MATH GRADE 8 UNIT 6

TRIANGLES AND BEYOND

ANSWERS FOR EXERCISES

PEARSON

LESSON 2: TRANSVERSALS

ANSWERS

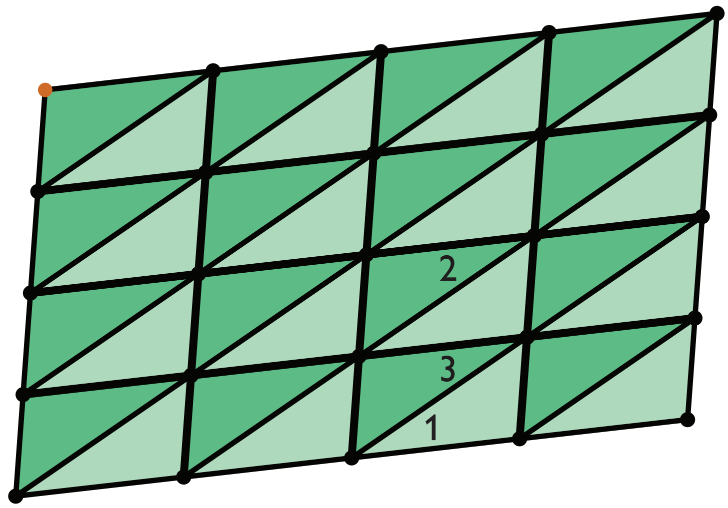
ANSWERS

1. Ⓓ 120°

   Angle 3 and angle 5 are co-interior angles and are supplementary; so, the measure of angle 5 is 120° (60° + 120° = 180°). Angle 5 and angle 8 are vertical angles; so the measure of angle 8 is 120°.

2. Ⓐ 28°

   If a third angle is named, it can be shown that angle 1 and angle 2 are congruent, given that there are parallel lines in the tessellation.



   Angle 1 and angle 3 are alternate interior angles and are congruent. Angle 3 and angle 2 are corresponding angles and are congruent. Therefore angle 1 and angle 2 are congruent, and both measure 28°.

3. Ⓒ A pair of co-interior angles that measure 75° and 105°

   Any two lines that intersect will have adjacent supplementary angles and vertical angles, which tell nothing about whether the lines are parallel. Alternate interior angles are congruent but not supplementary. Co-interior angles are supplementary; so, 75° + 105° = 180°, which means the lines are parallel.

LESSON 2: TRANSVERSALS

ANSWERS

4. Three of the other angles will measure 124°, and four will measure 56°. Two parallel lines that cross a transversal create eight angles. One intersection has two pairs of vertical angles, and the other intersection has the same four angles. So, there are two sets of four congruent angles. At the same two intersections, the two different angles are supplementary to each other. So, one set of congruent angles will be supplementary to the other set of congruent angles. Four angles measure 124°, and the other four measure 56° (124° + 56° = 180°).

5. The measure of angle 2 is 126°, the measure of angle 4 is 126°, and the measure of angle 3 is 54°.

   You learned the relationships of the angles in a parallelogram in seventh grade; in this case, think of the opposite lines as parallel lines (remember, opposite sides of a parallelogram are parallel).

   If you think of the top side of the parallelogram as a transversal, then angle 1 and angle 2 are co-interior angles and are supplementary. Likewise, angle 1 and angle 4 are same-side interior angles, with the left side being the transversal. So, angles 2 and 4 each measure 126° (54° + 126° = 180°).

   Angle 4 and angle 3 are also co-interior angles, with the bottom side as the transversal. So angle 3 measures 54°.

Challenge Problem

6. The measure of angle 1 is 36°, the measure of angle 2 is 71°, and the measure of angle 3 is 73°.

   Angle 2 measures 71° because it is an alternate interior angle with the other 71° angle, with the diagonal as the transversal.

   The 73° angle and the sum of the measures of angle 1 and angle 2 are supplementary because they are same-side interior angles. So, the measure of angle 1 is 36° (73° + 71° + 36° = 180°). The measure of angle 3 is 73° because it is a same-side interior angle with the sum of angle 1 and angle 2.

LESSON 3: TRIANGLE ANGLES

ANSWERS

ANSWERS

1. (C) 110°

   The sum of the angle measures must be 180°: 45° + 25° + 110° = 180°.

2. (A) 70° and 70°

   (D) 40° and 100°

   In an isosceles triangle, two of the angles have the same measure. One angle measure is known (40°), and the sum of the angle measures is 180°:

   70° + 70° + 40° = 180°

   40° + 40° + 100° = 180°

3. (A) 48° and 84°

   (C) 90° and 42°

   The sum of the angle measures must be 180°, including a 48° angle:
   48° + 48° + 84° = 180°
   48° + 90° + 42° = 180°

4. 50° and 70°

   If an exterior angle measures 130°, the interior angle adjacent to it is 50°, because they are supplementary. One interior angle is 60°; so the third angle must be 70° (50° + 60° + 70° = 180°). Notice that the two interior angles not adjacent to the 30° exterior angle add to 130° (60° + 70° = 130°).

