

LESSON 1: CHANGING TIDES

EXERCISES

EXERCISES

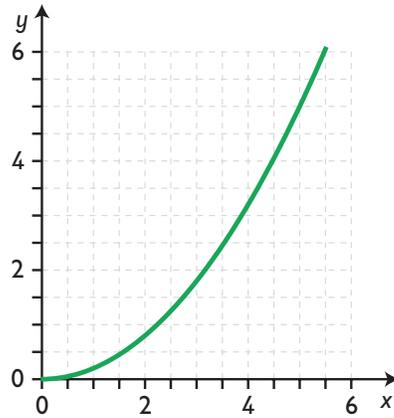
- Review your Unit Assessment from the previous unit.
- Write your wonderings about functions.
- Write a goal stating what you plan to accomplish in this unit.
- Based on your previous work, write three things you will do differently during this unit to increase your success.

LESSON 2: WHAT A GRAPH CAN TELL YOU

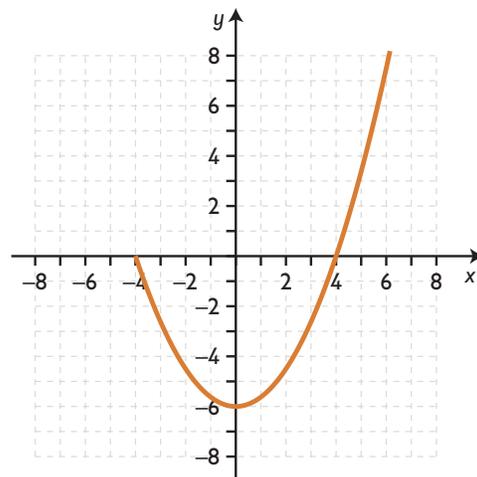
EXERCISES

EXERCISES

1. Which two qualities describe the following graph?



- A Increasing and nonlinear
 - B Decreasing and nonlinear
 - C Increasing and linear
 - D Decreasing and linear
2. On the graph shown for which values of x is the curve decreasing?

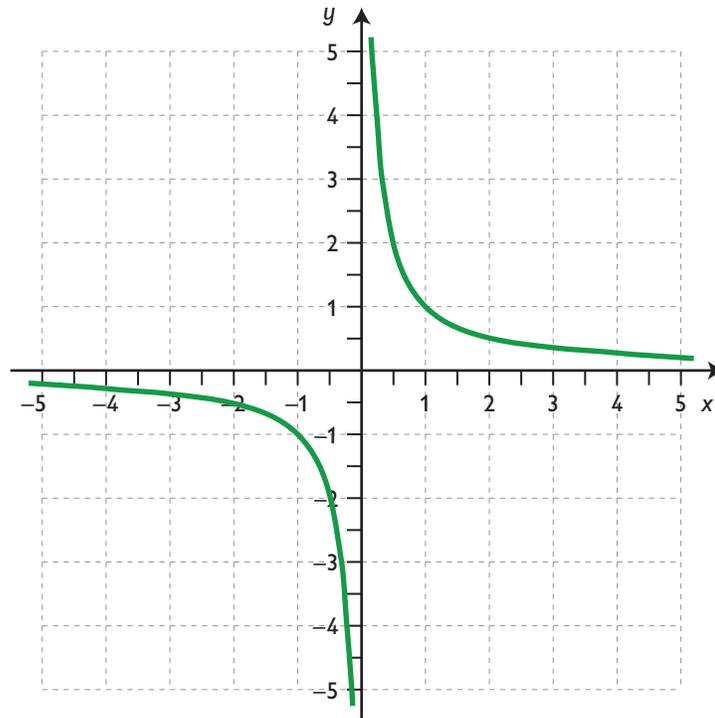


- A -6 to 6
- B -2 to 4
- C 0 to 6
- D -4 to 0

LESSON 2: WHAT A GRAPH CAN TELL YOU

EXERCISES

3. This is the graph of $y = \frac{1}{x}$.



- a. Complete the table using the equation or the graph.

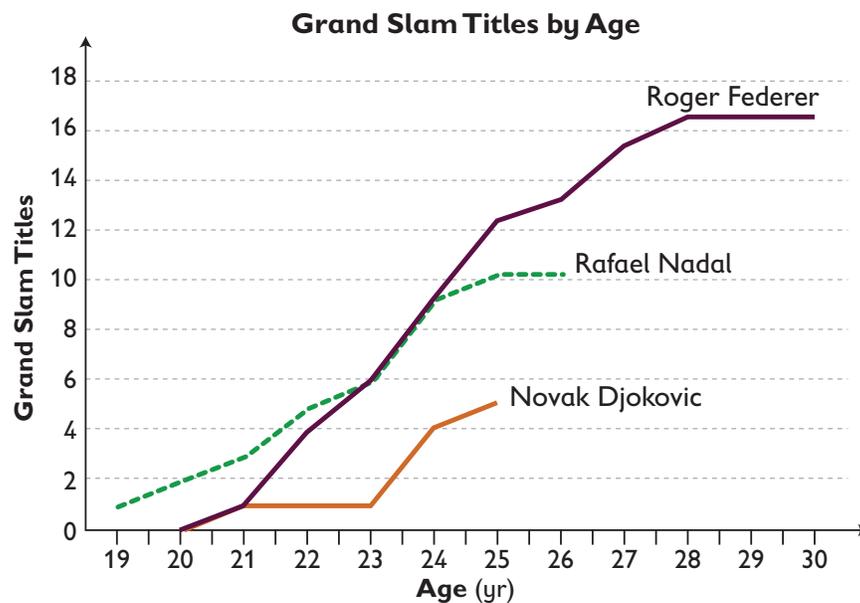
x (input)	-4	-3	-2	-1	0	1	2	3	4
y (output)		$-\frac{1}{3}$			Not defined	1			

