pairs of corresponding angles are equal, so the other corresponding angles are equal.

8.G.5 **3.** The ratio of the corresponding side lengths is $\frac{4}{3}$.

Since the triangles are similar, the ratio of the triangles is the same as the ratio of corresponding side lengths $\overline{AB}$ and $\overline{DE}$.

$$\frac{16}{12} = \frac{4}{3}$$

8.G.5 **4.** $\overline{DF}$ is 9 cm long.

8.G.5 **5.** $\angle B = 105°$

$\angle C = 21°$

$\angle D = 54°$

$\angle F = 21°$

$AC = 24$ units

$DE = 4.5$ units

8.G.5 **6.** $\Delta ABC \sim \Delta MNO$; SAS similarity
$\Delta JKL \sim \Delta GHI$; AA similarity (calculate the third angle for either triangle to see that the criterion is AA: $90° + 35° + 55° = 180°$)
$\Delta DEF \sim \Delta PQR$; SSS similarity

8.G.5 **7.** $\angle A = \angle BCD$, so these angles are congruent.

If you compare the sum of the angles of both triangles, you get these equations. ΔABC:
$\angle A + \angle B + \angle C = 180°$
ΔBCD:
$\angle B + \angle BCD + \angle BDC = 180°$

As a result:
$\angle A + \angle B + \angle C = \angle B + \angle BCD + \angle BDC$

Since $\angle A = \angle BCD$ and $\angle B$ is in both triangles and is congruent to itself, the equation can be simplified to:
$\overline{\angle A} + \overline{\angle B} + \angle C = \overline{\angle B} + \overline{\angle BCD} + \angle BDC$
$\angle C = \angle BDC$
Thus, by AA similarity, $\Delta ABC \sim \Delta CBD$.

LESSON 10: SIMILAR TRIANGLES

ANSWERS

8.G.5 **8.** $\Delta AEB \sim \Delta DEC$

$\angle DEC$ and $\angle AEB$ are opposite angles of two intersecting lines, and so they are congruent by definition. The ratio of *CE* to *BE* is 2, which is the same as the ratio of *DE* to *AE*.

Since there is one pair of corresponding angles and the lengths of the adjacent corresponding sides have the same ratio, the triangles are similar. The similarity condition used to determine the similarity of these triangles is SAS similarity.

8.G.5 **9.** $\Delta ABC \sim \Delta DEC$
It is given that $\angle BAE = \angle EDC = 90°$; the angles are therefore congruent. $\angle C$ is in both triangles and is congruent to itself.

Since there are two congruent angles, the triangles are similar. The similarity condition used to determine the similarity of these triangles is AA similarity.

8.G.5 **10.** $\Delta ABC \sim \Delta ADE$
It is given that $\angle ADE = \angle ABC = 50°$; the angles are therefore congruent. $\angle A$ is in both triangles and is congruent to itself.

Since there are two congruent angles, the triangles are similar. The similarity condition used to determine the similarity of these triangles is AA similarity.

Challenge Problem

8.G.5 **11.** Both triangles are right triangles, so they both have a 90° angle. In addition, they have the same hypotenuse length, which is the side length opposite the right angle.

Pedra does not have enough information to conclude that the triangles are congruent. She needs at least one more side length to assess whether there is SAS congruence or one more angle measure to assess ASA congruence.

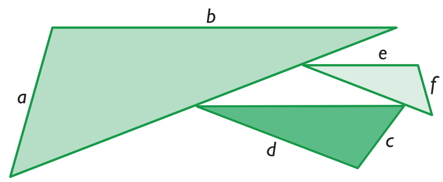
LESSON 11: USING SIMILAR TRIANGLES

ANSWERS

ANSWERS

8.G.5 1. D $\dfrac{c+e}{d+f}=\dfrac{a}{b}$

8.G.5 2. B



8.G.5 3. Since the triangles are similar, $\overline{BC}$ and $\overline{EF}$ are corresponding sides. (They are the longest sides.) Their ratio is $\dfrac{105}{28}=3.75$, which is also the scale factor.
The scale factor from ΔDEF to ΔABC is 3.75.