5. 90° and 53°

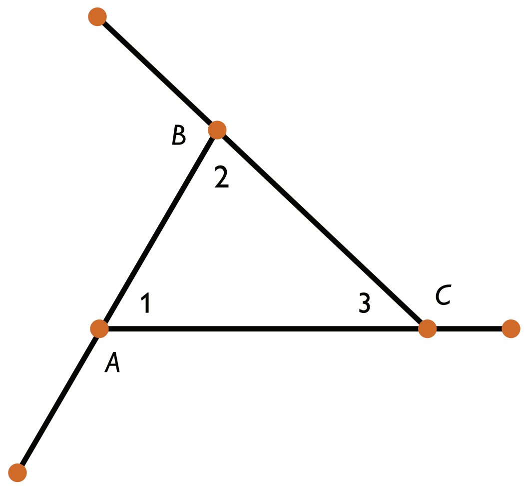
   Since the triangle is a right triangle, one angle must measure 90°. The angle sum must be 180°: 180° – (37° + 90°) = 180° – 127° = 53°.
   Notice that if one angle measures 90°, the sum of the other two angles will also be 90° (37° + 53° = 90°), because 90° + 90° = 180°.

LESSON 3: TRIANGLE ANGLES

ANSWERS

Challenge Problem

6. 360°



Angles A, B, and C are the exterior angles. Angles 1, 2, and 3 are the interior angles.

$\angle A = 180° - \angle 1$
$\angle B = 180° - \angle 2$
$\angle C = 180° - \angle 3$
$\angle 1 + \angle 2 + \angle 3 = 180°$

$$\begin{aligned}\angle A + \angle B + \angle C &= (180° - \angle 1) + (180° - \angle 2) + (180° - \angle 3)\\ &= (180° + 180° + 180°) - (\angle 1 + \angle 2 + \angle 3)\\ &= 180° + 180° + 180° - 180°\\ &= 360°\end{aligned}$$

LESSON 4: THE PYTHAGOREAN THEOREM ANSWERS

ANSWERS

1. C 13 in.

   $a^2 + b^2 = c^2$
   $(5 \text{ in.})^2 + (12 \text{ in.})^2 = c^2$
   $25 \text{ in}^2 + 144 \text{ in}^2 = c^2$
   $169 \text{ in}^2 = c^2$
   $\sqrt{169 \text{ in}^2} = \sqrt{c^2}$
   $13 \text{ in.} = c$

2. A 6 in., 8 in., 10 in.

   C 8 ft, 15 ft, 17 ft

   $a^2 + b^2 = c^2$
   $(6 \text{ in.})^2 + (8 \text{ in.})^2 = (10 \text{ in.})^2$
   $36 \text{ in}^2 + 64 \text{ in}^2 = 100 \text{ in}^2$
   $100 \text{ in}^2 = 100 \text{ in}^2$
   $\sqrt{100 \text{ in}^2} = \sqrt{c^2}$
   $10 \text{ in.} = c$

   $a^2 + b^2 = c^2$
   $(8 \text{ ft})^2 + (15 \text{ ft})^2 = (17 \text{ ft})^2$
   $64 \text{ ft}^2 + 225 \text{ ft}^2 = 289 \text{ ft}^2$
   $289 \text{ ft}^2 = 289 \text{ ft}^2$
   $\sqrt{289 \text{ ft}^2} = \sqrt{c^2}$
   $17 \text{ ft} = c$

   The other two do not work:
   $a^2 + b^2 = c^2$
   $(3 \text{ cm})^2 + (5 \text{ cm})^2 = (6 \text{ cm})^2$
   $9 \text{ cm}^2 + 25 \text{ cm}^2 = 36 \text{ cm}^2$
   $34 \text{ cm}^2 \neq 36 \text{ cm}^2$

   $a^2 + b^2 = c^2$
   $(9 \text{ km})^2 + (9 \text{ km})^2 = (13 \text{ km})^2$
   $81 \text{ km}^2 + 81 \text{ km}^2 = 169 \text{ km}^2$
   $162 \text{ km}^2 \neq 169 \text{ km}^2$

3. Marshall's fence should be about 65 ft long.

   $a^2 + b^2 = c^2$
   $(18 \text{ ft})^2 + (20 \text{ ft})^2 = c^2$
   $324 \text{ ft}^2 + 400 \text{ ft}^2 = c^2$
   $724 \text{ ft}^2 = c^2$
   $\sqrt{724 \text{ ft}^2} = \sqrt{c^2}$
   $c \approx 26.9$ ft, or about 27 ft