- b. Describe the characteristics of this graph.

LESSON 2: WHAT A GRAPH CAN TELL YOU

EXERCISES

4. This graph shows the ages and the number of Grand Slam tennis titles won by Novak Djokovic, Rafael Nadal, and Roger Federer as of January 30, 2012. (The four Grand Slam tournaments are the Australian Open, the French Open, Wimbledon, and the U.S. Open.)



- Based on the graph, who do you think was the best tennis player at the time? Use the graph to support your reasoning.
- An article published along with the graph states that Nadal's Grand Slam pace is "falling behind" Federer's pace. Explain this statement.
- Explain the logic behind the fact that all three lines are increasing or are remaining constant.

Challenge Problem

- Research how many Grand Slam titles Novak Djokovic, Rafael Nadal, and Roger Federer have won since January 30, 2012.
 - Find the age of each player when he won each tournament. Then complete the graph from problem 4.
 - Based on your new graph, who do you think is the best tennis player now?

LESSON 3: LINEAR VERSUS NONLINEAR GRAPHS

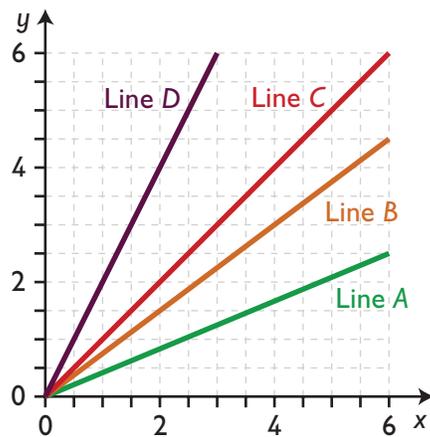
EXERCISES

EXERCISES

1. Which set of points makes a linear graph?

- A (1, 3), (2, 0), (3, 3)
- B (1, 3), (2, 6), (3, 9)
- C (1, 3), (2, 9), (3, 27)
- D (1, 3), (2, 5), (3, 4)

2. Which line has the greatest rate of change?

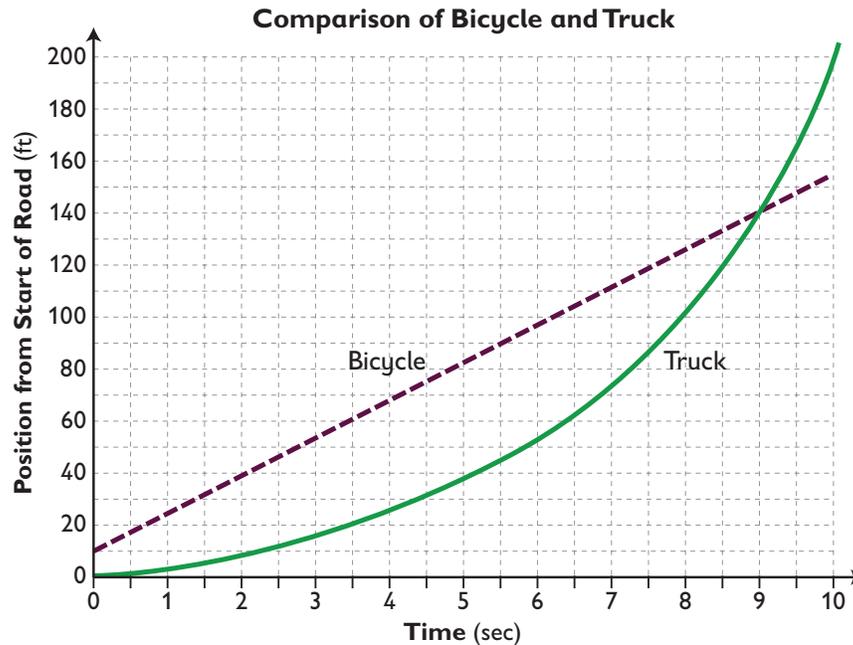


- A Line A
- B Line B
- C Line C
- D Line D

LESSON 3: LINEAR VERSUS NONLINEAR GRAPHS

EXERCISES

3. A bicycle and a truck are moving along a road in the same direction. The graph shows their positions as a function of time.