8.G.5 4. Calculation of the missing side lengths $\overline{AB}$ and $\overline{DF}$:

$$\frac{BC}{EF}=\frac{105}{28}=\frac{AB}{DE}=\frac{AB}{16}$$

$$AB=\frac{105}{28}\bullet 16=60$$

$$\frac{BC}{EF}=\frac{105}{28}=\frac{AC}{DF}=\frac{84}{DF}$$

$$DF=\frac{28}{105}\bullet 84=22.4$$

$AB=60$ cm $\qquad DF=22.4$ cm

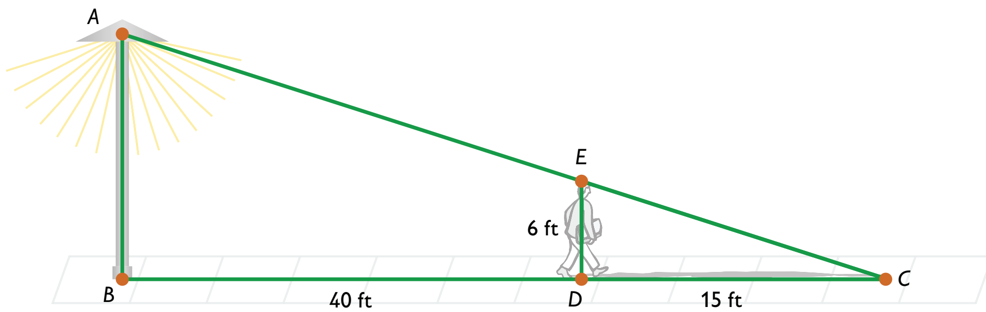
Calculation of the missing angle measures $\angle A$, $\angle C$, $\angle E$, and $\angle F$:
Two angles are known, so to find the third subtract those two from 180:
180 – 53 – 92 = 35.

$\angle A = \angle D = 92°$
$\angle C = \angle F = 35°$
$\angle E = \angle B = 53°$

LESSON 11: USING SIMILAR TRIANGLES

ANSWERS

8.G.5 **5.** The man's position in front of the streetlight creates two similar triangles.



$\overline{DE}$, which approximates the man's height, is perpendicular to his shadow, forming a right angle. Assuming the streetlight is also perpendicular to the ground, $\angle ABD$ is congruent to $\angle EDC$.

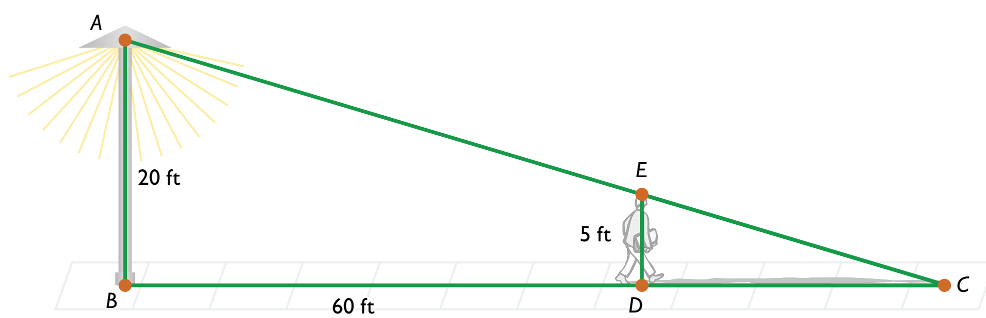
$\angle C$ is congruent to itself. According to AA similarity, two similar triangles can be constructed: $\Delta ABC \sim \Delta EDC$. The ratio of DE to AB is the same as that of DC to BC.

$$\frac{AB}{6} = \frac{40+15}{15} = \frac{55}{15}$$

$$AB = \frac{55}{15} \bullet 6 = 22$$

The streetlight is 22 ft high.

