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Illustrative Mathematics

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# Unit 3 Practice Problems

Unit 3 Practice Problems

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## Lesson 1

### Problem 1

Estimate the side length of a square that has a 9 cm long diagonal.

### Solution

6.3 cm, because the perimeter of the square is approximately  $9 \cdot 2.8$  or 25.2 cm and  $\frac{25.2}{4} = 6.3$  cm.

### Problem 2

Select **all** quantities that are proportional to the diagonal length of a square.

- A. Area of a square
- B. Perimeter of a square
- C. Side length of a square

### Solution

B, C

### Problem 3

Diego made a graph of two quantities that he measured and said, "The points all lie on a line except one, which is a little bit above the line. This means that the quantities can't be proportional." Do you agree with Diego? Explain.

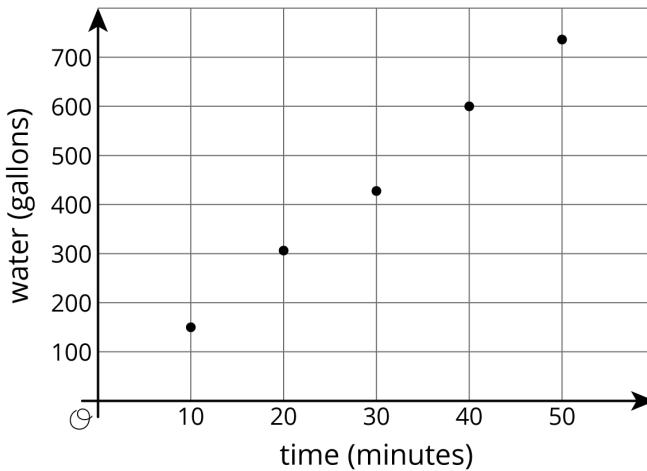
### Solution

Answers vary. Sample response: I don't agree with Diego, since the quantities could be proportional if the line goes through the origin. Measurements are not perfect and the relationship could be proportional.

### Problem 4

The graph shows that while it was being filled, the amount of water in gallons in a swimming pool was approximately proportional to the time that has passed in minutes.

1. About how much water was in the pool after 25 minutes?
2. Approximately when were there 500 gallons of water in the pool?
3. Estimate the constant of proportionality for the number of gallons of water per minute going into the pool.



### Solution

1. About 380 gallons
2. After about 35 minutes
3. About 15

## Lesson 2

### Problem 1

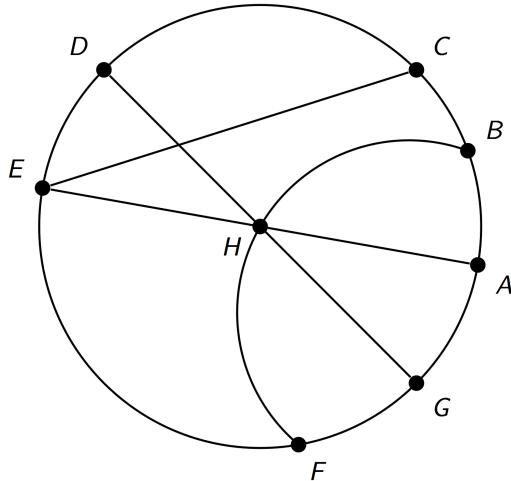
Use a geometric tool to draw a circle. Draw and measure a radius and a diameter of the circle.

### Solution

Answers vary.

## Problem 2

Here is a circle with center  $H$  and some line segments and curves joining points on the circle.



Identify examples of the following. Explain your reasoning.

1. Diameter

2. Radius

## Solution

1. Segments  $AE$  and  $DG$ . They are line segments that go through the center of the circle with endpoints on the circle.
2. Segments  $AH$ ,  $DH$ ,  $EH$ , and  $GH$  are radii. They are line segments that go from the center to the circle.

## Problem 3

Lin measured the diameter of a circle in two different directions. Measuring vertically, she got 3.5 cm, and measuring horizontally, she got 3.6 cm. Explain some possible reasons why these measurements differ.

## Solution

Two diameters of a circle should have the same length. Explanations vary. Possible explanations:

- These measurements could be rounded, not exact.
- The thickness of the circle could have affected the measurements.
- Lin did not measure across the widest part when measuring vertically.
- The shape is not quite a circle, because a perfect circle is very hard to draw.

## Problem 4

(from Unit 2, Lesson 1)

A small, test batch of lemonade used  $\frac{1}{4}$  cup of sugar added to 1 cup of water and  $\frac{1}{4}$  cup of lemon juice. After confirming it tasted good, a larger batch is going to be made with the same ratios using 10 cups of water. How much sugar should be added so that the large batch tastes the same as the test batch?

### Solution

2.5 cups since the larger batch is 10 times larger (for the water  $10 \div 1 = 10$ ) and  $10 \cdot \frac{1}{4} = 2.5$ .

### Problem 5

(from Unit 2, Lesson 13)

The graph of a proportional relationship contains the point with coordinates  $(3, 12)$ . What is the constant of proportionality of the relationship?

### Solution

4

## Lesson 3

### Problem 1

Diego measured the diameter and circumference of several circular objects and recorded his measurements in the table.

object	diameter (cm)	circumference (cm)
half dollar coin	3	10
flying disc	23	28
jar lid	8	25
flower pot	15	48

One of his measurements is inaccurate. Which measurement is it? Explain how you know.