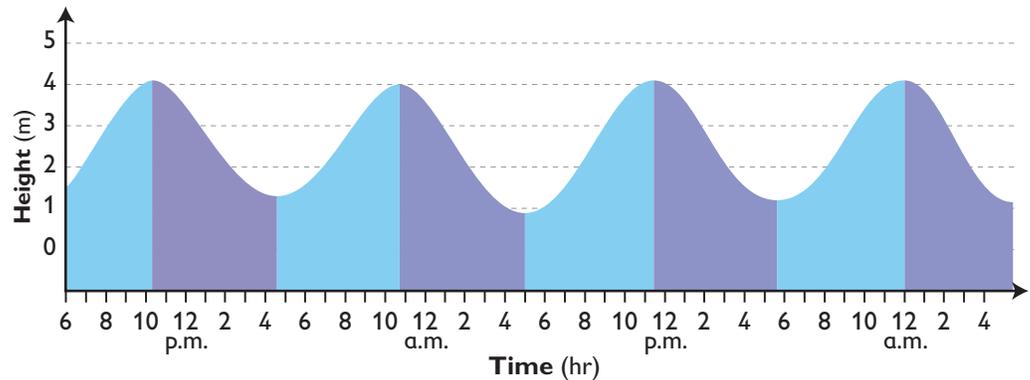
- After how many seconds does the truck pass the bicycle?
 - What is the speed of the bicycle?
 - Another bicycle travels at a speed of 10 mph. The second bicycle starts in the same position as the first. Which bicycle goes faster?
 - A third bicycle travels according to the following formula, with x representing time in seconds and y the position from the start of the road in feet:
 $y = 10x + 20$. Compare this third bike with the other two.
 - When is the truck going roughly the same speed as the first bike?
4. Consider the equation $y = ax + 4$. Another line has an x -intercept of 4 and a y -intercept of 6. If the two lines are parallel, what must be the value of a ?

LESSON 3: LINEAR VERSUS NONLINEAR GRAPHS

EXERCISES

Challenge Problem

5. Look at the tide graph shown.



- What is the approximate height of high tide (maximum) and of low tide (minimum)?
- About how many hours are there between two maxima?
- The tides at a second location farther along the coast are related to this tide—the wave of high tide reaches that location 4 hr later. The maximum is 3 m and the minimum is 2 m. The time between high tides (a period) remains the same. Draw the tide graph for this second location.

LESSON 4: WHAT IS A FUNCTION?

EXERCISES

EXERCISES

- Which set of ordered pairs could not represent a function?

A (1, 2), (2, 3), (3, 4) B (2, 5), (3, 10), (4, 10)
 C (1, 3), (1, 4), (1, 5) D (3, 4), (2, 1), (1, 0)
- Suppose you have a function f that takes a number for the input, triples it, then adds 4. What would $f(6)$ be?

A 12 B 24 C 28 D 22
- You can measure temperature in degrees Celsius as well as in degrees Fahrenheit. There are input-output machines on the Internet that can convert one temperature into the other.

Temperature Conversion

Convert what quantity?

<p>From:</p> <div style="border: 1px solid #ccc; padding: 2px; margin-bottom: 5px;">degree Celsius</div> <div style="border: 1px solid #ccc; padding: 2px; margin-bottom: 5px;">degree Fahrenheit</div> <div style="border: 1px solid #ccc; padding: 2px; margin-bottom: 5px;">degree Rankine</div> <div style="border: 1px solid #ccc; padding: 2px; margin-bottom: 5px;">degree Reaumur</div> <div style="border: 1px solid #ccc; padding: 2px;">kelvin</div>	<p>To:</p> <div style="border: 1px solid #ccc; padding: 2px; margin-bottom: 5px;">degree Celsius</div> <div style="border: 1px solid #ccc; padding: 2px; margin-bottom: 5px;">degree Fahrenheit</div> <div style="border: 1px solid #ccc; padding: 2px; margin-bottom: 5px;">degree Rankine</div> <div style="border: 1px solid #ccc; padding: 2px; margin-bottom: 5px;">degree Reaumur</div> <div style="border: 1px solid #ccc; padding: 2px;">kelvin</div>
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Convert

30 degrees Celsius = 86 degrees Fahrenheit

An input-output machine with the temperature in Celsius as the input and the temperature in Fahrenheit as the output uses the following formula, where f is the temperature in degrees Fahrenheit and c is the temperature in degrees Celsius:

$$f = \frac{9}{5}c + 32$$

- Check that 30°C is the same as 86°F , or $f(30) = 86$.
- Is $f = \frac{9}{5}c + 32$ a function? Explain.
- Draw the graph of $f = \frac{9}{5}c + 32$. (c is the independent variable; f is the dependent variable.)
- What is the constant rate of change and what does it mean in this context?
- Does it make sense to connect the dots in this graph?

LESSON 4: WHAT IS A FUNCTION?

EXERCISES

4. The temperature conversion machine can also work in the other direction: You can compute the temperature in Celsius starting with the temperature in Fahrenheit. Find this formula and check your answer with this calculator result: $100^{\circ}\text{F} = 37.\bar{7}^{\circ}\text{C}$.

Challenge Problem

5. Another interesting temperature scale is Kelvin. Try to find out the relationship among Celsius, Fahrenheit, and Kelvin temperatures.