   The fence's length is the sum of the three side lengths:
   $18 \text{ ft} + 20 \text{ ft} + 27 \text{ ft} \approx 65 \text{ ft}$

LESSON 4: THE PYTHAGOREAN THEOREM

ANSWERS

4. $\sqrt{3}$ ft, or about 1.7 ft

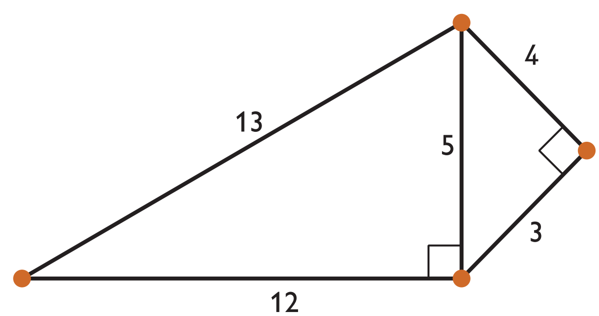
   The height divides the base in half; so the right triangle has a base of 1 ft and a hypotenuse of 2 ft.
   $a^2 + b^2 = c^2$
   $(1 \text{ ft})^2 + b^2 = (2 \text{ ft})^2$
   $1 \text{ ft}^2 + b^2 = 4 \text{ ft}^2$
   $b^2 = 3 \text{ ft}^2$
   $\sqrt{b^2} = \sqrt{3\text{ft}^2}$
   $b = \sqrt{3}$ ft, or about 1.7 ft

5. $4^2 + 11^2 = c^2$
   $16 + 121 = c^2$
   $187 = c^2$
   $c = \sqrt{187} \approx 13.67$

   The length of the hypotenuse is about 13.67 cm.

Challenge Problem

6. $A = 5$, $B = 4$



LESSON 5: APPLYING THE THEOREM

ANSWERS

ANSWERS

1. (A) 41 in.

   $a^2 + b^2 = c^2$
   $(9 \text{ in.})^2 + (40 \text{ in.})^2 = c^2$
   $81 \text{ in}^2 + 1{,}600 \text{ in}^2 = c^2$
   $1{,}681 \text{ in}^2 = c^2$
   $\sqrt{1{,}681 \text{ in}^2} = \sqrt{c^2}$
   $41 \text{ in.} = c$

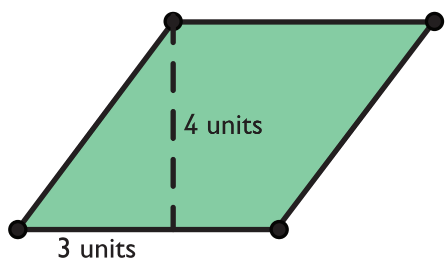
2. (C) 60 cm

   $a^2 + b^2 = c^2$
   $(11 \text{ cm})^2 + b^2 = (61 \text{ cm})^2$
   $121 \text{ cm}^2 + b^2 = 3{,}721 \text{ cm}^2$
   $b^2 = 3{,}600 \text{ cm}^2$
   $\sqrt{b^2} = \sqrt{3{,}600 \text{ cm}^2}$
   $b = 60 \text{ cm}$

3. a. 26
   b. 50
   c. $\sqrt{149} \approx 12.2$

4. 20 units

   The top and bottom side of the parallelogram are both 5 units long. To find the side lengths, use the Pythagorean theorem. The parallelogram has a height of 4 units, and the base of the right triangle is 3 units.



   $a^2 + b^2 = c^2$
   $3^2 + 4^2 = c^2$
   $9 + 16 = c^2$
   $25 = c^2$
   $c = 5$ units
   So, all four sides are 5 units long (In fact, the figure is a rhombus.), so the perimeter is 20 units: 5 + 5 + 5 + 5 = 20 units.

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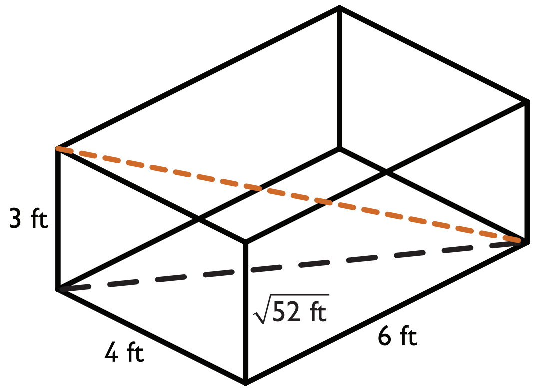
LESSON 5: APPLYING THE THEOREM

ANSWERS

5. About 7.8 ft

First, the diagonal of the base needs to be found in order to know the second side of the triangle that is being solved for:



$a^2 + b^2 = c^2$
$(4\text{ ft})^2 + (6\text{ ft})^2 = c^2$
$16\text{ ft}^2 + 36\text{ ft}^2 = c^2$
$52\text{ ft}^2 = c^2$
$\sqrt{52\text{ ft}^2} = \sqrt{c^2}$
$\sqrt{52}\text{ ft} = c$

Now the a and b sides are known for the hypotenuse to be solved for:
$a^2 + b^2 = c^2$
$(3\text{ ft})^2 + (\sqrt{52}\text{ ft})^2 = c^2$
$9\text{ ft}^2 + 52\text{ ft}^2 = c^2$
$61\text{ ft}^2 = c^2$
$\sqrt{61\text{ ft}^2} = \sqrt{c^2}$
$c \approx 7.8$ ft.

Challenge Problem

6. Four Pythagorean triples are:

(5, 12, 13)
(8, 15, 17)
(7, 24, 25)
(20, 21, 29)

There do exist formulas to generate a triple based on an arbitrary pair or positive integers, m and n, where $m > n$.

$a = m^2 - n^2,\ b = 2mn,\ c = m^2 + n^2,$

a, b, and c will form a Pythagorean triple.

LESSON 6: FINDING LENGTHS

ANSWERS

ANSWERS

1. D $\sqrt{244}$ units

   The difference between the x-coordinates is 10, (8 – (–2) = 10). The difference between the y-coordinates is 12, (5 – (–7) = 12).

   $a^2 + b^2 = c^2$
   $10^2 + 12^2 = c^2$
   $100 + 144 = c^2$
   $244 = c^2$
   $\sqrt{244} = \sqrt{c^2}$
   $c = \sqrt{244}$

2. C (2, 2), (8, –1)

   (2, 2), (8, –1)
   x: 2 – 8 = –6
   y: 2 – (–1) = 3

3. C $\sqrt{113}$ units

   D ≈ 10.6 units

   Using the coordinates, or just counting on the grid, the sides of the right triangle are 7 units and 8 units:

   $a^2 + b^2 = c^2$
   $7^2 + 8^2 = c^2$
   $49 + 64 = c^2$
   $113 = c^2$
   $\sqrt{113} = \sqrt{c^2}$
   $c = \sqrt{113}$ units, or about 10.6 units

   The square root answer is the exact answer, while the decimal answer is an approximate answer.

4. $80\sqrt{2}$ mi, or about 113 mi.

   There is a 1 : 1 ratio to the triangle. Using this ratio, the hypotenuse is $\sqrt{2}$:
   $a^2 + b^2 = c^2$
   $1^2 + 1^2 = c^2$
   $1 + 1 = c^2$
   $2 = c^2$
   $\sqrt{2} = \sqrt{c^2}$
   $c = \sqrt{2}$

(continues)

LESSON 6: FINDING LENGTHS

ANSWERS

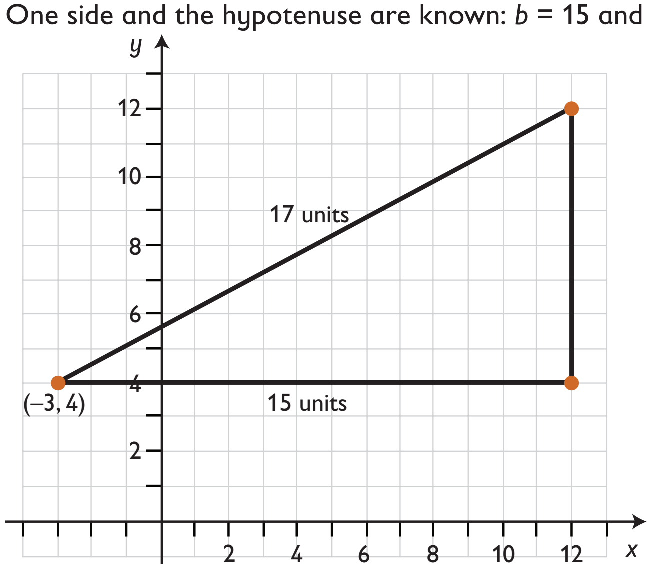
(continued)

Since the triangle is 80 times larger, the hypotenuse will be 80 times longer; $80\sqrt{2}$ mi, or about 113 mi.