### Solution

The measurement for the flying disc is very inaccurate. It should be about 3 times the diameter (or a little more).

### Problem 2

Complete the table. Use one of the approximate values for  $\pi$  discussed in class (for example 3.14,  $\frac{22}{7}$ , 3.1416). Explain or show your reasoning.

object	diameter	circumference
hula hoop	35 in	
circular pond		556 ft
magnifying glass	5.2 cm	
car tire		71.6 in

## Solution

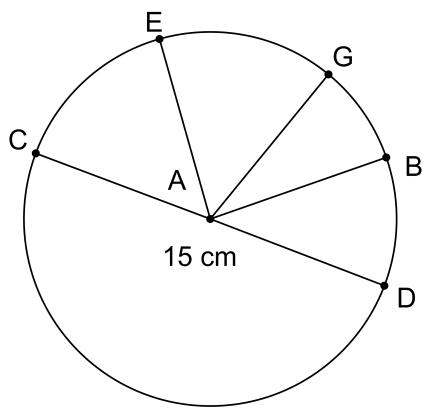
object	diameter	circumference
hula hoop	35 in	110 in
circular pond	177 ft	556 ft
magnifying glass	5.2 cm	16 cm
car tire	22.8 in	71.6 in

The constant of proportionality is about 3.14. The given diameters are multiplied by 3.14 to find the missing circumferences. The given circumferences are divided by 3.14 to find the missing diameters. Both the missing circumferences and the missing diameters have been rounded.

## Problem 3

(from Unit 3, Lesson 2)

1. Name a segment that is a radius. How long is it?
2. Name a segment that is a diameter. How long is it?



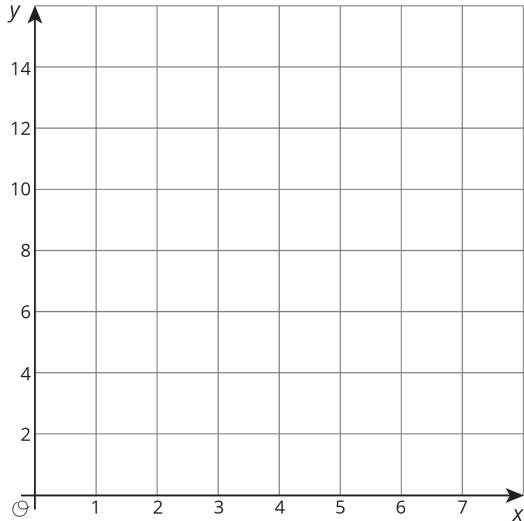
## Solution

1. Answers vary. Sample responses:  $AC$ ,  $AD$ ,  $AB$ ,  $AE$ ,  $AG$ , 7.5 cm
2.  $CD$ , 15 cm

## Problem 4

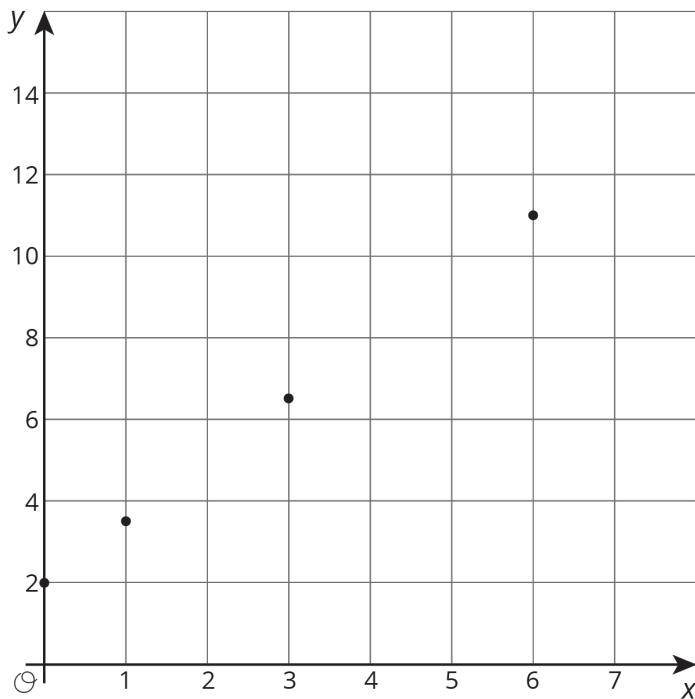
(from Unit 2, Lesson 10)

1. Consider the equation  $y = 1.5x + 2$ . Find four pairs of  $x$  and  $y$  values that make the equation true. Plot the points  $(x, y)$  on the coordinate plane.
2. Based on the graph, can this be a proportional relationship? Why or why not?



## Solution

1. Answers vary. Sample response:



2. Answers vary. Sample response: No, this relationship could not be proportional because the graph does not go through (0, 0).