8.G.5 **6.** The man's position in front of the streetlight creates two similar triangles.



$$\frac{AB}{DE} = \frac{20}{5} = \frac{BD+CD}{CD} = \frac{60+CD}{CD}$$

$$4 \bullet CD = 60 + CD$$

$$3 \bullet CD = 60$$

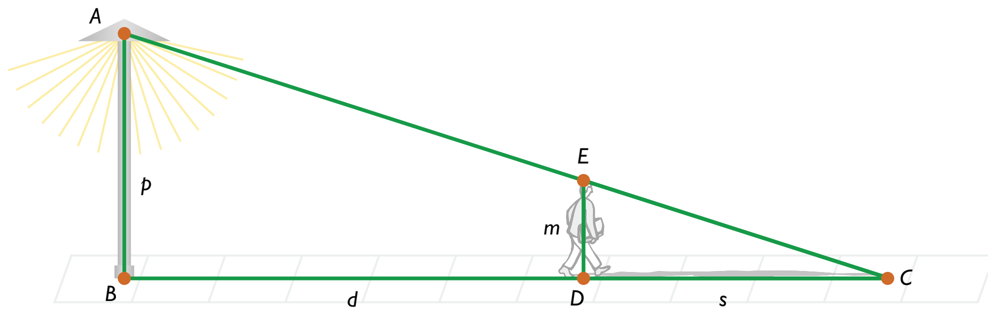
$$CD = 20$$

The length of the shadow when the man is 60 ft away from the streetlight is 20 ft.

LESSON 11: USING SIMILAR TRIANGLES

ANSWERS

8.G.5 **7.**



The two triangles formed by the streetlight, the man, and his shadow are similar because they form two right triangles with a common $\angle C$. With two congruent angles, the triangles are similar by AA similarity. As a result:

$$\frac{AB}{DE}=\frac{p}{m}=\frac{BD+CD}{CD}=\frac{d+s}{s}$$

As the man continues to walk the length of his shadow, the hypotenuse changes, but his height and the height of the streetlight do not. The two triangles are still similar, because $\angle EDC$ and $\angle ABC$ are right angles and $\angle C$ is congruent to itself.

If *d*' is the new distance from the streetlight and s' the new length of the shadow, the equation becomes:

$$\frac{AB}{DE}=\frac{p}{m}=\frac{BD+CD}{CD}=\frac{d'+s'}{s'}$$

You want to find out how much farther the man has to walk for his shadow to be twice as long. Twice as long means s' = 2*s*.

$$\frac{p}{m}=\frac{d'+s'}{s'}=\frac{d'+2s}{2s}$$

The ratio $p:m$ has not changed; therefore:

$$\frac{d+s}{s}=\frac{p}{m}=\frac{d'+2s}{2s}$$

$$d+s=\frac{d'+2s}{2}=\frac{1}{2}d'+s$$

$$d=\frac{1}{2}d'$$

$$d'=2d$$

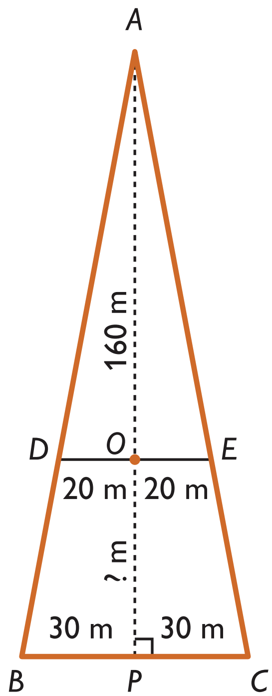
When the shadow is twice as long as it was originally, the distance *d*' from the streetlight is also twice as long as it was originally. Therefore, the man must walk the same distance as his original distance from the streetlight for his shadow's length to double.

LESSON 11: USING SIMILAR TRIANGLES

ANSWERS

8.G.5 **8.** The tree is 35 m tall.

8.G.5 **9.** The pylon can be represented as a triangle.



In this model, $\Delta AOE \sim \Delta APC$ because the triangles have two pairs of congruent angles. You can use this relationship to determine OP.

$$\frac{OE}{PC} = \frac{20}{30} = \frac{AO}{AO + OP} = \frac{160}{160 + OP}$$

$$20(160 + OP) = 30 \cdot 160$$

$$20 \cdot OP = 10 \cdot 160$$

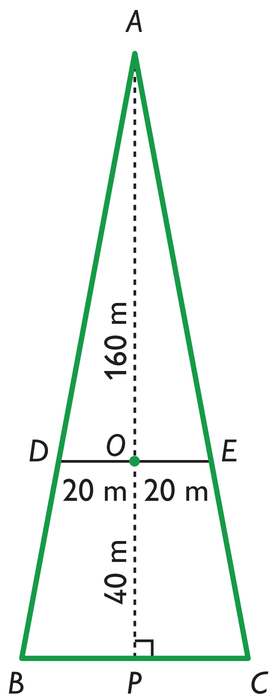
$$OP = 80$$

The road deck is 80 m above the water.