The problem can also be solved using the numbers shown:
$a^2 + b^2 = c^2$
$(80 \text{ mi.})^2 + (80 \text{ mi.})^2 = c^2$
$6{,}400 \text{ mi}^2 + 6{,}400 \text{ mi}^2 = c^2$
$12{,}800 \text{ mi}^2 = c^2$
$\sqrt{12{,}800 \text{ mi}^2} = \sqrt{c^2}$
$c \approx 113$ mi $\qquad \left(\sqrt{12{,}800} = \sqrt{6{,}400} \cdot \sqrt{2} = 80\sqrt{2}\right)$

5. (12 12)

One side and the hypotenuse are known: $b = 15$ and $c = 17$:



$a^2 + b^2 = c^2$
$a^2 + 15^2 = 17^2$
$a^2 + 225 = 289$
$a^2 = 64$
$\sqrt{a^2} = \sqrt{64}$
$a = 8$ units

Now that the other side of the triangle is known, 15 can be added to the x-coordinate of the first endpoint, and 8 can be added to the y-coordinate of the first endpoint:
(–3, 4); –3 + 15 = 12, 4 + 8 = 12
So, the other endpoint is at (12, 12).

LESSON 6: FINDING LENGTHS

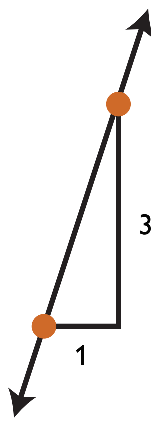
ANSWERS

Challenge Problem

6. (3, 9)

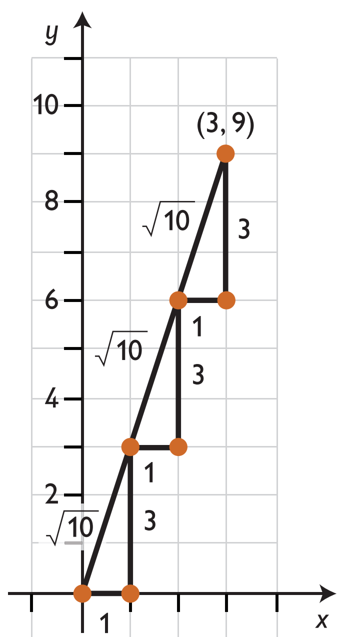
If the slope of the line segment is 3, the ratio of the sides is 3 : 1. One strategy would be to try combinations of this ratio until one works: 1 and 3, 2 and 6, 3 and 9, etc.

Another strategy is to look at the simplest case:



$a^2 + b^2 = c^2$
$1^2 + 3^2 = c^2$
$1 + 9 = c^2$
$10 = c^2$
$\sqrt{10} = \sqrt{c^2}$
$c = \sqrt{10}$ units

The line segment is $3\sqrt{10}$ units long, which is 3 times longer than the unit triangle; so the coordinates will be 3 times larger (since the first endpoint is at the origin), or 3 and 9: (3, 9).



Using the first method, 3 and 9 should work:

$a^2 + b^2 = c^2$
$3^2 + 9^2 = c^2$
$9 + 81 = c^2$
$90 = c^2$
$\sqrt{90} = \sqrt{c^2}$
$c = \sqrt{90} = \sqrt{9} \cdot \sqrt{10} = 3\sqrt{10}$ units

LESSON 7: CYLINDER VOLUME

ANSWERS

ANSWERS

1. D $\approx 1{,}130\text{ m}^3$

   $V = \pi r^2 h$
   $V = \pi(6\text{ m})^2(10\text{ m})$
   $V = \pi(36\text{ m}^2)10\text{ m}$
   $V = \pi(360\text{ m}^3)$
   $V \approx \pi(360\text{ m}^3) \approx 1{,}130.4\text{ m}^3 \approx 1{,}130\text{ m}^3$

2. A 5 m

   A guess-and-check strategy would work, trying each radius in the equation. However, the equation can also be used to solve for the radius:
   $V = \pi r^2 h$
   $785\text{ m}^3 = \pi r^2(10\text{ m})$
   $78.5\text{ m}^2 = \pi r^2$
   $\dfrac{78.5\text{m}^2}{\pi} = r^2$
   $25\text{ m}^2 = r^2$
   $r = 5\text{ m}$

3. C A cylinder: 6 cm wide, 5 cm in height

   The larger of the two prisms has a volume ($l \bullet w \bullet h$) of 125 cm^3. The other cylinder is smaller because the radius is smaller, and the 1 cm change in height is not enough to compensate.
   $V = \pi r^2 h$
   $V = \pi(3\text{ cm})^2(5\text{ cm})$
   $V = \pi(9\text{ cm}^2)5\text{ cm}$
   $V = \pi(45\text{ cm}^3)$
   $V \approx \pi(45\text{ cm}^3) \approx 141.3\text{ cm}^3 \approx 141\text{ cm}^3$

4. 186 in^3

   The volume of the prism is 6 in. • 12 in. • 12 in. = 864 in^3.