## Lesson 4

### Problem 1

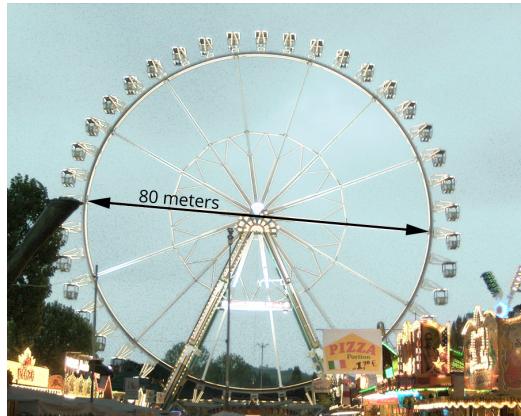
Here is a picture of a Ferris wheel. It has a diameter of 80 meters.



1. On the picture, draw and label a diameter.
2. How far does a rider travel in one complete rotation around the Ferris wheel?

### Solution

1. Answers vary. Possible response:



2. In one complete rotation, a rider travels the circumference of the Ferris wheel. This distance is  $80 \cdot \pi$ , or about 251 meters. Since the gondola where the rider is seated is a little bit further from the center of the Ferris wheel than 40 meters, the distance the rider travels is actually a little more.

### Problem 2

Identify each measurement as the diameter, radius, or circumference of the circular object. Then, estimate the other two measurements for the circle.

1. The length of the minute hand on a clock is 5 in.
2. The distance across a sink drain is 3.8 cm.
3. The tires on a mining truck are 14 ft tall.
4. The fence around a circular pool is 75 ft long.
5. The distance from the tip of a slice of pizza to the crust is 7 in.
6. Breaking a cookie in half creates a straight side 10 cm long.
7. The length of the metal rim around a glass lens is 190 mm.

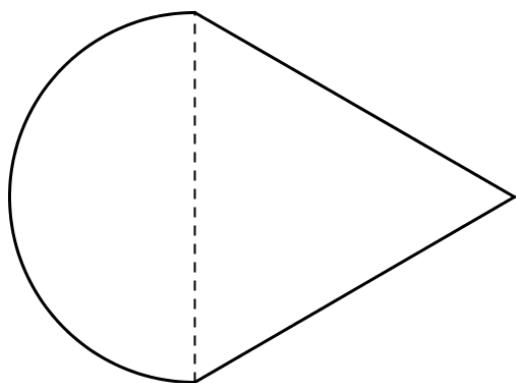
8. From the center to the edge of a DVD measures 60 mm.

### Solution

1. Radius; diameter: 10 in, circumference: about 31 in
2. Diameter; radius: 1.9 cm, circumference: about 12 cm
3. Diameter; radius: 7 ft, circumference: about 44 ft
4. Circumference; diameter: about 24 ft, radius: about 12 ft
5. Radius; diameter: 14 in, circumference: about 44 in
6. Diameter; radius: 5 cm, circumference: about 31 cm
7. Circumference; diameter: about 60 mm, radius: about 30 mm
8. Radius; diameter: 120 mm, circumference: about 380 mm

### Problem 3

A half circle is joined to an equilateral triangle with side lengths of 12 units. What is the perimeter of the resulting shape?



### Solution

about 42.84 units. The two sides of the triangle each contribute 12 units and the semi-circle has a perimeter of  $6 \cdot \pi$  or about 18.84 units.

### Problem 4

Circle A has a diameter of 1 foot. Circle B has a circumference of 1 meter. Which circle is bigger? Explain your reasoning. (1 inch = 2.54 centimeters)

### Solution

Circle B is bigger. Answers vary. Possible explanation: There are 12 inches in 1 foot. The circumference of Circle A is about 95.8 cm because  $1 \cdot 12 \cdot 2.54 \cdot \pi \approx 95.8$ . The circumference of Circle B is 100 cm because there are 100 cm in 1 m.

### Problem 5

(from Unit 3, Lesson 3)

The circumference of Tyler's bike tire is 72 inches. What is the diameter of the tire?

## **Solution**

$72 \div \pi$  or about 23 inches.

# **Lesson 5**

## **Problem 1**

The diameter of a bike wheel is 27 inches. If the wheel makes 15 complete rotations, how far does the bike travel?

## **Solution**

$405 \cdot \pi$  or about 1,272 inches (106 feet)

## **Problem 2**

The wheels on Kiran's bike are 64 inches in circumference. How many times do the wheels rotate if Kiran rides 300 yards?

## **Solution**

About 169 times. There are 36 inches in a yard so 10,800 inches in 300 yards and  $10,800 \div 64 \approx 169$ .

## **Problem 3**

(from Unit 3, Lesson 4)

The numbers are measurements of radius, diameter, and circumference of circles A and B. Circle A is smaller than circle B. Which number belongs to which quantity? 2.5, 5, 7.6, 15.2, 15.7, 47.7

## **Solution**

Circle A: radius 2.5, diameter 5, circumference 15.7 Circle B: radius 7.6, diameter 15.2, circumference 47.7

## **Problem 4**

(from Unit 3, Lesson 3)

Circle A has circumference  $2\frac{2}{3}$  m. Circle B has a diameter that is  $1\frac{1}{2}$  times as long as Circle A's diameter. What is the circumference of Circle B?