LESSON 5: CHOOSING A FUNCTIONS PROJECT

EXERCISES

- Based on what your classmates shared about their projects, revise your project proposal.
- Make a plan for how you will complete your project in 2 weeks.
- If you have a partner or are working with a group, make a list of assignments for each person.
- Complete any exercises that you have not finished from this unit.

LESSON 6: WHAT IS A LINEAR FUNCTION?

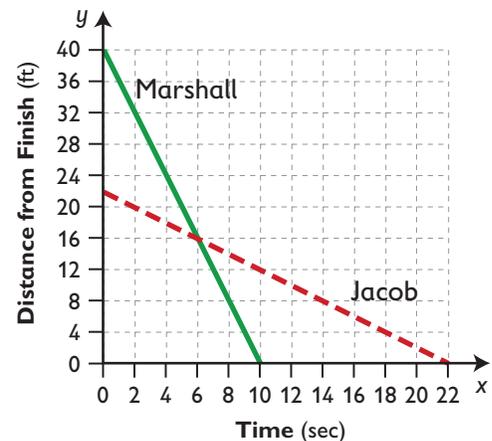
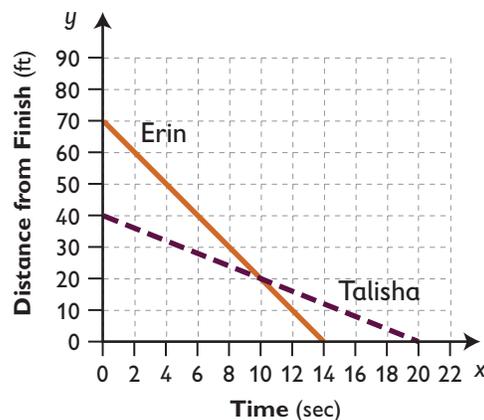
EXERCISES

EXERCISES

1. What is the slope of the linear function represented by these points?

x	0	1	2	3
y	3	7	11	15

- (A) 2 (B) 4 (C) -2 (D) -4
2. Which linear function is parallel to $y = 3x - 6$?
- (A) $y = 6x - 6$ (B) $y = -3x + 6$
 (C) $y = 3x + 18$ (D) $y = -6x + 12$
3. Look at the two graphs. They represent a fun run for two girls and two boys.



- Talisha is how far ahead of Erin at $t = 0$?
- Tell the story of the girls' graph.
- Tell the story of the boys' graph.
- Are the boys faster than the girls?
- Find the formulas for all four lines, with time (t) as the independent variable and the distance (d) from the finish as the dependent variable.
- Compute the coordinates of the points of intersection for each graph using the equations of the lines.

LESSON 6: WHAT IS A LINEAR FUNCTION?

EXERCISES

Challenge Problem

4. In order to regulate taxicab rates a city has a set of rules that all taxi companies must follow.

The maximum rate to be charged for taxicab service shall be as follows:

- For the first two (2) miles, or fraction thereof, four dollars and no cents (\$4.00); for each succeeding one-fifth ($\frac{1}{5}$) mile or fraction thereof, thirty-five cents (\$0.35).
 - A one-dollar (\$1.00) surcharge may be charged per passenger above the first passenger.
- a. Draw a graph of the taxi fares that follow the maximum rate. The independent variable is distance. The dependent variable is price in dollars. Make a graph for one passenger.
- b. Explain how the graph changes if you take three friends with you.

LESSON 7: COMPARING LINEAR FUNCTIONS

EXERCISES

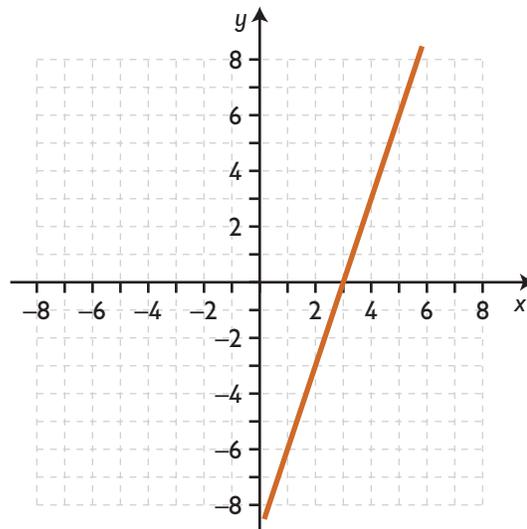
EXERCISES

1. Which linear relationship has the greatest rate of change?

A	x	0	1	2	3
	y	0	3	6	9

- B** $y = 4x - 18$
C $y = -2x + 3$
D A fraction with a rise-to-run ratio of 18:6.

2. Does this graph represent the function $y = 3x + 3$?



- A** Yes, because the slope and intercepts are the same.
B No, because the slope and intercepts are different.
C No, because the slope is the same and the intercepts are different.
D No, because the slope is different and the intercepts are the same.

LESSON 7: COMPARING LINEAR FUNCTIONS

EXERCISES

3. Tickets for a large rock festival can be bought by the day or for the whole three-day festival. The tickets are \$80 per day or \$200 for three days.

- a. Complete the table for day tickets.