   The cake has a diameter of 12 in., so its radius is 6 in. Its height is also 6 in.
   $V = \pi r^2 h$
   $V = \pi(6\text{ in.})^2(6\text{ in.})$
   $V = \pi(36\text{ in}^2)6\text{ in.}$
   $V = \pi(216\text{ in}^3)$
   $V \approx \pi(216\text{ in}^3) \approx 678.24\text{ in}^3 \approx 678\text{ in}^3$

   The volume of the cake can be subtracted from the volume of the whole box to find the remaining volume:
   $864\text{ in}^3 - 678\text{ in}^3 = 186\text{ in}^3$

LESSON 7: CYLINDER VOLUME

ANSWERS

5. ≈ 17 mm^3

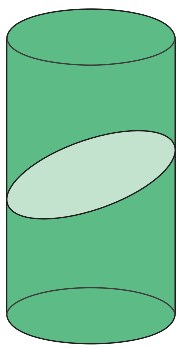
   $V = \pi r^2 h$
   $V = \pi(9\text{ mm})^2(50 \times 1.35\text{ mm})$
   $V = \pi(9\text{ mm})^2(67.5\text{ mm})$
   $V = \pi(81\text{ mm}^2)(67.5\text{ mm})$
   $V \approx \pi(5{,}467.5\text{ mm}^3) \approx 17{,}167.95\text{ mm}^3$

Challenge Problem

6. The volume of the figure would be a little more than 250 m^3.

   The trick in solving the problem is to realize that if the figure were doubled and the sliced surfaces matched, it would be a cylinder with a height of 10 m (6 m + 4 m).

   So, the volume will be half the volume of a cylinder with a height of 10 m and a radius of 4 m.



$V = \pi r^2 h$
$V = \pi(4\text{ m})^2(10\text{ m})$
$V = \pi(16\text{ m})^2(10\text{ m})$
$V = \pi(160\text{ m}^3)$
$V \approx \pi(160\text{ m}^3) \approx 502.4\text{ m}^3 \approx 502\text{ m}^3$

Then, divide by 2:
$502\text{ m}^3 \div 2 = 251\text{ m}^3$

So, the volume of the figure would be a little more than 250 m^3.

LESSON 8: CONE VOLUME

ANSWERS

ANSWERS

1. C $\approx 377\ \text{m}^3$

$V = \frac{1}{3}\pi r^2 h$

$V = \frac{1}{3}\pi(6\ \text{m})^2(10\ \text{m})$

$V = \frac{1}{3}\pi(36\ \text{m})^2(10\ \text{m})$

$V = \frac{1}{3}\pi(360\ \text{m}^3)$

$V = \pi(120\ \text{m}^3)$

$V \approx \pi \times 120\ \text{m}^3 \approx 377\ \text{m}^3$

2. D A cone: 8 cm diameter, 4 cm height

The volume of the cone (with a radius of 4 cm) will be a little more than the volume of the cube, which is 64 cm^3 (4 cm × 4 cm × 4cm = 64 cm^3), because $\frac{\pi}{3}$ is a little more than 1. The volume of the cylinder is about 63 cm^3, and the volume of the other cone is about 47 cm^3.

$V = \frac{1}{3}\pi r^2 h$

$V = \frac{1}{3}\pi(4\ \text{cm})^2(4\ \text{cm})$

$V = \frac{1}{3}\pi(16\ \text{cm}^2)(4\ \text{cm})$

$V = \frac{1}{3}\pi(64)\ \text{cm}^3$

$V = \pi\left(\frac{1}{3}\right)(64)\ \text{cm}^3$

$V \approx \pi(21.3)\ \text{cm}^3$

$V \approx \pi \times 21.3\ \text{cm}^3 \approx 67\ \text{cm}^3$

LESSON 8: CONE VOLUME

ANSWERS

3. Ⓒ 12 mm

A guess-and-check strategy would work, trying each height in the equation. However, the equation can also be used to solve for the height:

$V = \frac{1}{3}\pi r^2 h$

$314\text{ mm}^3 = \frac{1}{3}\pi(5\text{ mm})^2 h$

$314\text{ mm}^3 = \frac{1}{3}\pi(25\text{ mm}^2)h$

$942\text{ mm}^3 = \pi(25\text{ mm}^2)h$

$942\text{ mm}^3 \approx \pi(25\text{ mm}^2)h$

$300\text{ mm}^3 \approx (25\text{ mm}^2)h$

$12\text{ mm} \approx h$

4. 471 in^3

The volume of the cone is one-third the volume of the cylinder. So, with that volume removed, the remaining volume must be two-thirds the volume of the cylinder:

$V = \frac{2}{3}\pi r^2 h$

$V = \frac{2}{3}\pi(5\text{ in.})^2(9\text{ in.})$

$V = \frac{2}{3}\pi(25\text{ in}^2)9\text{ in.}$

$V = \frac{2}{3}\pi(225\text{ in}^3)$

$V = \pi(150\text{ in}^3)$

$V \approx \pi \times 150\text{ in}^3 = 471\text{ in}^3$

5. The larger cone will have eight times the volume.

When the formula is used to represent the smaller cone, the 2r and 2h replace h and r in the formula for the larger cone:

$V = \frac{1}{3}\pi r^2 h$

$V = \frac{1}{3}\pi(2r)^2(2h)$

$V = \frac{1}{3}\pi(4r^2)(2h)$

$V = \frac{1}{3}\pi 8r^2 h$

The equations are the same, except for the 8, so the second cone has eight times the volume.