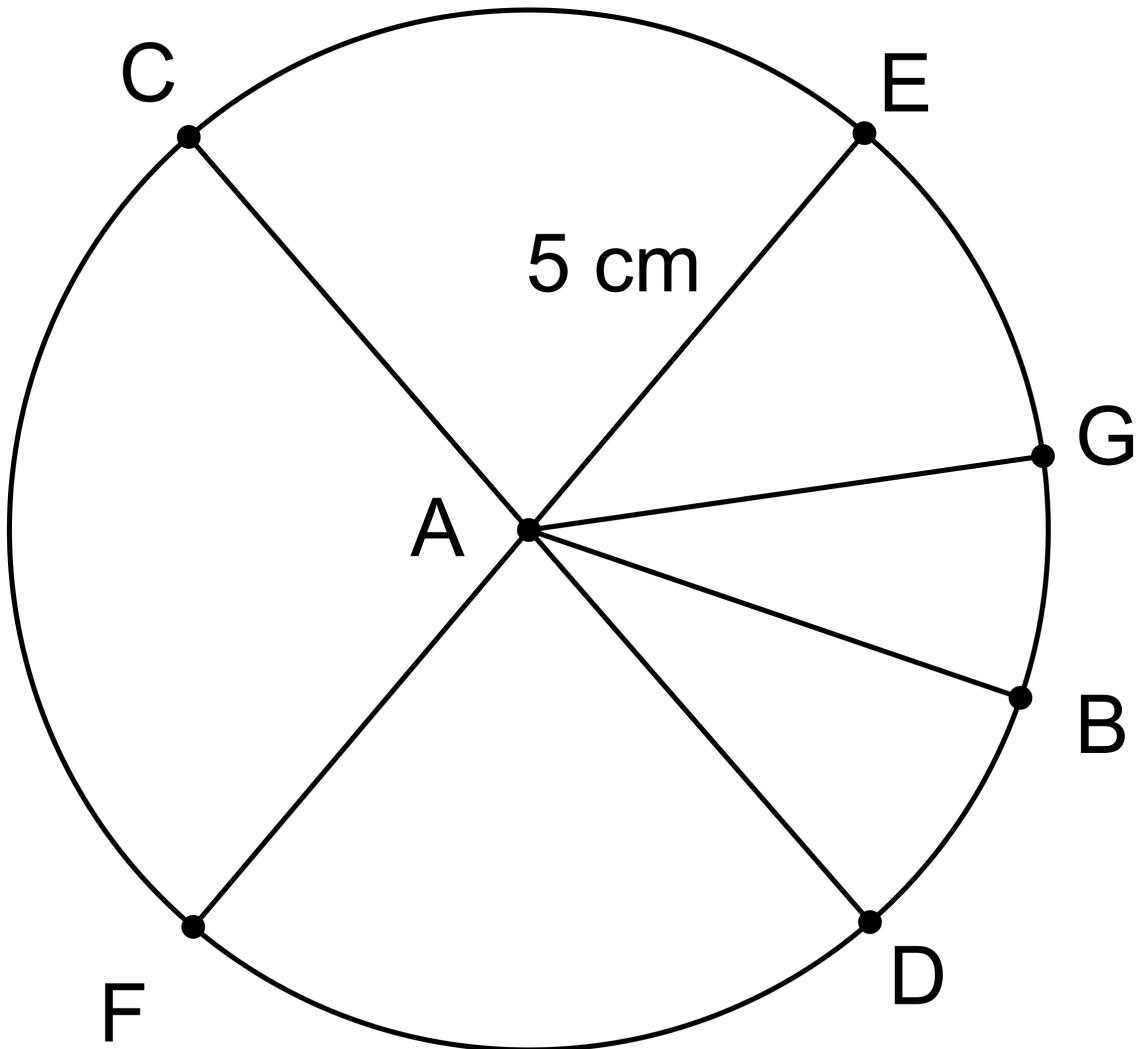
## **Solution**

4 m. If the diameter of Circle B is  $1\frac{1}{2}$  times larger than Circle A, its circumference must be as well. We can rewrite to calculate:  $\left(\frac{8}{3}\right) \left(\frac{3}{2}\right) = 4$ .

## **Problem 5**

(from Unit 3, Lesson 2)

The length of segment  $AE$  is 5 centimeters.



1. What is the length of segment  $CD$ ?
2. What is the length of segment  $AB$ ?
3. Name a segment that has the same length as segment  $AB$ .

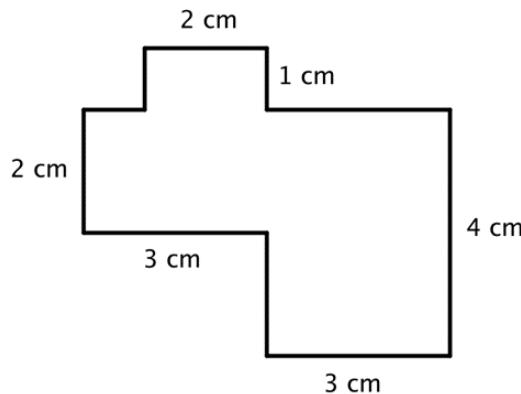
### Solution

1. 10 cm
2. 5 cm
3. Answers vary. Sample responses:  $CA, AF, AD, AG, AE$

## Lesson 6

### Problem 1

Find the area of the polygon.



## Solution

$20 \text{ cm}^2$  since the shape can be divided (vertically) into rectangles of area 2, 6, and 12 square centimeters.