LESSON 11: USING SIMILAR TRIANGLES ANSWERS

Challenge Problem

8.G.5 **10.** The pylon in the new design can also be represented as a triangle.



In this model, $\Delta AOE \sim \Delta APC$ because the triangles have two pairs of congruent angles. You can use this relationship to determine *PC*.

$$\frac{AP}{AO} = \frac{PC}{OE}$$

$$\frac{200}{160} = \frac{PC}{20}$$

$$4{,}000 = 160 \cdot PC$$

$$PC = 25$$

The width of the base is *BP* + *PC*. Since this triangle was constructed to be a reflection across the perpendicular bisector, *BP* = *PC* and the base is equal to 50 m.

The pylon's base is 50 m.

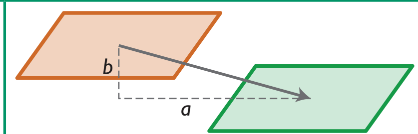
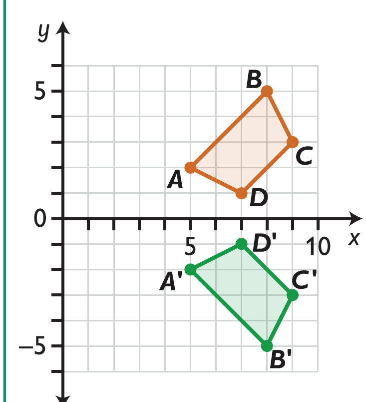
LESSON 12: PUTTING IT TOGETHER

ANSWERS

ANSWERS

8.G.1 8.G.2, 8.G.3, 8.G.4

2. Definitions and examples will vary. Here are some examples.

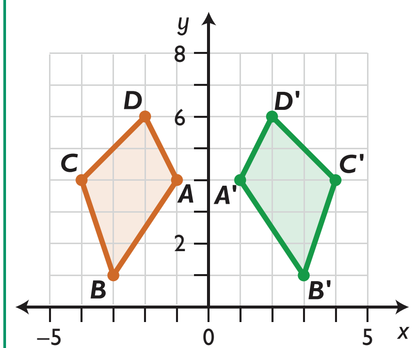
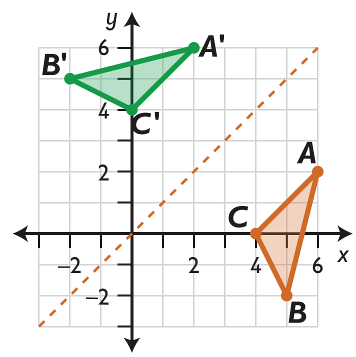
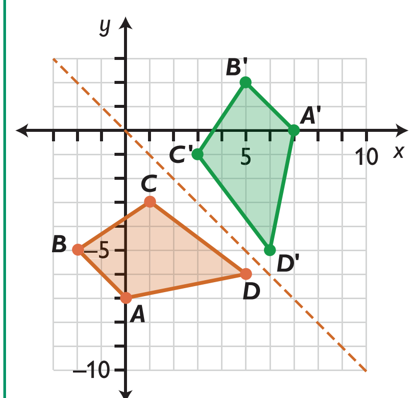
Word or Phrase	Definition	Examples
translation	a transformation that involves the movement of a figure from one location to another in such a way that every point of the figure moves in the same direction and over the same distance, without any rotation, reflection, or change in size A translation of a units horizontally and b units vertically results in this coordinate change of any given point of the figure: $(x, y) \rightarrow (x + a, y + b)$. Translations do not affect the side lengths, angle measures, or parallelism of lines of the figure. The translated figure is congruent to its original.	
reflection	a transformation that involves the inversion of a figure across a line, called a line of reflection, in such a way that every point of the figure has transferred perpendicularly across the line to a point the same distance away from the line on the other side of the line A reflection results in these coordinate changes of any given point of the figure. • Vertical reflection across the x-axis ($y = 0$): $(x, y) \rightarrow (x, -y)$	Vertical reflection across x-axis: 

LESSON 12: PUTTING IT TOGETHER

ANSWERS

8.G.1
8.G.2,
8.G.3,
8.G.4

2.