Day Tickets						
Number of Tickets	20,000	25,000	30,000	35,000	40,000	45,000
Income (\$)	1,600,000					

- b. Give the equation for this relationship.
 c. Draw the graph. The independent variable is the number of tickets and the dependent variable is income.
 d. Complete the table for full festival tickets.

Full Festival Tickets						
Number of Tickets	20,000	25,000	30,000	35,000	40,000	45,000
Income (\$)		5,000,000				

- e. Give the equation for this relationship.
 f. Draw the graph in the same coordinate system as the previous graph.
 g. What is the constant rate of change (including units) in each graph?

Challenge Problem

4. Notice that if you sell a total of 45,000 day tickets for the three-day festival you will probably have 15,000 people per day. Maximum festival capacity is 50,000 people. What will be the best mix of tickets to assure a good profit, assuming that no more than 60,000 day tickets will be sold in total? What will be the total earnings from ticket sales for the three-day festival?

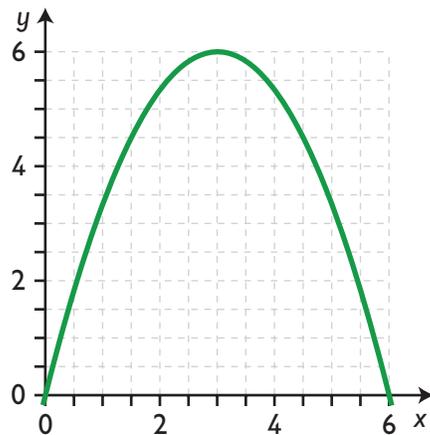
LESSON 8: GRAPHS AND FORMULAS

EXERCISES

EXERCISES

1. A line goes through the point $(4, 6)$ and has a y -intercept of 2. What is the slope?
 Ⓐ 1 Ⓑ 2 Ⓒ 3 Ⓓ 4

2. Which statement about the following graph is not true?



- Ⓐ The graph decreases from $x = 0$ to $x = 6$.
 Ⓑ The graph increases from $x = 0$ to $x = 3$.
 Ⓒ The graph decreases from $x = 3$ to $x = 6$.
 Ⓓ The maximum of the graph is $y = 6$.
3. At $x = 0$ the value of a linear function is 12. For $x = 4$ the value is 9.
- When will this value be 0?
 - What is the equation for this relationship?
4. $f(x) = 6x - 11$
- This function has a domain of $[0, 10]$. What is the range?
 - Show that the point $(6, 26)$ does not fit the line.
 - $g(x)$ is parallel to $f(x)$ and goes to $(6, 26)$. Give the equation of $g(x)$.
5. A large tank is being drained. At 6 a.m. the height in the water tank is 12 ft. By 8 a.m. the height of the water has dropped to 9 ft. Predict the time when the tank will be empty by first constructing the equation for this linear relationship.

LESSON 8: GRAPHS AND FORMULAS

EXERCISES

Challenge Problem

6. Dutch cheese, such as Gouda and Edam, is well known in the United States. In 1995 some 700,000,000 kg of Dutch cheese was produced. The growth rate of cheese production was around 36,000,000 kg/yr.
- Find a linear equation fitting this situation.
 - What would the production be in 2013 if this model still fits?