## Problem 2

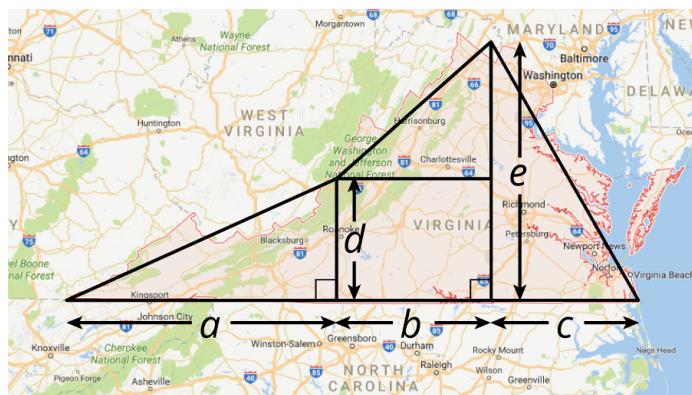
1. Draw polygons on the map that could be used to approximate the area of Virginia.



2. Which measurements would you need to know in order to calculate an approximation of the area of Virginia? Label the sides of the polygons whose measurements you would need. (Note: You aren't being asked to calculate anything.)

## Solution

1. Answers vary. There are many possible ways to draw polygons that would approximate the area of Virginia. One sample response is shown below. Other choices could be made to yield a more or less precise approximation.



2. Answers vary. For rectangles, parallelograms, and triangles, you need both base and height. In the example above, the variables represent measurements needed to find the area of the polygons.

### **Problem 3**

(from Unit 3, Lesson 5)

Jada's bike wheels have a diameter of 20 inches. How far does she travel if the wheels rotate 37 times?

### **Solution**

$37 \cdot 20 \cdot \pi$  or about 2,325 in.

### **Problem 4**

(from Unit 3, Lesson 4)

The radius of the earth is approximately 6400 km. The equator is the circle around the earth dividing it into the northern and southern hemisphere. (The center of the earth is also the center of the equator.) What is the length of the equator?

### **Solution**

$6400 \cdot 2 \cdot \pi$  is about 40,000 km

### **Problem 5**

(from Unit 2, Lesson 1)

Here are several recipes for sparkling lemonade. For each recipe describe how many tablespoons of lemonade mix it takes per cup of sparkling water.

Recipe 1: 4 tablespoons lemonade mix and 12 cups of sparkling water

Recipe 2: 4 tablespoons of lemonade mix and 6 cups of sparkling water

Recipe 3: 3 tablespoons of lemonade mix and 5 cups of sparkling water

Recipe 4:  $\frac{1}{2}$  tablespoon os lemonade mix and  $\frac{3}{4}$  cups of sparkling water

### **Solution**

Recipe 1:  $\frac{4}{12}$  or  $\frac{1}{3}$  tablespoons lemonade mix per cup of sparkling water

Recipe 2:  $\frac{4}{6}$  or  $\frac{2}{3}$  tablespoons lemonade mix per cup of sparkling water

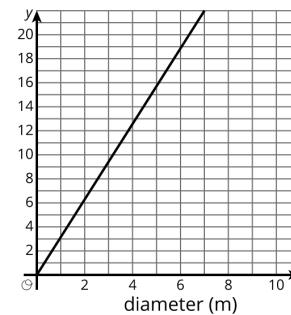
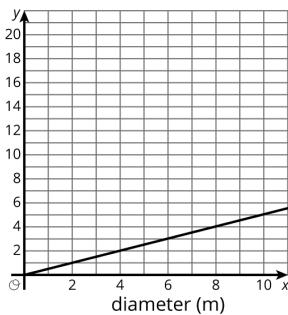
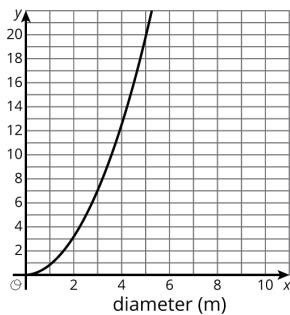
Recipe 3:  $\frac{3}{5}$  or 0.6 tablespoons of lemonade mix per cup of sparkling water

Recipe 4:  $\frac{1}{2} \div \frac{3}{4}$  or  $\frac{2}{3}$  tablespoon of lemonade mix per cup of sparkling water

## **Lesson 7**

### **Problem 1**

The  $x$ -axis of each graph has the diameter of a circle in meters. Label the  $y$ -axis on each graph with the appropriate measurement of a circle: radius (m), circumference (m), or area ( $m^2$ ). Explain how you know.

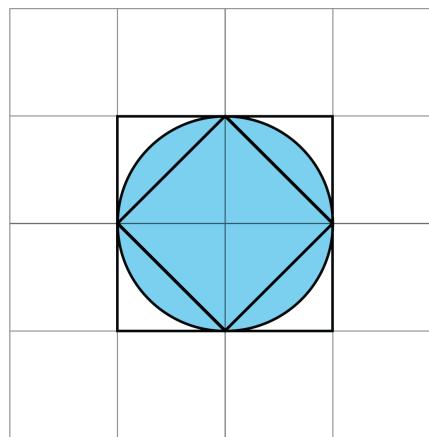


## Solution

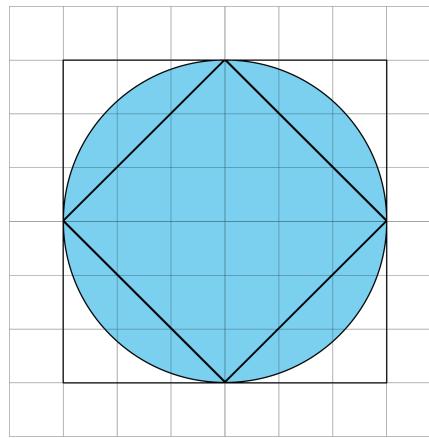
The first graph shows the relationship between the diameter and area of a circle, because it is not a proportional relationship. The second graph shows the relationship between the diameter and the radius, because it is proportional and the constant of proportionality is  $\frac{1}{2}$ . The third graph shows the relationship between the diameter and the circumference, because it is proportional and the constant of proportionality is  $\pi$ .