LESSON 8: CONE VOLUME

ANSWERS

Challenge Problem

6. The cup will hold 356 ml.

The volume of the original cone can be found, and then the volume of the sliced-off cone can be subtracted:

Original cone:	Part sliced off:
$V = \frac{1}{3}\pi r^2 h$	$V = \frac{1}{3}\pi r^2 h$
$V = \frac{1}{3}\pi(5\text{ cm})^2(20\text{ cm})$	$V = \frac{1}{3}\pi(4\text{ cm})^2(10\text{ cm})$
$V = \frac{1}{3}\pi(25\text{ cm})^2(20\text{ cm})$	$V = \frac{1}{3}\pi(16\text{ cm})^2(10\text{ cm})$
$V = \frac{1}{3}\pi(500\text{ cm}^3)$	$V = \frac{1}{3}\pi(160\text{ cm}^3)$
$V \approx \pi(167\text{ cm}^3)$	$V \approx \pi(53\text{ cm}^3)$
$V \approx \pi \times (167\text{ cm}^3) \approx 523\text{ cm}^3$	$V \approx \pi \times (53\text{ cm}^3) \approx 167\text{ cm}^3$

$523\text{ cm}^3 - 167\text{ cm}^3 = 356\text{ cm}^3 = 356$ ml, which is about 12 oz.

LESSON 9: SPHERE VOLUME

ANSWERS

ANSWERS

1. (D) ≈ 904 m^3

   $V = \frac{4}{3}\pi r^3$
   $V = \frac{4}{3}\pi(6\text{ m})^3$
   $V = \frac{4}{3}\pi(216\text{ m}^3)$
   $V = \pi(288\text{ m}^3)$
   $V \approx \pi \times 288\text{ m}^3 \approx 904\text{ m}^3$

2. (A) A cube: 4 cm wide, 4 cm long, 4 cm in height

   The cube (64 cm^3) will be larger than the sphere because the sphere will fit inside of it. The cylinder will be smaller than the sphere because its height is half of the sphere's, or less than two-thirds its height. This leaves the hemisphere to check:
   $V = \left(\frac{1}{2}\right)\left(\frac{4}{3}\right)\pi r^3$
   $V = \frac{2}{3}\pi(3\text{ cm})^3$
   $V = \frac{2}{3}\pi(27\text{ cm}^3)$
   $V = \pi(18\text{ cm}^3)$
   $V \approx \pi \times (18\text{ cm}^3) \approx 56\text{ cm}^3 < 64\text{ cm}^3$

3. (A) 1.5 mm

   A guess-and-check strategy would work, trying each radius in the equation. However, the equation can also be used to solve for the radius:
   $\frac{9}{2}\pi\text{mm}^3 = \frac{4}{3}\pi r^3$
   $\frac{9}{2}\text{ mm}^3 = \frac{4}{3}r^3$
   $\frac{27}{8}\text{ mm}^3 = r^3$
   $r = \frac{3}{2} = 1.5$ [3 is the cube root of 27, and 2 is the cube root of 8.]

4. The larger sphere will have eight times the volume. If r is one radius, $2r$ will be the other radius. When the radius is cubed, it will be $8r^3$ ($2r \times 2r \times 2r = 8r^3$), while the first sphere will be r^3.

LESSON 9: SPHERE VOLUME

ANSWERS

$$\frac{\frac{4}{3}\pi 8r^3}{\frac{4}{3}\pi r^3} = \frac{8}{1}$$

5. About 236 cm^3

   One strategy is to find the volume of the cylinder, and then subtract the volume of the balls.

   However, one ball will take up two-thirds of the space in its part of the cylinder; so this ratio will be true for the whole can. In fact, the cylinder's volume can be found in terms of *r* because the can is three balls (6*r*) in height.

   Cylinder:
   $V = \pi r^2 h$
   $V = \pi r^2 6r$
   $V = 6\pi r^3$

   Spheres:
   $V = 3\left(\frac{4}{3}\right)\pi r^3$
   $V = 4\pi r^3$

   $$\frac{4\pi r^3}{6\pi r^3} = \frac{2}{3}$$

   So, the empty space is one-third of the cylinder's volume, or $2\pi r^3$.
   $2\pi r^3 = 2(\pi)(6.7\text{ cm} \div 2)^3$
   $\approx 2(\pi)(3.35\text{ cm})^3$
   $\approx 6.28 \times 37.6\text{ cm}^3 \approx 236\text{ cm}^3$

Challenge Problem

6. The volume of the cylinder will be equal to the volume of the cone plus the volume of the sphere (the height for the cone and the cylinder will be 2*r*).