LESSON 9: MODEL SITUATIONS

EXERCISES

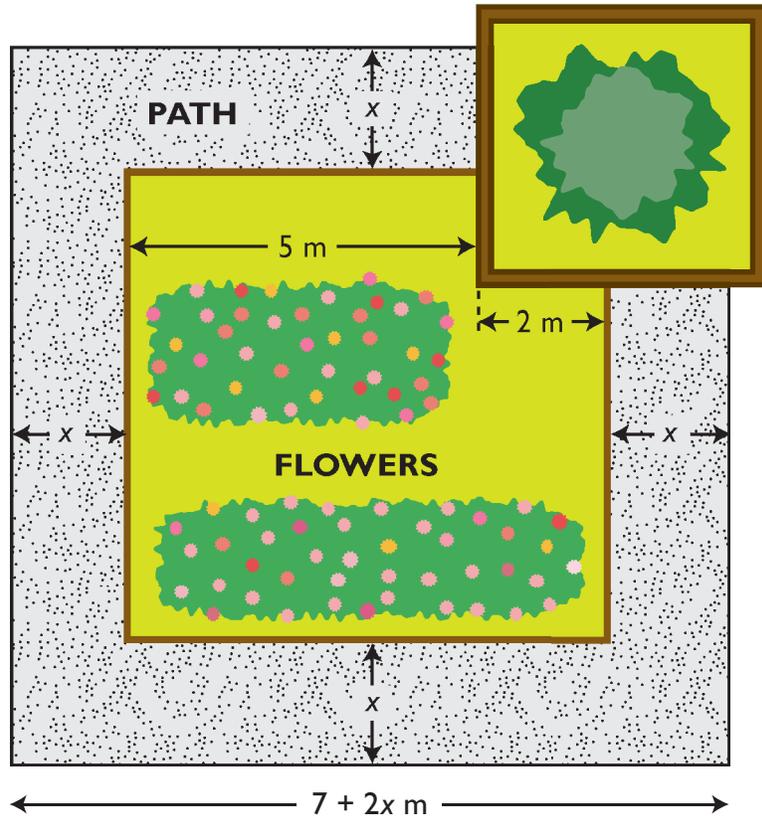
EXERCISES

- An amusement park is offering season passes for \$200. Sodas only cost \$1 if you own a season pass. Regular daily tickets are \$20 and sodas are \$5. Which deal is cheaper if you know you will go to the park 9 times in a season and buy 1 soda on each visit?
 - Season pass
 - Regular tickets
 - They would be the same price.
 - All of the above
- A magician charges \$50 to show up at a party and \$10 an hour after that. Which function represents the magician's total cost, y , where x is number of hours that the magician is at the party?
 - $y = 50x - 10$
 - $y = 50x + 10$
 - $y = 50 - 10x$
 - $y = 10x + 50$
- Consider a bicyclist who travels at a constant speed of 10 mi/hr. Distance d is a function f of time t .
 - Write a function $f(t)$ for this situation.
 - What is the domain of the function?
 - How far can the bicyclist travel in 3 hr—that is, what is $f(3)$?
 - Choose two more input values and determine the outputs from the function.
 - Sketch a graph of $d = f(t)$ using the three points you determined in parts c and d.
- Using the same situation as in problem 3 you can also model time t as a function of distance d .
 - Write a function g in the form $g(d) = ad + b$ for this situation.
 - What is the domain of the function?
 - How long did it take the bicyclist to travel 30 mi—that is, what is $g(30)$?
 - Choose two more input values and determine the outputs from the function.
 - Sketch a graph of $t = g(d)$ using the three points you determined in parts c and d.

LESSON 9: MODEL SITUATIONS

EXERCISES

5. This is a geometrical garden design that was very popular in the 18th century.



To order gravel the owner needs to know the total area of the path as shown in this sketch. The green flower area is a square. The width of the path is unknown, x . Luckily the owner is good at problem solving. She says the total area is:

$$A(x) = 3x^2 + 24x$$

- Check whether this equation is correct.
- Compute the total area of the path if its width is 1 m.
- Compute the total area of the path if its width is 1.5 m.

LESSON 9: MODEL SITUATIONS

EXERCISES

Challenge Problem

6. Plumbers have different ways of charging for their services. Many of them charge a fee for making a service call and then they charge by the hour. Plumber Jack charges \$50 for the service call and then charges \$50 per hour. His colleague Marta has an hourly rate of \$40 but charges \$65 for the service call.
- Find the essential mathematics: one rule for each plumber.
 - Express each rule as an equation.
 - Represent both plumbers' prices as graphs on the same coordinate system.
 - Which plumber is the best choice in terms of cost?



LESSON 10: PROJECT WORK DAY I

EXERCISES

- Continue to work on your project.
- Check that you have addressed all the rubric criteria for your project.
- Complete any exercises that you have not finished from this unit.

LESSON II: MORE MODELING

EXERCISES

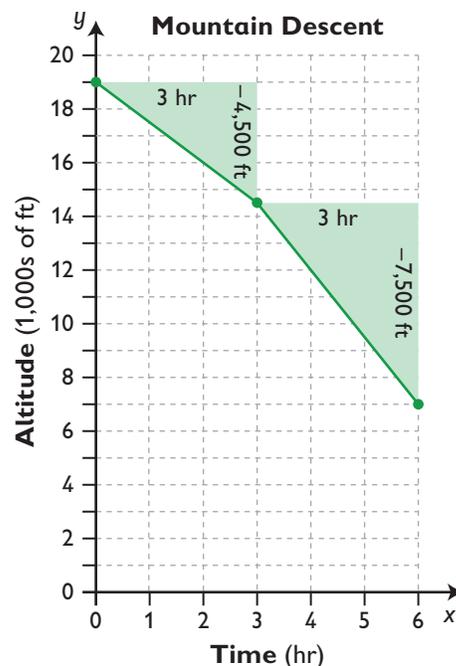
EXERCISES

1. A mountain climber is descending a 19,000 ft mountain. Her descent during the first 3 hr was at an average speed of 1,500 ft/hr. Her descent from there to the base camp at 7,000 ft was at an average speed of 2,500 ft/hr. This graph represents the situation.

- Write a function f that models distance y as a function of time x for the first 3 hr.
- What is the y -intercept of the graph of $y = f(x)$?
- What is the domain and range of f ?
- Let the function g define the relationship during the period the mountain climber was descending at a rate of 2,500 ft/hr.