## Problem 2

1. Here is a picture of two squares and a circle. Use the picture to explain why the area of this circle is more than 2 square units but less than 4 square units.



2. Here is another picture of two squares and a circle. Use the picture to explain why the area of this circle is more than 18 square units and less than 36 square units.



## Solution

1. The square inside the circle has an area of 2 square units because it is made of 4 triangles each with area  $\frac{1}{2}$  square unit, and  $\frac{4}{2} = 2$ . The square outside the circle has

an area of 4 square units, because  $2^2 = 4$ .

2. The square inside the circle has an area of 18 square units because  $12 + \frac{12}{2} = 18$  (the square inside the circle contains 12 full grid squares and 12 half grid squares). The square outside the circle has an area of 36 square units because  $6^2 = 36$ .

### Problem 3

Circle A has area  $500 \text{ in}^2$ . The diameter of circle B is three times the diameter of circle A. Estimate the area of circle B.

### Solution

About  $4,500 \text{ in}^2$ . If the diameter is 3 times greater, the area must be  $3^2$ , or 9 times greater.

### Problem 4

(from Unit 3, Lesson 5)

Lin's bike travels 100 meters when her wheels rotate 55 times. What is the circumference of her wheels?

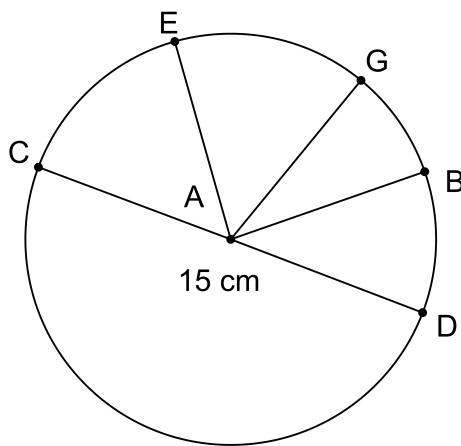
### Solution

About 1.82 meters because  $100 \div 55 \approx 1.82$

### Problem 5

(from Unit 3, Lesson 3)

Find the circumference of this circle.



### Solution

About 47 cm because  $15 \cdot \pi \approx 47$

### Problem 6

(from Unit 3, Lesson 3)

Priya drew a circle whose circumference is 25 cm. Clare drew a circle whose diameter is 3 times the diameter of Priya's circle. What is the circumference of Clare's circle?

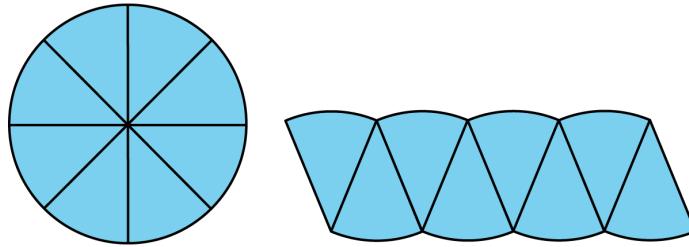
### Solution

75 cm

## Lesson 8

## Problem 1

The picture shows a circle divided into 8 equal wedges which are rearranged.



The radius of the circle is  $r$  and its circumference is  $2\pi r$ . How does the picture help to explain why the area of the circle is  $\pi r^2$ ?

## Solution

The rearranged shape looks more and more like a rectangle as the circle is cut into more pieces. The length of the rectangle is about half of the circumference of the circle or  $\pi r$ , and its height is roughly the radius  $r$ . So the area of the rectangle (and of the circle) is  $\pi r^2$ .

## Problem 2

A circle's circumference is approximately 76 cm. Estimate the radius, diameter, and area of the circle.

## Solution

The radius is approximately 12 cm. The diameter is approximately 24 cm. The area is approximately  $460 \text{ cm}^2$ .

## Problem 3

Jada paints a circular table that has a diameter of 37 inches. What is the area of the table?

## Solution

About  $1,075 \text{ in}^2$

## Problem 4

(from Unit 3, Lesson 4)

The Carousel on the National Mall has 4 rings of horses. Kiran is riding on the inner ring, which has a radius of 9 feet. Mai is riding on the outer ring, which is 8 feet farther out from the center than the inner ring is.

1. In one rotation of the carousel, how much farther does Mai travel than Kiran?
2. One rotation of the carousel takes 12 seconds. How much faster does Mai travel than Kiran?