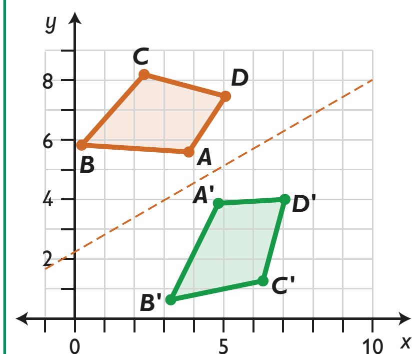
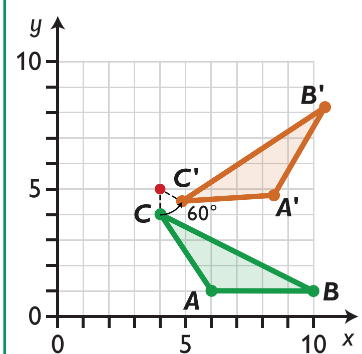
Word or Phrase	Definition	Examples
reflection	• Horizontal reflection across the y-axis ($x = 0$): $(x, y) \rightarrow (-x, y)$ • Reflection across the line $y = x$: $(x, y) \rightarrow (y, x)$ • Reflection across the line $y = -x$: $(x, y) \rightarrow (-y, -x)$ Reflections do not affect the side lengths, angle measures, or parallelism of lines of the figure. However, reflections invert the orientation of the vertices so that it is no longer possible to make the figures overlap unless another reflection is applied. The reflected figure is congruent to its original.	Horizontal reflection across y-axis:  Reflection across the line $y = x$:  Reflection across the line $y = -x$: 

LESSON 12: PUTTING IT TOGETHER

ANSWERS

8.G.1
8.G.2,
8.G.3,
8.G.4

2.

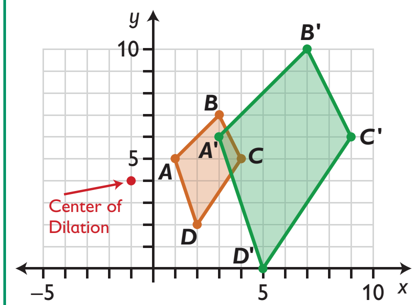
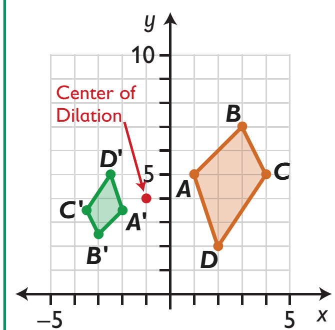
Word or Phrase	Definition	Examples
reflection		Reflection across the line $y = ax + b$: 
rotation	a transformation that makes a figure turn according to a given angle around a fixed center point called the center of rotation By convention, positive angle rotations make the figure move counterclockwise around the center of rotation and negative rotations make the figure rotate clockwise. A rotation results in these coordinate changes of any given point of the figure. • –90° rotation around (0, 0): $(x, y) \rightarrow (y, -x)$ • 90° rotation around (0, 0): $(x, y) \rightarrow (-y, x)$ • 180° rotation around (0, 0): $(x, y) \rightarrow (-x, -y)$ Rotations do not affect the side lengths, angle measures, or parallelism of lines of the figure. The rotated figure is congruent to its original, yet it can no longer overlap the original figure unless transformed again.	Rotation of 60° around the point (4, 5): 

LESSON 12: PUTTING IT TOGETHER

ANSWERS

8.G.1
8.G.2,
8.G.3,
8.G.4

2.