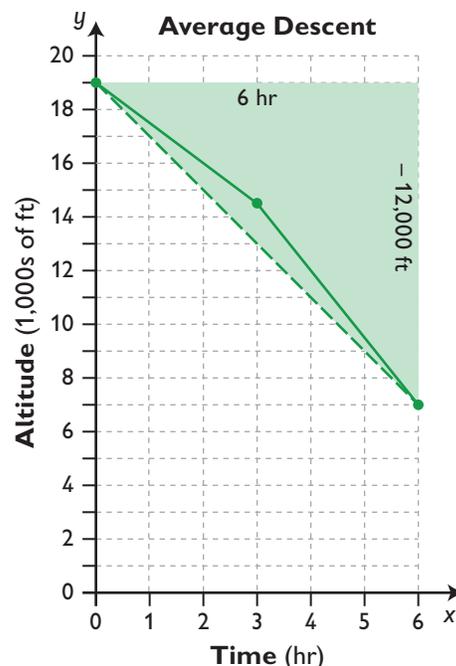
$$g(x) = 14,500 - 2,500x$$

What altitude above sea level will she reach after her second 3 hr descent—that is, what is $g(3)$?
- What is the domain and range of g ?
- How long did it take her in total to descend from 19,000 ft to the base camp?



The dotted line on the second graph has a slope equal to the average speed of descent for the climber from 19,000 ft down to the base camp at 7,000 ft.

- What was the climber's average speed for the entire descent to the base camp?
- Write a new function h in the form $h(x) = ax + b$ to model this situation using her average speed of descent.
- To observers watching from the valley floor, at what altitude above sea level would they expect the climber to be at after 4 hr using her average speed—that is, what is $h(4)$?



LESSON II: MORE MODELING

EXERCISES

2. In 2012 taxi fares in Miami were \$2.50 for the first $\frac{1}{6}$ mi and then \$0.40 for each additional $\frac{1}{6}$ mi.
In the same year taxi fares in New York were \$2.50 for the first 0.2 mi and then \$2.00 for each additional 0.2 mi.
- Find mathematical models for each fare.
 - Draw graphs for both fare models.
 - Determine which fare is cheaper for which distances.
3. A smart phone app programmer is designing the vector graphics for a laser effect in his new mobile game. The lasers fly across the screen according to the following two functions. (The restricted domain indicates the left and right edges of the phone screen.)
- $$f(x) = 4x + 15 \quad \text{Domain } [-5, 5]$$
- $$g(x) = 7x - 3 \quad \text{Domain } [-5, 5]$$
- Draw both graphs in one coordinate system.
 - Does the point of intersection belong to the domain?
 - What is the range of each function?

Challenge Problem

4. Two submarine captains are reviewing their previous dive. They were rising from deep below sea level. Something critical happened at time = 0 so they are analyzing the situation with negative time, which indicates the time before the critical incident.

The two submarines' depths can be modeled with the following functions.
(Note that negative numbers indicate meters below sea level.)

$$f(x) = 3.8x + 15.4 \quad \text{and} \quad g(x) = 0.7x - 3.8$$

Graph the two functions and find the point of intersection.

LESSON 12: PROJECT WORK DAY 2

EXERCISES

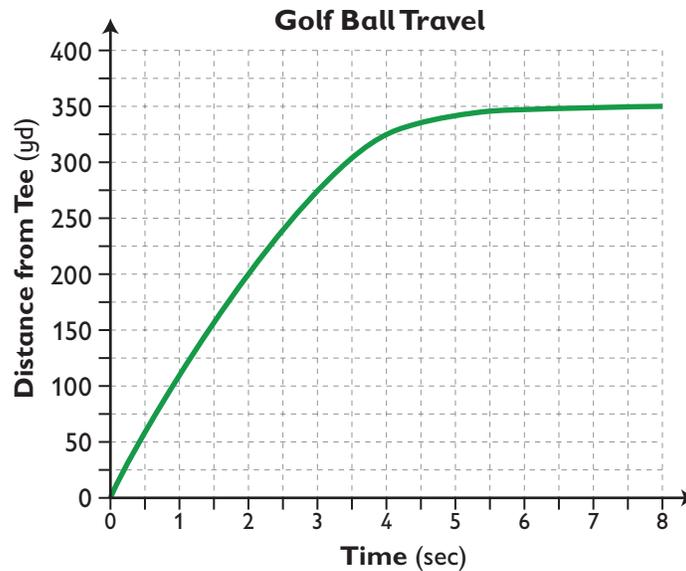
- Continue to work on your project.
- Check that you have addressed all the rubric criteria for your project.
- Complete any exercises that you have not finished from this unit.

LESSON 13: USING FUNCTIONS TO PREDICT

EXERCISES

EXERCISES

1. A golfer hits a golf ball from the tee. The ball's distance is shown in the following graph. When did the golf ball reach a distance of 200 yd from the tee?