## Solution

1. about  $106.8 - 56.5$ , or 50.3 feet farther
2. about  $50.3 \div 12$ , or 4.2 feet per second faster

## Problem 5

(from Unit 3, Lesson 5)

Here are the diameters of four coins:

coin	penny	nickel	dime	quarter
diameter	1.9 cm	2.1 cm	1.8 cm	2.4 cm

1. A coin rolls a distance of 33 cm in 5 rotations. Which coin is it?
2. A quarter makes 8 rotations. How far did it roll?
3. A dime rolls 41.8 cm. How many rotations did it make?

### Solution

1. Nickel because  $33 \div 5 \div \pi \approx 2.1$
2. About 60.3 cm because  $2.4 \cdot \pi \cdot 8 \approx 60.3$
3. About 7 because  $41.8 \div \pi \div 1.8 \approx 7$

## Lesson 9

### Problem 1

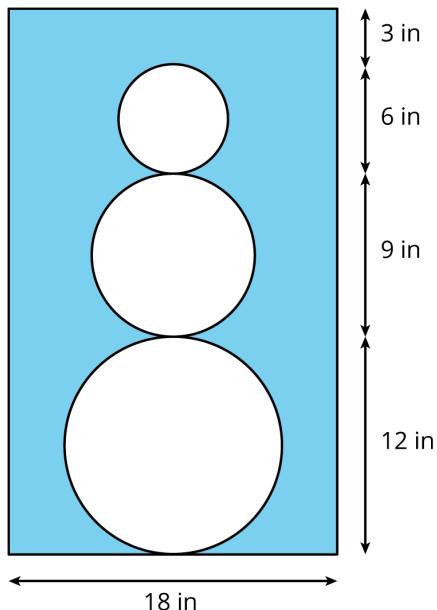
A circle with a 12 inch diameter is folded in half and then folded in half again. What is the area of the resulting shape?

### Solution

$9\pi$  in<sup>2</sup>, or about 28 in<sup>2</sup>, because  $\frac{1}{4} \cdot 6^2\pi = 9\pi$

### Problem 2

Find the area of the shaded region. Express your answer in terms of  $\pi$ .



### Solution

$540 - 65.25\pi$  in $^2$ . Find the area of the rectangle by multiplying  $(18)(30) = 540$ . Find the radii of the circles, square them, and add them together.  $62 + 4.52 + 32 = 65.25$ . Multiply 65.25 by  $\pi$  to get the total area of the circles. Subtract  $65.25\pi$  from 540 to find the area of the shaded region.

### Problem 3

(from Unit 3, Lesson 8)

The face of a clock has a circumference of 63 in. What is the area of the face of the clock?

### Solution

About 316 in $^2$ . Divide 63 by  $\pi$  and by 2 to determine the radius of the clock.  $63 \div 2 \div \pi \approx 10$ . To find the area of the face of the clock multiply  $\pi$  by  $10^2$ .

### Problem 4

(from Unit 3, Lesson 7)

Which of these pairs of quantities are proportional to each other? For the quantities that are proportional, what is the constant of proportionality?

1. Radius and diameter of a circle
2. Radius and circumference of a circle
3. Radius and area of a circle
4. Diameter and circumference of a circle
5. Diameter and area of a circle

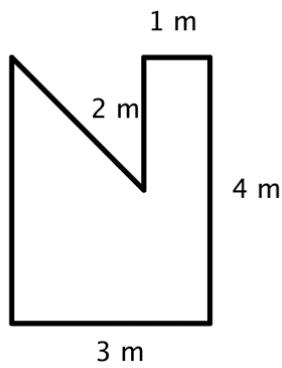
### Solution

1. Yes. The diameter is twice the radius so the constant of proportionality is either 2 or  $\frac{1}{2}$ .
2. Yes. The circumference is  $2\pi$  times the radius so the constant of proportionality is either  $2\pi$  or  $\frac{1}{2\pi}$ .
3. No
4. Yes. The circumference is  $\pi$  times the diameter so the constant of proportionality is either  $\pi$  or  $\frac{1}{\pi}$ .
5. No

### Problem 5

(from Unit 3, Lesson 6)

Find the area of this shape in two different ways.



## Solution

$10 \text{ m}^2$ . Explanations vary. Sample responses:

1. It is a rectangle of area  $12 \text{ m}^2$  with a triangle of area  $2 \text{ m}^2$  missing.
2. It is a rectangle of area  $6 \text{ m}^2$  plus a rectangle of area  $2 \text{ m}^2$  plus a triangle of area  $2 \text{ m}^2$ .

## Problem 6

(from Unit 2, Lesson 5)

Elena and Jada both read at a constant rate, but Elena reads more slowly. For every 4 pages that Elena can read, Jada can read 5.

1. Complete the table.

pages read by Elena	pages read by Jada
4	5
1	
9	
$s$	
	15
	$j$

2. Here is an equation for the table:  $j = 1.25e$ . What does the 1.25 mean?
3. Write an equation for this relationship that starts  $e = \dots$

## Solution

- 1.

pages read by Elena	pages read by Jada
4	5
1	$\frac{5}{4}$ or 1.25
9	$\frac{45}{4}$ or 11.25
$s$	$\frac{5}{4}s$ or 1.25s
12	15
$\frac{4}{5}j$ or $0.8j$	$j$

2. For every one page that Elena reads, Jada reads 1.25 pages.

3.  $e = \frac{4}{5}j$  or  $e = 0.8j$

## Lesson 10

### Problem 1

For each problem, decide whether the circumference of the circle or the area of the circle is most useful for finding a solution. Explain your reasoning.