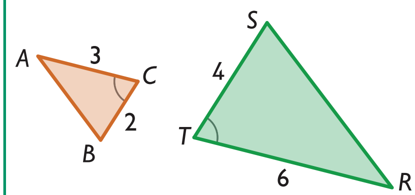
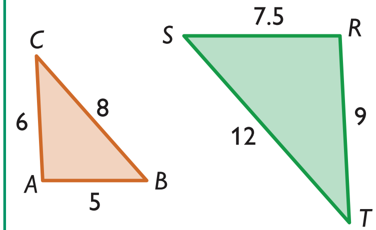
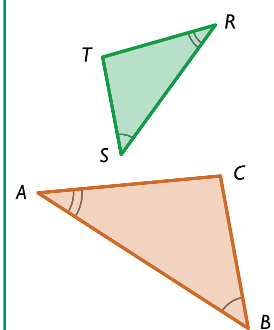
Word or Phrase	Definition	Examples
dilation	a transformation that results in a figure that is the same shape as the original, but that is a different size (The dilated figure is stretched or shrunk compared to the original figure.) A dilation includes a scale factor *k* and a center of the dilation. • $-1 < k < 1$: the figure is smaller than the original • $k = -1$ or $k = 1$: the figure is congruent with the original • $k < -1$ or $k > 1$: the figure is larger than the original • $k < 0$: the figure is rotated 180° around the center of dilation A dilation with a scale factor *k* around (0, 0) results in this coordinate change of any given point of the figure: $(x, y) \rightarrow (kx, ky)$. Dilations do not affect angle measures or parallelism of lines of the figure. However, when the scale factor is different from –1 or 1, dilations do alter the side lengths of the figure. Under such conditions, the dilated figure is similar to, but not congruent with, the original. For a dilation to be congruent, the scale factor needs to be either –1 or 1.	$k = 2$  $k = -0.5$ 

LESSON 12: PUTTING IT TOGETHER

ANSWERS

8.G.1
8.G.2,
8.G.3,
8.G.4

2.

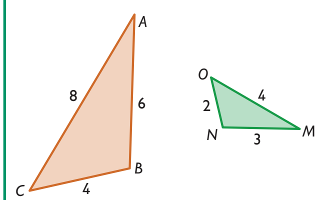
Word or Phrase	**Definition**	**Examples**
triangle similarity	Triangles are similar when their angles are congruent and their side lengths are proportionally scaled by the same factor k (ratio of the sides). $a : k(a), b : k(b), c : k(c)$ The triangle with side lengths a, b, and c is similar to the triangle with side lengths $k(a)$, $k(b)$, and $k(c)$. • **SAS Similarity** (side-angle-side): If the ratios of the lengths of two pairs of sides are equal and the included angles are equal, then the triangles are similar. • **SSS Similarity** (side-side-side): If the ratios of the lengths of the three pairs of sides are equal, then the triangles are similar. • **AA Similarity** (angle-angle): If the corresponding angles are equal, then the triangles are similar. Since the sum of the three angle measures of a triangle is always equal to 180°, if two pairs of angle measures are equal, then the third pair will also be equal. As a result, the minimum requirement for similarity with angles is that two pairs of corresponding angles must be equal.	$\Delta ABC \sim \Delta RST$: SAS similarity  $\Delta ABC \sim \Delta RST$: SSS similarity  $\Delta ABC \sim \Delta RST$: AA similarity 

LESSON 12: PUTTING IT TOGETHER

ANSWERS

8.G.1
8.G.2,
8.G.3,
8.G.4

2.

Word or Phrase	Definition	Examples
triangle congruence	Similar triangles are congruent when the ratio of the sides (scale factor) equals 1. Both SAS and SSS Similarity Theorems can be extended to verify whether two triangles are congruent in addition to being similar. The AA Similarity Theorem cannot be used to confirm congruency because it does not use side lengths, and therefore you cannot confirm the value of k.	ΔABC is similar but not congruent to ΔMNO: $k = \frac{AB}{MN} = \frac{BC}{NO} = \frac{CA}{OM} = 2$  ΔABC is similar and congruent to ΔRST: $k = \frac{AB}{MN} = \frac{BC}{NO} = \frac{CA}{OM} = 1$