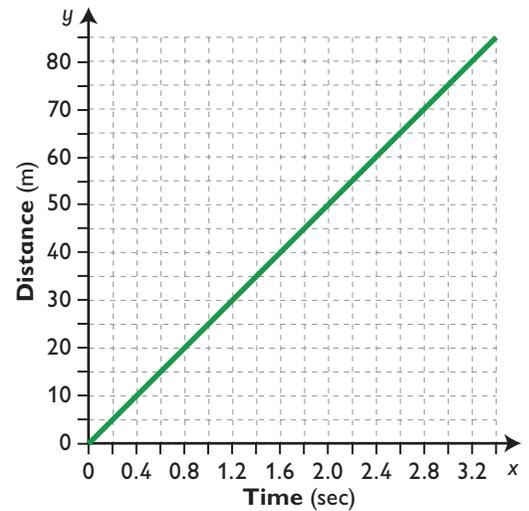
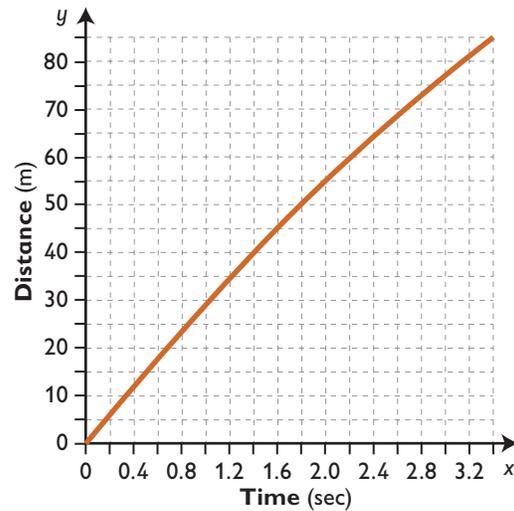


- A 8 sec
 B 1 sec
 C 5 sec
 D 2 sec
2. How many coordinate pairs do you need to determine whether a function is linear?
- A 1 pair
 B 2 pairs
 C 3 pairs
 D You may need more than three coordinate pairs.

LESSON 13: USING FUNCTIONS TO PREDICT

EXERCISES

For problems 3–5 look at these two graphs, which indicate the time it takes for a driver to completely stop the car after hitting the brake pedal.



3. Do you think it is correct to say that the two relationships are about the same on this domain? What seems to be the difference?
4. For the graph on the left, how many meters does the car go after 2 sec of braking?
5. These equations represent the same functions as shown on the graphs. Compare these functions over the domain $[0, 10]$.

$$f(x) = 25x$$

$$g(x) = -1.5x^2 + 30x$$

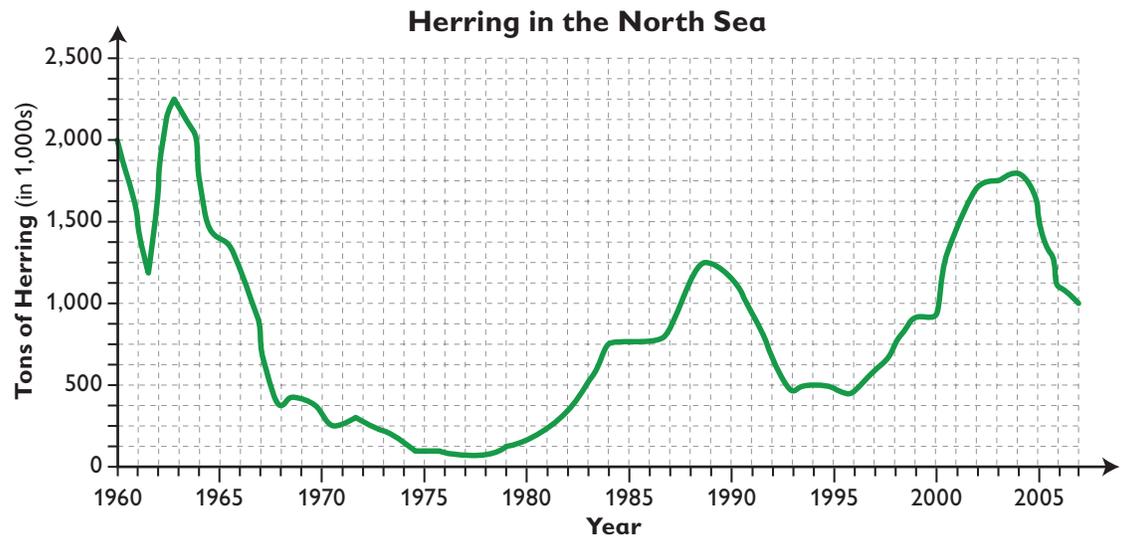
- a. Describe the differences.
- b. Which graph tells you when the car has actually stopped?
- c. What is the range of $f(x)$ and $g(x)$ on the domain?

LESSON 13: USING FUNCTIONS TO PREDICT

EXERCISES

Challenge Problem

6. Herring is a popular fish to eat raw in many countries. It is so popular that people are concerned about its survival. Reports in the 1990s were especially worrisome. In 1989, there were still approximately 1.4 million tons of herring swimming in the North Sea. Then the number of herring decreased by about 0.1 million tons per year for 7 years.
- Draw the graph for the years after 1989.
 - Use this graph to predict what the numbers for 2005 might have been.
 - A graph of the real data is shown. Compare your predictions to the real data. Explain your observations.



LESSON 14: PUTTING IT TOGETHER

EXERCISES

- Read through your Self Check and think about your work in this lesson.
- Write down what you have learned during the lesson.
- What would you do differently if you were starting the Self Check task now?
- Which method would you prefer to use if you were doing the task again? Why?
- Compare the new approaches you learned about with your original method.
- Record your ideas—keep track of problem-solving strategies.
- Complete any exercises from this unit you have not finished.