1. A car's wheels spin at 1000 revolutions per minute. The diameter of the wheels is 23 inches. You want to know how fast the car is travelling.
2. A circular kitchen table has a diameter of 60 inches. You want to know how much fabric is needed to cover the table top.
3. A circular puzzle is 20 inches in diameter. All of the pieces are about the same size. You want to know about how many pieces there are in the puzzle.
4. You want to know about how long it takes to walk around a circular pond.

### Solution

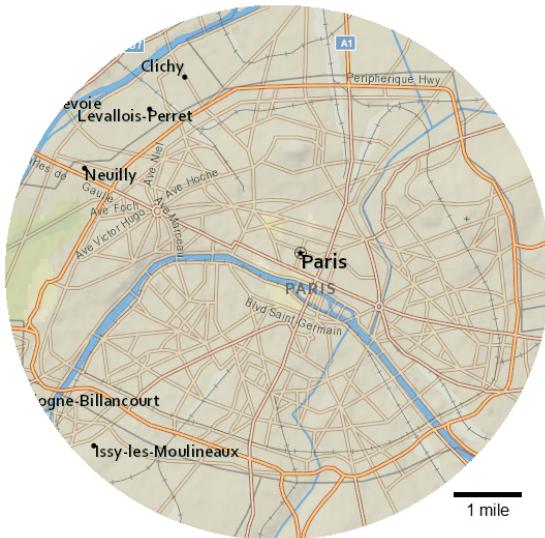
1. Circumference. The circumference of the wheels and the number of revolutions per minute tells you how far the car is traveling and this can be used to calculate the speed.
2. Area. The fabric covers the surface of the table and it is this area that is needed.
3. Area. The area of the puzzle divided by the area of a puzzle piece will give an estimate of the number of pieces.
4. Circumference. You need to know the distance around the pond which is its circumference.

### Problem 2

The city of Paris, France is completely contained within an almost circular road that goes around the edge. Use the map with its scale to:

1. Estimate the circumference of Paris.

2. Estimate the area of Paris.



## Solution

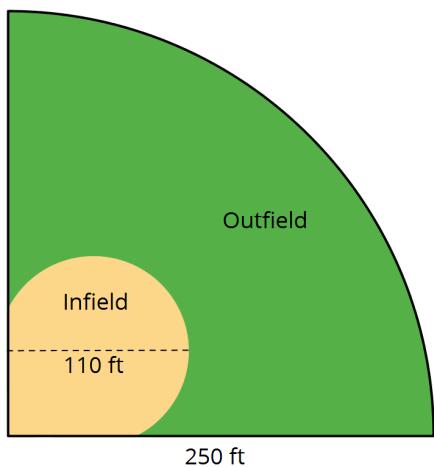
Answers vary. Sample response:

1. About  $6\pi$  miles (or about 20 miles)
2. About  $(3)^2\pi \text{ mi}^2$  (or about 30  $\text{mi}^2$ )

## Problem 3

Here is a diagram of a softball field:

1. About how long is the fence around the field?
2. About how big is the outfield?



## Solution

Answers vary. Sample responses:

1.  $500 + 125\pi$  (or about 893 ft): This estimate assumes that the curved boundary of the outfield is modeled by a quarter circle.

2.  $12,600\pi$  (or about  $39,600 \text{ ft}^2$ ): The area of the full softball field, modeled by a quarter circle, is  $\frac{1}{4} \cdot \pi \cdot 250^2$  or  $15,625\pi$  square feet. The infield, which needs to be subtracted, has about the same area as a circle of radius 55 feet or  $3,025\pi$  square feet. The difference is  $12,600\pi$  square feet. Note that if we draw a circle with diameter 110 feet (where the 110 foot measurement is marked), it misses some of the lower left part of the infield but also contains some extra area below the softball field so this is a good estimate.

## Problem 4

(from Unit 2, Lesson 5)

While in math class, Priya and Kiran come up with two ways of thinking about the proportional relationship illustrated in the table below.

$x$	$y$
2	?
5	1750

Both students agree that they need to solve this equation to get to the constant of proportionality:

$$5k = 1750$$

Read each student's explanation and answer the questions that follow.

- Priya says, "I can solve this equation by dividing 1750 by 5."
- Kiran says, "I can solve this equation by multiplying 1750 by  $\frac{1}{5}$ ."

1. What value of  $k$  would each student get if they use their own method?
2. How are Priya and Kiran's approach related?
3. Explain how each student might approach the following equation:  $\frac{2}{3}k = 50$ .

## Solution

1. 350
2. Priya used an inverse operation to solve the equation. Seeing that the operation that binds 5 and  $k$  is multiplication, she is using division to get the coefficient of  $k$  to be 1. Meanwhile, Kiran multiplied by the reciprocal of 5 to solve for  $k$ .
3. Priya divides by  $\frac{2}{3}$  since  $k$  is being multiplied by  $\frac{2}{3}$ . Her equation is  $k = 50 \div \frac{2}{3}$ . Kiran multiplies by the reciprocal of  $\frac{2}{3}$ . His equation is  $k = \frac{3}{2} \cdot 50$ .



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