   Cylinder:
   $V = \pi r^2 h$
   $V = \pi r^2(2r)$
   $V = 2\pi r^3$

   Cone:
   $V = \frac{1}{3}\pi r^2 h$
   $V = \frac{1}{3}\pi r^2(2r)$
   $V = \frac{2}{3}\pi r^3$

   Sphere:
   $V = \frac{4}{3}\pi r^3$

   $$\frac{2}{3}\pi r^3 + \frac{4}{3}\pi r^3 = \frac{6}{3}\pi r^3 = 2\pi r^3$$

LESSON 10: VOLUMES

ANSWERS

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1. **D** Change the height of the cone to 32 ft

The original figures have the following volumes:

Cone:	Sphere:
$V = \frac{1}{3}\pi 2^2(4)$	$V = \frac{4}{3}\pi(2)^3$
$V = 5.33\pi \text{ ft}^3$	$V = 10.66\pi \text{ ft}^3$

Here are the calculations for each potential change:

- Change sphere radius to 4 ft
  $V = \frac{4}{3}\pi(2)^3$
  $V = 85.33\pi \text{ ft}^3$
  Result: Sphere is 16 times larger than the cone.

- Change cone radius to 4 ft
  $V = \frac{1}{3}\pi(4)^2(4)$
  $V = 21.33\pi \text{ ft}^3$
  Result: Cone is twice the volume of the sphere

- Change height of cone to 16 ft
  $V = \frac{1}{3}\pi(2)^2(16)$
  $V = 21.33\pi \text{ ft}^3$
  Result: Cone is twice the volume of the sphere

- Change height of cone to 32 ft
  $V = \frac{1}{3}\pi(2)^2(32)$
  $V = 42.66\pi \text{ ft}^3$
  Result: Cone is four times the volume of the sphere.

2. **C** A cylinder: radius of 4 cm, 6 cm in height

The sphere and cone have the same dimensions, so the sphere has twice the volume of the cone. The cylinder has a height of 6 cm, or three-quarters of the sphere's height. Because $\frac{3}{4} > \frac{2}{3}$, the cylinder will have a larger volume than the sphere (and cone). The cube's volume is 216 cm^3 (6 cm × 6 cm × 6 cm = 216 cm^3), so the cylinder's volume needs to be greater than 216 cm^3. Use 3 for π to make the calculation quicker.

$V = \pi r^2 h$
$V = 3(4 \text{ cm})^2(6 \text{ cm})$
$V = 3(16 \text{ cm}^2)(6 \text{ cm})$
$V = 288 \text{ cm}^3 > 216 \text{ cm}^3$

LESSON 10: VOLUMES

ANSWERS

3. (B) A cone with $r = 8$ cm, $h = 32$ cm

   A cone with the same radius as a sphere has half the volume, so its height needs to be double: $16 \cdot 2 = 32$.

   Doubling the radius of the hemisphere does not double its volume.

4. $r = 12$ in., $h = 16$ in.

   If the sphere has a radius of 12 in., its diameter is 24 in. The sphere would be two-thirds as large as a cylinder with the same radius (12 in.) and height (24 in.). So, the cylinder's height needs to be two-thirds of the sphere's height, or 16 in. $\left(\frac{16}{24} = \frac{2}{3}\right)$. There could be other answers if the radius were adjusted carefully, or both the radius and height could change.

5. The cylinder has a larger volume.

   If the cone and cylinder were the same height, the cone would have one-third of the cylinder's volume. The cylinder is much shorter than the cone's 12-m height, but its height is still more than one-third of the cone's height $\left(\frac{5}{12} > \frac{1}{3}\right)$, so it will still have a greater volume.

Challenge Problem

6. 4π cubic units

   The volume of the cylinder is $\pi r^2 h$, or $\pi(1)^2(2) = 2\pi$ cubic units. The volume of the other two figures will add up to the cylinder's volume, so the volume will be $2 \times 2\pi$, or 4π cubic units.

   Cone:

   $V = \frac{1}{3}\pi r^2 h$

   $V = \frac{1}{3}\pi(1)^2(2)$

   $V = \frac{2}{3}\pi$ cubic units

   Sphere:

   $V = \frac{4}{3}\pi r^3$

   $V = \frac{4}{3}\pi(1)^3$

   $V = \frac{4}{3}\pi$ cubic units

   $\frac{2}{3}\pi + \frac{4}{3}\pi = \frac{6}{3}\pi = 2\pi$ cubic units